

Spin dependent disorder in a junction device with spin orbit couplings

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Abstract. Using the multi-probe Landauer-Büttiker formula and Green's function approach, we calculate the longitudinal conductance (LC) and spin Hall conductance (SHC) numerically in a two-dimensional junction system with the Rashba and Dresselhaus spin orbit coupling (SOC) and spin dependent disorder (SDD) in presence of both random onsite and hopping disorder strengths. It has been found that when the strengths of the RSOC and DSOC are same, the SHC vanishes. Further in presence of random onsite or hopping disorder, the SHC is still zero when the strengths of the two types of SOC, that is Rashba and Dresselhaus are the same. This indicates that the cancellation of SHC is robust even in the presence of random disorder. Only with the inclusion of SDD (onsite or hopping), a non-zero SHC is found and it increases as the strength of SDD increases. The physical implication of the existence of a non-zero SHC has been explored in this work. Finally, we have compared the effect of onsite SDD and hopping SDD on both longitudinal and spin Hall conductances.

1. Introduction

The notion of dissipationless spin current [1] has attracted considerable interest recently. The spin-orbit coupling produces a transverse force on a moving electron and this force tends to form a transverse spin current. In the simplest form, a spin current is about the flow of spin-up electrons in one direction, say $+x$, accompanied by the flow of an equal number of spin-down electrons in the opposite direction, $-x$. The total charge current in the x direction is therefore zero, $I_e = e(I_\uparrow + I_\downarrow) = 0$. The total spin current is finite: $I_s = e(I_\uparrow - I_\downarrow) \neq 0$.

In a two-dimensional electron gas (2DEG) lacking structure inversion symmetry of the confining potential and bulk inversion symmetry, the effective Hamiltonian is given by,

$$H = \frac{\mathbf{p}^2}{2m^*} + \alpha (\sigma^x p_y - \sigma^y p_x) + \beta (\sigma^x p_x - \sigma^y p_y) \quad (1)$$

where the second term is the Rashba spin-orbit coupling (RSOC) and the third term is the Dresselhaus spin-orbit coupling (DSOC). $\sigma^{x/y}$ is the x/y component of the Pauli matrices, and α and β are the coupling parameters which denote the strengths of RSOC and DSOC respectively.

In this paper, we investigate the effect of a spin dependent random disorder on a four terminal square lattice in presence of both Rashba and Dresselhaus spin orbit interactions. Recently it has become possible to create the spin dependent disorder within a new experimental context, namely that of ultracold fermionic and bosonic atomic gases confined to optical lattices [5, 6]. Few theoretical works have been done on this context [2, 3, 4]. However, till now it is not



possible to create experimentally which facilitate studying of such phenomena in mesoscopic physics. Nevertheless, the issue deserves to be studied theoretically.

We organize our paper as follows. The theoretical formalism leading to the expressions for the spin Hall and longitudinal conductances using Landauer Bütiker formula are presented in section II. Section III includes an elaborate discussion on the results obtained for the spin Hall and longitudinal conductances that are helpful in studying the effect of spin dependent disorder on four terminal junction device in presence of spin orbit interactions.

2. Theoretical formulation

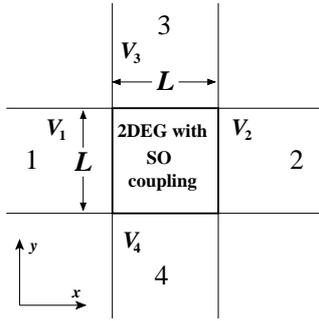


Figure 1. Cross-shaped device with four semi-infinite metallic leads. The spin-orbit coupling exists in the central region only, and the effect of the semi-infinite leads is treated exactly through the self-energy terms.

We choose a four-probe measuring set-up as shown in Fig.1 to observe the spin Hall effect. Here the four ideal semi-infinite leads are attached to the central conducting region which in our case is a square lattice with spin orbit interaction. An unpolarized charge current is allowed to pass through the longitudinal leads along lead-1 and lead-2 (see Fig.1) inducing spin Hall current in the transverse directions that is along lead-3 and lead-4. With the tight binding approximation for nearest neighbour interaction, Eq.(1) reads as,

$$H = \sum_{i,\sigma} \epsilon_{i\sigma} c_{i\sigma}^\dagger c_{i\sigma} + \sum_{\langle ij \rangle, \sigma} t_{ij,\sigma} c_{i\sigma}^\dagger c_{j\sigma} + V_R \sum_i \left[\left(c_{i\uparrow}^\dagger c_{i+\delta_x\downarrow} - c_{i\downarrow}^\dagger c_{i+\delta_x\uparrow} \right) - i \left(c_{i\uparrow}^\dagger c_{i+\delta_y\downarrow} + c_{i\downarrow}^\dagger c_{i+\delta_y\uparrow} \right) \right] + V_D \sum_i \left[(-i) \left(c_{i\uparrow}^\dagger c_{i+\delta_x\downarrow} + c_{i\downarrow}^\dagger c_{i+\delta_x\uparrow} \right) + \left(c_{i\uparrow}^\dagger c_{i+\delta_y\downarrow} - c_{i\downarrow}^\dagger c_{i+\delta_y\uparrow} \right) \right] \quad (2)$$

Here $\epsilon_{i\sigma}$ is the random on-site spin dependent potential and $t_{ij,\sigma}$ is the spin dependent hopping strength chosen from a uniform rectangular distribution ($-W$ to W), $V_R = \frac{\hbar\alpha}{a_0}$ and $V_D = \frac{\hbar\beta}{a_0}$ are the Rashba and Dresselhaus coupling strengths, respectively. $\delta_{x/y}$ is the unit vector along x/y direction.

For the four-probe case, where pure spin current is expected to flow through the transverse leads, due to the flow of charge current through the longitudinal leads, the longitudinal and spin Hall conductances are defined as [7],

$$G_L = \frac{I_2^q}{V_1 - V_2} \quad \text{and} \quad G_{SH} = \frac{\hbar}{2e} \frac{I_3^s}{V_1 - V_3} = \frac{\hbar}{2e} \frac{I_3^\uparrow - I_3^\downarrow}{V_1 - V_3} \quad (3)$$

where I_2^q and I_3^s are the charge and spin current flowing through the lead-2 and lead-3 respectively. V_m is the potential at the m -th lead. I_3^\uparrow and I_3^\downarrow are the up and down spin currents flowing in lead-3.

The calculation of electric and spin currents is based on the Landauer-Bütiker multi-probe formalism [8]. The charge and spin currents flowing through the lead m with potential V_m , can

be written in terms of spin resolved transmission probability as [9]

$$I_m^q = \frac{e^2}{h} \sum_{n \neq m, \sigma, \sigma'} \left(T_{nm}^{\sigma\sigma'} V_m - T_{mn}^{\sigma'\sigma} V_n \right) \quad (4)$$

$$I_m^s = \frac{e^2}{h} \sum_{n \neq m, \sigma'} \left[\left(T_{nm}^{\sigma'\sigma} - T_{nm}^{\sigma'-\sigma} \right) V_m + \left(T_{mn}^{-\sigma\sigma'} - T_{mn}^{\sigma\sigma'} \right) V_n \right] = \frac{e^2}{h} \sum_{n \neq m} \left[T_{nm}^{out} V_m - T_{mn}^{in} V_n \right] \quad (5)$$

where, we have defined two useful quantities as follows,

$$T_{pq}^{in} = T_{pq}^{\uparrow\uparrow} + T_{pq}^{\uparrow\downarrow} - T_{pq}^{\downarrow\uparrow} - T_{pq}^{\downarrow\downarrow} \quad \text{and} \quad T_{pq}^{out} = T_{pq}^{\uparrow\uparrow} + T_{pq}^{\downarrow\uparrow} - T_{pq}^{\uparrow\downarrow} - T_{pq}^{\downarrow\downarrow} \quad (6)$$

Physically, the term $\frac{e^2}{h} \sum_{n \neq m} T_{nm}^{out} V_m$ is the total spin current flowing from the m -th lead with potential V_m to all other n leads, while the term $\frac{e^2}{h} \sum_{n \neq m} T_{mn}^{in} V_n$ is the total spin current flowing into the m -th lead from all other n leads having potential V_n .

The zero temperature conductance, $G_{pq}^{\sigma\sigma'}$ that describes the spin resolved transport measurements, is related to the spin resolved transmission coefficient as [10, 11], $G_{pq}^{\sigma\sigma'} = \frac{e^2}{h} T_{pq}^{\sigma\sigma'}(E)$. The transmission coefficient can be calculated as [12, 13], $T_{pq}^{\sigma\sigma'} = \text{Tr} \left[\Gamma_p^\sigma G_R \Gamma_q^{\sigma'} G_A \right]$. Γ_p^σ 's are the coupling matrices representing the coupling between the central region and the leads, and they are defined by the relation [14], $\Gamma_p^\sigma = i \left[\Sigma_p^\sigma - (\Sigma_p^\sigma)^\dagger \right]$. Here Σ_p^σ is the retarded self-energy associated with the lead p . The self-energy contribution is computed by modeling each terminal as a semi-infinite perfect wire [15].

The retarded Green's function, G_R is computed as $G_R = \left(E - H - \sum_{p=1}^4 \Sigma_p \right)^{-1}$, where E is the electron Fermi energy and H is the model Hamiltonian for the central conducting region. The advanced Green's function is, $G_A = G_R^\dagger$.

Now, following the spin Hall phenomenology, in our set-up since the transverse leads are voltage probes, $I_3^q = I_4^q = 0$. Also, as the currents in various leads depend only on voltage differences among them, we can set one of the voltages to zero without any loss of generality. Here we set $V_2 = 0$. Finally, if we assume that the leads are connected to a geometrically symmetric ordered bridge, so, $\frac{V_3}{V_1} = \frac{V_4}{V_1} = \frac{1}{2}$. Now from Eq.(5), Eq.(3) and Eq.(4) we can write the expression of spin Hall and the longitudinal conductances as,

$$G_{SH} = \frac{e}{8\pi} \left(T_{13}^{out} + T_{43}^{out} + T_{23}^{out} - T_{34}^{in} - T_{31}^{in} \right) \quad \text{and} \quad G_L = \frac{e^2}{h} \left(T_{21} + \frac{1}{2} T_{32} + \frac{1}{2} T_{42} \right) \quad (7)$$

3. Results and Discussion

We have investigated the effect of spin dependent disorder (onsite and hopping) (W) in presence of spin orbit coupling (both Rashba and Dresselhaus terms) on the experimentally measurable quantities such as the longitudinal conductance (G_L) and the spin Hall conductance (G_{SH}).

Before we start computing the physical quantities, we briefly describe the values of different parameters used in our calculation. Throughout our work, we have considered lattice constant, $a = 1$, system size, $L = 20$, onsite term, $\epsilon = \epsilon_L = 0$, hopping term, $t = t_L = t_C = 1$. All the energies are measured in unit of t . Further we choose a unit where $c = h = e = 1$. The longitudinal conductance, G_L is measured in unit of $\frac{e^2}{h}$. The spin Hall conductance, G_{SH} is measured in unit of $\frac{e}{8\pi}$.

Disorder (onsite and hopping) in the system is modeled by a random rectangular distribution between $[-W, +W]$. We consider two different types of disorder:

- (i) Spin dependent disorder for the up spin (USDD) with $W_{\uparrow} = W$ and $W_{\downarrow} = 0$,
- (ii) Spin dependent disorder for the down spin (DSDD) with $W_{\uparrow} = 0$ and $W_{\downarrow} = W$.

All the results obtained below are averaged over 1000 disorder configurations. For most of our numerical calculations we have used KWANT [16].

There exists a special case at the symmetric point of $V_R = V_D$. The spin Hall conductance (SHC) vanishes at that point [17, 18]. In presence of onsite or hopping spin independent disorder (SID), SHC is still zero in the given symmetric case. However, the main finding of the paper is that when we introduce onsite or hopping spin dependent disorder, we found a non-zero SHC.

Fig.2 shows the variation of spin Hall conductance as a function of energy E , in presence of onsite and hopping SDD when Rashba and Dresselhaus spin orbit interaction have the same strengths ($V_R = V_D$). Here, $W_{O\sigma}$ stands for the onsite disorder with spin σ and $W_{H\sigma}$ is the hopping disorder with spin σ . In Fig.2(a), G_{SH} increases in the negative region with increasing the strength of USDD.

The same thing happens with DSDD, except the value of G_{SH} which acquires non zero values in the positive region as shown in Fig.2(b). Another interesting point is that the hopping SDD is much more efficient to increase SHC in magnitude than onsite SDD.

We can explain this non-zero SHC from the definition of the spin current (Eq.(3)). As we introduce SDD in the up spin electron, I_3^{\uparrow} (up spin current flowing in terminal 3) will decrease and most of I_3^{\downarrow} (down spin current flowing in terminal 3) will reach lead 3, resulting negative G_{SH} . The same argument will render positive G_{SH} in case of DSDD.

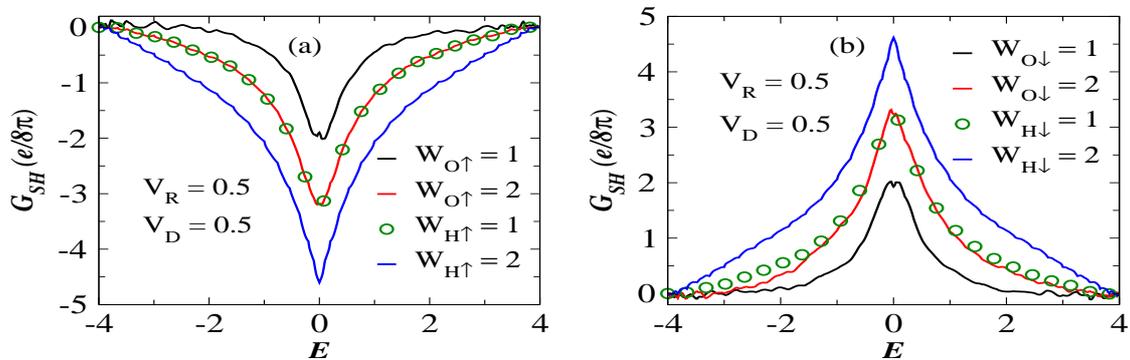


Figure 2. The spin Hall conductance, G_{SH} (in units of $e/8\pi$) is plotted as a function of energy E when both the interactions have the same strength, *i.e.* $V_R = V_D$. (a) Onsite and hopping USDD, (b) onsite and hopping DSDD.

In Fig.3, we have studied the variation of longitudinal conductance, G_L as a function of energy (E) for different spin dependent disorder strengths (W). Fig.3(a) shows G_L vs E plot for onsite and hopping disorder with different USDD strengths. For $W_{O\uparrow} = 2$ and $W_{H\uparrow} = 1$, the value of G_L remains almost same, implying that the hopping disorder is more efficient to decrease the longitudinal conductance. Fig.3(b) shows G_L vs E plot for the DSDD case. Hopping disorder is again found to be more efficient to suppress longitudinal conductance than onsite disorder. Hence we can say that the variation of G_L is independent of SDD, *i.e.*, G_L remains same for both USDD and DSDD irrespective of the onsite and hopping disorder.

We can explain why the longitudinal conductance is independent of spin dependent disorder from (Eq.(3)). G_L is just the ratio between I_3^s (the total charge current flowing in terminal 3) and V_3 (the voltage at terminal 3). Since the expressions of G_L contains the total charge

current, no matter what type of SDD is introduced, the total charge current at terminal 3, I_3^q will always be the same. As a result, G_L is independent of SDD.

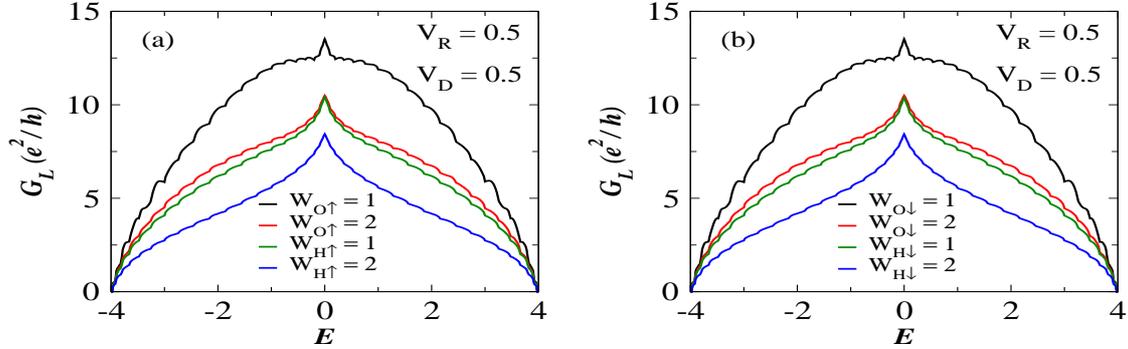


Figure 3. The longitudinal conductance, G_L (in units of e^2/h) is plotted as a function of energy E when both the interactions have the same strength, *i.e.* $V_R = V_D$. (a) Onsite and hopping USDD, (b) onsite and hopping DSDD.

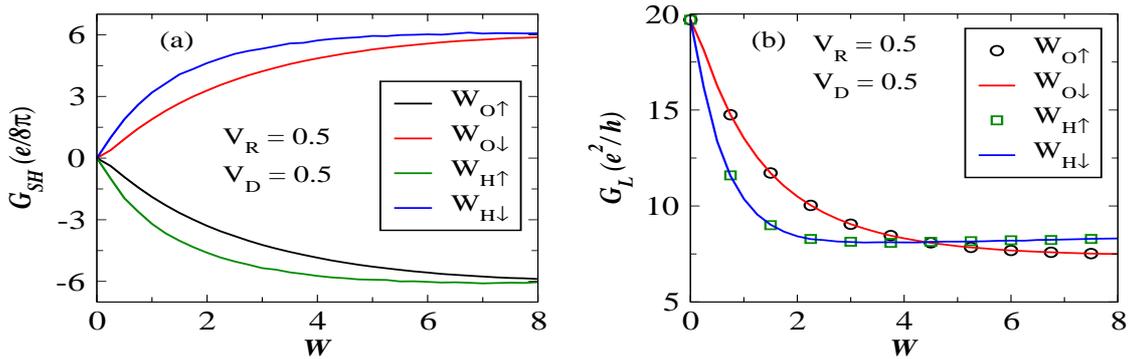


Figure 4. (a) G_{SH} is plotted as a function of disorder strength, W . (b) G_L is plotted as a function of disorder strength.

The variation of G_{SH} as a function of disorder, W at $E = 0$ is shown in Fig.4. Here we set the energy at $E = 0$. From Fig.4(a) we see that for USDD, G_{SH} starts to increase with W_{\uparrow} and tends to saturate in the negative region. For DSDD, G_{SH} shows a behaviour which looks like a mirror reflection of the corresponding feature for USDD. From Fig.4(a) it is clear that G_{SH} increases more in case of hopping disorder than that of onsite disorder. Fig.4(b) shows the variation of G_L as a function of disorder strength. For both USDD and DSDD, the variation of G_L is exactly the same as we saw in Fig.3. Here G_L decreases with increasing disorder strength and finally tends to saturate in the higher region of W . In case of hopping disorder, G_L decreases more abruptly than the onsite case. G_{SH} becomes negative for USDD and positive for DSDD. In presence of USDD, I_3^{\uparrow} will be suppressed and for larger value of W_{\uparrow} , $I_3^{\uparrow} \simeq 0$. As a result G_{SH} tends to saturate in the higher region of W_{\uparrow} .

In order to study the effect of system size on G_{SH} and G_L in presence of SDD, we fixed width of the scattering region to lattice of size $L = 20$ and set energy, $E = 0$. Fig.5(a) shows the variation of G_{SH} as a function of the system size, L . Here G_{SH} is essentially constant except when the width is lower than the length of the scattering region. G_{SH} is almost same for $W_{O\uparrow} = 2$ and $W_{H\uparrow} = 1$. Fig.5(b) shows the variation of G_L as a function of the system size, L . G_L decreases with length and then becomes almost constant.

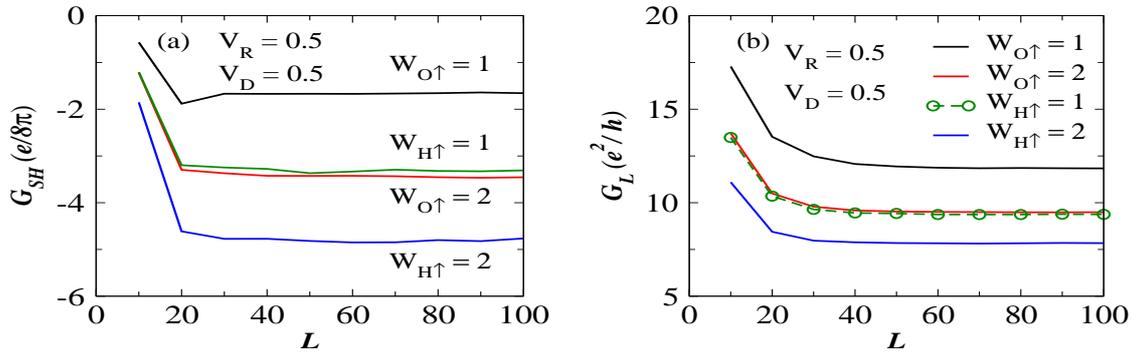


Figure 5. (a) G_{SH} is plotted as a function of system size, L . (b) G_L is plotted as a function of system size.

After a certain value of L , both G_{SH} and G_L remain constant with L . G_{SH} takes appropriate sign depending upon the nature of the SDD as we discussed earlier. Whereas, the variation of G_L with L is found to be independent of the nature of SDD.

4. Summary and Conclusions

In summary, in the present work we have studied the effect of spin dependent disorder effect on a four terminal junction device in presence of Rashba and Dresselhaus spin orbit coupling, specifically when the strengths of the two types of the spin orbit coupling are the same.

Spin dependent disorder can produce a non zero G_{SH} even if the strengths of RSOC and DSOC are the same. Higher the SDD, larger is the G_{SH} (in magnitude). G_{SH} is symmetric about $E = 0$ and takes opposite signs for different types of SDD. Hopping disorder is more efficient to increase G_{SH} than the onsite disorder. G_L is independent of the nature of SDD. With the same disorder strength, G_L decreases more in case of hopping disorder than the onsite case. With increasing the strength of SDD, both G_L and G_{SH} tend to saturate in the higher region of SDD. Further, we have found that essentially G_L and G_{SH} are independent of length (L) of the sample, at least for larger values of L .

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