Lattice field theory applications in high energy physics

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Abstract. Lattice gauge theory was formulated by Kenneth Wilson in 1974. In the ensuing decades, improvements in actions, algorithms, and computers have enabled tremendous progress in QCD, to the point where lattice calculations can yield sub-percent level precision for some quantities. Beyond QCD, lattice methods are being used to explore possible beyond the standard model (BSM) theories of dynamical symmetry breaking and supersymmetry. We survey progress in extracting information about the parameters of the standard model by confronting lattice calculations with experimental results and searching for evidence of BSM effects.

1. Introduction

In 1974, Kenneth Wilson invented lattice Quantum Chromodynamics (QCD), a non-perturbative approach to Nature's strong force [1]. Wilson's formulation was based on using elements of the Lie group $SU(3)$, rather than elements of the Lie algebra, which is used in the continuum formulation of the theory. This approach allowed Wilson to exactly preserve gauge invariance, which was not possible when formulating the theory in terms of finite difference operators applied to elements of the Lie algebra. Gauge invariance is familiar to us from electromagnetism, but in QCD it is much richer, as it is based on the non-Abelian group $SU(3)$, not the Abelian group $U(1)$. Gauge symmetry determines three of Nature's forces: electromagnetic, strong, and weak.

In the years since Wilson's initial paper, which discussed quark confinement based on a strong coupling expansion, there have been monumental advances in algorithms, formulations of the theory, and computer power. In the early days, it was necessary to neglect the contribution of quantum fluctuations related to quark-antiquark production and annihilation in the vacuum. This is called the quenched approximation and cannot be systematically improved. However, it is now possible to include the quantum fluctuations of the four lightest quarks: up, down, strange, and charm. The still heavier bottom and top quarks have negligible effect at current precision. We can even make the up and down quarks as light as in Nature, which has traditionally been a difficult computing challenge. Work on lattice QCD has progressed to the stage that a number of interesting quantities can be calculated to sub-percent level, and there have been predictions of particle masses and decay properties, not just postdictions.

Lattice QCD is now extensively used in theoretical studies of elementary particle and nuclear physics. The proton-neutron mass difference, the spectrum of excited baryons, parton moment distributions, and properties of light nuclei have all been studied with varying degrees of precision. The properties of QCD at non-zero temperature have been studied to understand the quark-gluon plasma produced in heavy-ion collisions. In particle theory, the quark masses for up, down, strange, charm, and bottom, i.e., all except top, have been calculated. So, has the strong coupling α_s and many weak decays that are needed to determine the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Lattice field theory has also been used to study theories with dynamical symmetry breaking as an alternative to the spontaneous symmetry breaking of the simple Higgs boson theory. As exciting as it was to discover the Higgs boson at the LHC, knowing the mass of the Higgs boson does not solve the mysteries of the Standard Model, and we would dearly love to find evidence of Beyond the Standard Model physics. This might come from seeing new particles at the LHC, but it could just as easily come from observing anomalies in high precision experiments, where the anomalies come from small interactions caused by new (virtual) particles that are too heavy to be produced at the LHC. Lattice QCD has an important role to play here in determining the elements of the CKM matrix. Another area of significant recent progress is in the formulation of lattice theories with supersymmetry.

Unfortunately, I cannot cover all of these topics, so I shall point the interested reader to the annual International Symposium on Lattice Field Theory. The most recent one was held in July, 2015 in Kobe, Japan [2]. There were over a dozen plenary talks relevant to nuclear and particle physics. In this talk, I have relied heavily either on results from my own collaborations, MILC and Fermilab Lattice-MILC, (referred to as FNAL/MILC below), or state-of-the-art summaries prepared by the Flavor Lattice Averaging Group (FLAG) [3] an international group of scientists who critically review work on a large number of quantities in lattice QCD and prepare averages for ease of use by a wider (non-lattice) community. The last FLAG review appeared in 2013 [3] and a new one will appear in early 2016. I am pleased to be a member of FLAG.

2. The Standard Model and lattice QCD

The Standard Model is a theory of quarks, leptons, gauge bosons, and the Higgs boson. It describes only three of the known forces, as gravity is not included. The model is described by its symmetries and the matter content. The symmetry is $SU(3) \times SU(2) \times U(1)$. The group $SU(3)$ is the symmetry of QCD and $SU(2) \times U(1)$ is that of the electroweak interactions. Spin-1 particles are the force carriers. They are called gluons (for QCD), and the photon and weak bosons (specifically, $W^{\pm}$ and Z) for the electromagnetism and the weak force, respectively. The Higgs boson has no intrinsic spin. The quarks and leptons are spin-1/2 matter particles. The quarks interact with all the force carriers. The charged and neutral leptons don't interact with the gluons, but they do interact with the weak force carriers. The charged leptons interact with the photon, but the neutral ones (neutrinos) do not. One of the reasons we think there is physics beyond the the Standard Model is that the model has many undetermined parameters. These parameters must be determined from experiment (with various inputs from theory). In a more fundamental theory, there might be relations between the parameters, so they would not seem as arbitrary as they do now.

For each of the three symmetry groups there is a coupling constant. For $SU(3)$, it is called g_s. For $SU(2) \times U(1)$, the two couplings are g and g'. There are six quark masses. There are three masses for the charged leptons. Now that we know neutrinos have mass, there are also three neutrino masses. The Cabibbo-Kobayashi-Maskawa quark mixing matrix (detailed in the next section) is complex and unitary. An arbitrary 3×3 complex matrix would have 18 real parameters; however, because of unitarity and our ability to chose some phases of the quark fields, there are only four independent parameters that determine the CKM matrix. These are commonly described as three angles and a complex phase factor that determines CP violation. The combination of discrete symmetries charge conjugation C, and parity is denoted by CP. There is a similar matrix for the neutrinos called PMNS for Pontecorvo, Maki, Nakagaw and Sakata. However, since the neutrinos do not interact strongly, we will have no more to say about that. Lattice QCD input is important for determination of ten parameters of the Standard Model: $\alpha_s = g_s^2/(4\pi)$, the four parameters that determine the CKM matrix, and m_u, m_d, m_s,

m_c, and m_b. The sixth quark, the top quark, decays weakly before it can form bound states, so we do not need lattice QCD to study its mass.

Lattice QCD provides a nonperturbative treatment of the quantum field theory that describes the strong interaction. Because the coupling is strong, many phenomena cannot be calculated perturbatively. Quantum field theories require regularization and renormalization. The lattice technique provides one such regularization. However, numerical errors must be carefully controlled. Errors come from the non-zero lattice spacing (continuum limit), finite volume (infinite volume limit), and unphysical light quark masses (chiral extrapolation). In parentheses, we have the limit or operation that must be done to control the systematic error from each effect. In addition, there are statistical errors. Groups are increasingly able to work with up and down quark masses very close to their physical value, which greatly reduces errors from the chiral extrapolation that were seen in earlier calculations. There are at least five popular ways to deal with the quarks in lattice QCD. They go by the names: Wilson/Clover, staggered, domain wall, twisted mass, and overlap. Each method has different systematic errors at nonzero lattice spacing, so it is useful to use different methods and compare the final results after all errors are controlled. The number of dynamical flavors used also varies by collaboration. The most phenomenologically relevant calculations use dynamical up, down, and strange quarks, or those plus charm. These are denoted $N_f = 2+1$ or $2+1+1$, respectively, because the up and down quarks are usually treated as if their masses were identical. (Their average mass is used.)

3. CKM matrix

It has been observed for many years that the Universe contains much more matter than antimatter. This is known as the baryon asymmetry. Kobayashi and Maskawa won the Nobel prize for their realization that with three (or more) generations we can have CP violation, which might explain the baryon asymmetry of the Universe. However, we now know that the CP violation in the strong interaction is probably too weak for this purpose, and it may be the CP violation appearing in the PMNS matrix for neutrino mixing that accounts for the baryon asymmetry. Here is the CKM mixing matrix (bold notation) augmented with some of the decay or mixing processes that can be used to determine each matrix element:

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to l\nu & K \to l\nu & B \to l\nu \\ & K \to \pi l\nu & B \to \pi l\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to l\nu & D_s \to l\nu & B_c \to l\nu \\ D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \bar{B}_d & B_s \leftrightarrow \bar{B}_s & \end{pmatrix}. \tag{1}$$

In the second and fifth rows, we have meson decays called leptonic, because only a charged lepton and a neutrino appear in the final state. The third and six rows contain decays called semi-leptonic because there is also a meson in the final state. The last row contains two meson mixing processes that determine the CKM matrix elements just above them. Since the CKM matrix is unitary, each row and each column is a complex unit vector. Also, each row (column) is orthogonal to the other rows (columns) leading to the so-called unitarity triangle in the complex plane. Violations of unitarity are evidence of non-standard-model physics. Further, if two different processes are used to determine an element of the matrix and they do not agree, that is evidence for new BSM physics, which we would dearly love to find. We will examine both these tests of the SM.

If we could do experiments on free quarks, it would be easy to determine mixing; however, confinement means we need to deal with bound states. Thus, LQCD input for decay constants and form factors is needed to determine elements of the CKM matrix. For example, the branching fraction for the leptonic decay of a $D_{(s)}$ meson is given by

$$\mathcal{B}(D_{(s)} \to \ell\nu_\ell) = \frac{G_F^2 |V_{cq}|^2 \tau_{D_{(s)}}}{8\pi} f_{D_{(s)}}^2 m_\ell^2 m_{D_{(s)}} \left(1 - \frac{m_\ell^2}{m_{D_{(s)}}^2}\right)^2 \tag{2}$$

where the unknowns are $|V_{cq}|$, the absolute value of the CKM matrix element with $q = d$ or $q = s$ for the D or D_s meson, respectively, and $f_{D_{(s)}}$ is the corresponding decay constant, which needs to be calculated in LQCD. The other quantities, such as masses, lifetimes and the Fermi constant are easily found from experiment.

3.1. Light quarks and the first row

We will begin our discussion with results for mesons that contain only the three lightest quarks up, down, and strange. The ground states are called pions and kaons. In Fig. 1(L), we see the FLAG summary of LQCD results stretching back over a decade by various groups. Results in red are deemed to have insufficient control of all systematic errors. Results with a solid green symbol are included in the FLAG estimate (black point with error bar and vertical gray bands). Points with light green have been superseded or not been refereed, hence not included in the FLAG estimate. Blue points come from the Particle Data Group (PDG) [4] or a non-LQCD method. Calculations with different numbers of dynamical quarks are considered separately. In some cases, f_π has been used to set the lattice spacing (or scale), so only f_K is shown. We see excellent agreement with the values from the PDG. The ratio f_K/f_π can be calculated accurately and used to determine $|V_{us}/V_{ud}|$ from precise measurements of the ratio of pion and kaon decay rates which show that $\left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2758(5)$. Figure 1(R) shows results for the decay constant ratio.

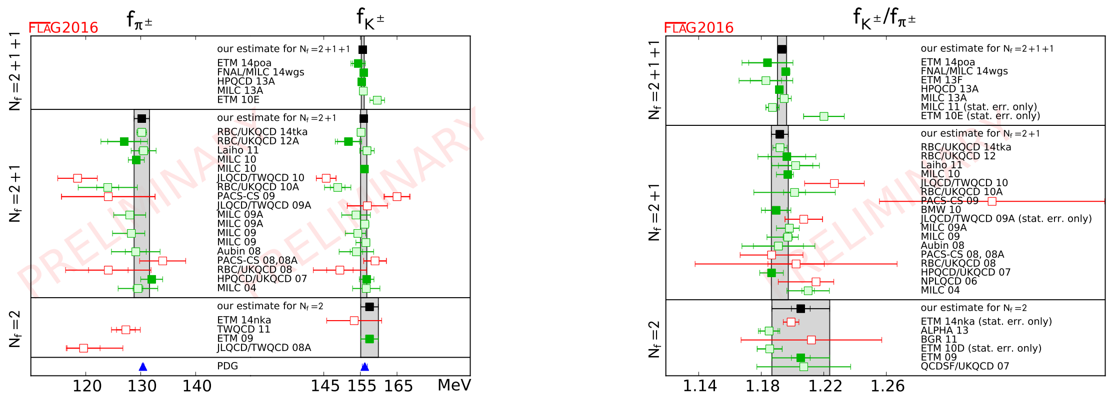

Figure 1. (L) FLAG [3] summary of LQCD results for the charged pion and kaon decay constants. (R) The ratio of charged kaon to charged pion decay constants. The meaning of the colors, symbols and bands is explained in the text.

Let's turn to the semileptonic kaon decay. Semileptonic decays have three-body final states, so there is one kinematic variable, usually denoted q^2, which is the square of the momentum transfer to the leptons. From 4-momentum conservation, $p_K = p_\pi + q_l + q_\nu$ and $q = q_l + q_\nu$, where we have used p for hadron momenta and q for lepton momenta, with the subscript denoting the particle. To extract $|V_{us}|$, we just need the form factor at zero momentum transfer, i.e., $f_+(0)$ as experiment tells us that $|V_{us}|f_+(0) = 0.2163(5)$. This can be combined with the FNAL/MILC

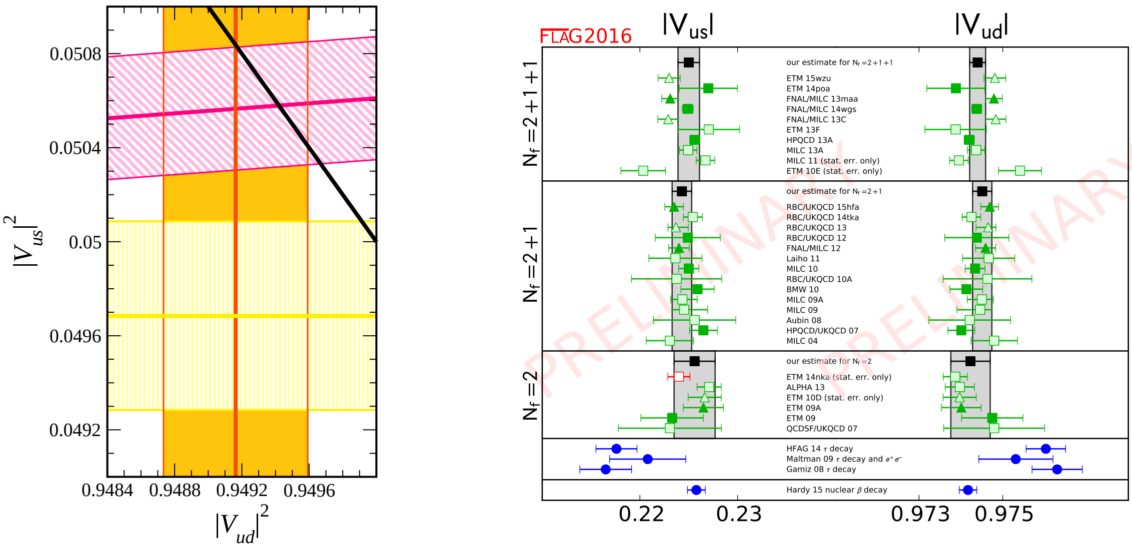

Figure 2. (L) Unitarity test for the first row of the CKM matrix from Ref. [6]. Black line shows unitarity constraint. Colored bands explained in text. (R) Subset of FLAG [3] summary for $|V_{us}|$ and $|V_{ud}|$. Most red points eliminated for readability.

$N_f = 2+1+1$ result [5] $f_+(0) = 0.9704(24)(220$ to determine an error band for $|V_{us}|$. First row unitary states $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$. However, as we will see below $|V_{ub}| \approx 4 \times 10^{-3}$, so the last term can be neglected as the errors on the first two terms are a few times 10^{-4}. Thus, the unitarity constraint will be a straight line in the $|V_{ud}|^2$ - $|V_{us}|^2$ plane. In Fig. 2(L), we show the unitary constraint as a black line, along with the angled error band from leptonic pion and kaon decay, the horizontal error band from kaon semileptonic decay, and a vertical error band from nuclear β-decay that is independent of LQCD calculations. We see that there is some tension between the two types of decay studied in LQCD, but that unitarity, leptonic decays, and β-decay are in good agreement.

A summary of many determinations of $|V_{ud}|$ and $|V_{us}|$, based on either leptonic or semileptonic decays, has been prepared by FLAG. This is shown in Fig. 2(R) where squares indicate leptonic decays and triangles indicate semileptonic. The blue points, as usual, are from non-lattice calculations. Careful inspection of the $N_f = 2+1+1$ 2014 results from MILC and FNAL/MILC shows the tension we have seen above between leptonic and semileptonic decays. Other calculations do not yet have the precision to confirm the discrepancy or rule it out. The errors on the FLAG estimates for $N_f = 2$ and $2+1$ are larger than for $2+1+1$ and they do not have a tension with unitarity. The FLAG estimate for $2+1+1$ does show some tension with unitarity. First row unitarity is not the first place we would expect to find evidence for BSM flavor physics, so it will be interesting to improve these calculations, particularly to calculate the kaon semileptonic form factor over its entire kinematic range. There is also the interesting tension between the results near the bottom of the figure, based on τ decays, and those for pion and kaon decay.

3.2. Charm decays and the second row

Leptonic and semileptonic decays of the D and D_s mesons have been studied on the lattice, but not as extensively as for the pion and kaon. It has been about a decade since decay constant predictions of FNAL/MILC were tested at CLEO-c [7]. Initial errors were about 10%, but current errors from FNAL/MILC are only 0.6%. A great deal of the improvement is due to the use of highly improved staggered quarks (HISQ) that were developed by the HPQCD collaboration. Figure 3(L) compares the 2014 FNAL/MILC results (labeled "This work") with prior calculations, mostly by the European Twisted Mass collaboration and HPQCD. The FNAL/MILC results are $f_{D^+} = 212.6(0.4)(^{+1.0}_{-1.2})$ MeV, $f_{D_s} = 249.0(0.3)(^{+1.1}_{-1.5})$ MeV, and $f_{D_s}/f_{D^+} = 1.1712(10)(^{+29}_{-32})$ for the decay constant ratio, for which there is some cancellation of the systematic errors. For references to the other work and the HISQ action see Ref. [6].

To make use of these decay constants, we rely on the work of Rosner and Stone [8] to summarize the experimental results. They find $f_D|V_{cd}| = 46.06(1.11)$MeV, and $f_{D_s}|V_{cs}| = 250.66(4.48)$MeV. The experimental errors are 1.8–2.4%.

Combining the experimental results and the FNAL/MILC decay constants gives $|V_{cd}| = 0.217(1)(5)(1)$, and $|V_{cs}| = 1.010(5)(18)(6)$, where the errors are lattice, experiment and structure-dependent electromagnetic, respectively. Thus, the experimental errors are currently dominant. In Fig. 3R, we see evidence for an $\approx 1.8\sigma$ tension with unitarity for the two leptonic charm decays. The black line is the unitarity constraint. The horizontal blue band is for D_s decay and the vertical green band is for D^+ decay. Once again, the third element of the row V_{cb} is too small to make a difference at the current level of precision. The semileptonic form factors for $D_{(s)}$ mesons are much less studied than for light quarks; however, there should be some updates in the coming year. We refer the reader to FLAG [3] for details.

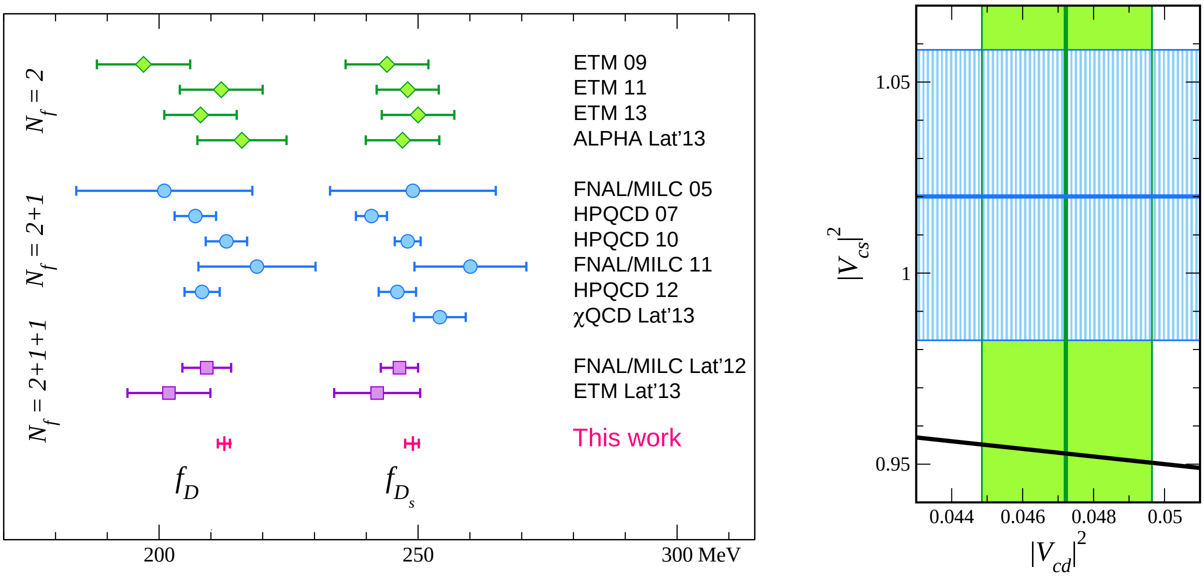

Figure 3. (L) Summary of results for the charm meson decay constant prepared by FNAL/MILC [6]. "This work" refers to the calculation presented there. (R) Unitarity test for the second row of the CKM matrix from Ref. [6].

3.3. B meson decays

The b quark is the heaviest one that forms bound states that can be studied with LQCD. Both leptonic and semileptonic decays of B and B_s mesons have been studied. In addition to the usual decays that produce a charged lepton and a neutrino, there are a number of rare decays that involve the so-called flavor changing neutral current (FCNC). In the SM, the FCNC vanishes at the tree level, so small quantum loop effects from new physics may be visible. That makes the rare decays a promising topic to study. In fact, there have been some recent results from the LHCb experiment at the Large Hadron Collider that show tensions with the SM predictions.

These rare decays also present an alternative way to determine $|V_{td}|$ and $|V_{ts}|$ that can be compared with the meson mixing processes indicated in our CKM matrix.

FLAG has summarized results for decay constants f_B and f_{B_s} [3]. The errors on these decay constants are about 2% for $N_f = 2+1$ and $2+1+1$. For $N_f = 2+1$, $f_B = 190.5(4.2)$MeV and $f_{B_s} = 227.7(4.5)$MeV. Unfortunately only $B \to \tau\nu$ has been observed so far and the error is about 20%. So, in this case the LQCD calculation is ahead of the measurement. This allows a determination of $|V_{ub}|$, but it is not competitive with the value from semileptonic decay.

The semileptonic decays $B \to \pi\ell\nu$ and $B_s \to K\ell\nu$ have been studied on the lattice. The former has been observed at BaBar and Belle, but the latter has not been observed. Another way to determine $|V_{ub}|$ is from inclusive decays. There is a long standing tension between that determination and the one from $B \to \pi\ell\nu$. The central value of $|V_{ub}|$ based on the SM analysis of the leptonic decay is between that from the semileptonic exclusive decay and the inclusive method. However, its error bar, limited by experiment, is too large to help clarify the situation. Belle II will improve the $B \to \tau\nu$ measurement, which should really help resolve these issues. Figure 4(L) shows the FLAG summary from 2013. There are new results for $B \to \pi\ell\nu$ from FNAL/MILC and RBC/UKQCD. There is also a new determination of $|V_{ub}|$ based on decay of the Λ_B baryon recently seen at LHCb. These determinations are compared in Fig. 4(R) [9]. FLAG computed values of $|V_{ub}|$ based on the BaBar and Belle experiments. They found $|V_{ub}| = 3.37(21) \times 10^{-3}$ and $3.47(22) \times 10^{-3}$, respectively, based on 2+1 flavor LQCD. The new FNAL/MILC result which uses results from both BaBar and Belle is $3.72(16) \times 10^{-3}$ which reduces, but does not eliminate the tension between exclusive and inclusive decays.

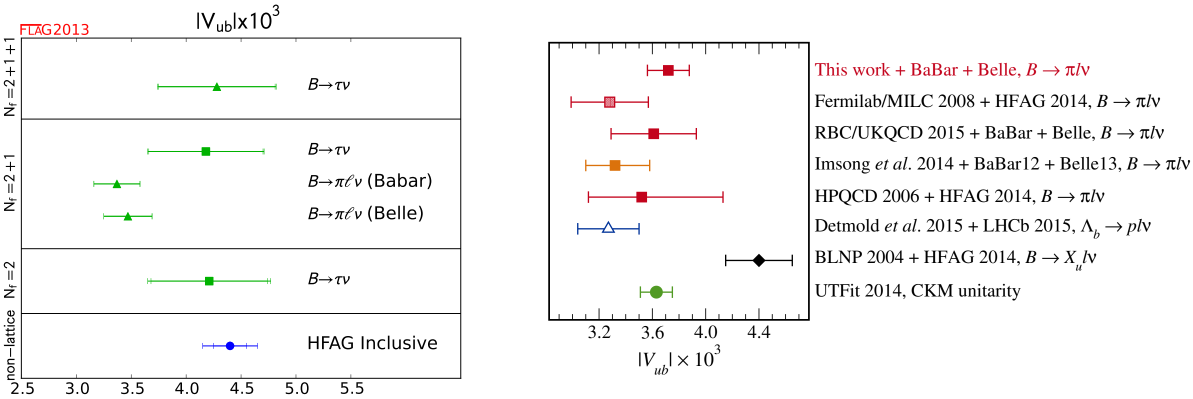

Figure 4. (L) FLAG [3] summary for $|V_{ub}|$ including results for $N_f = 2$, 2+1, and 2+1+1, as well as the inclusive result. (R) FNAL/MILC [9] summary of results for $|V_{ub}|$, including determination from Λ_b decay. See Ref. [9] for original sources.

The last entry in the second row of the CKM matrix, V_{cb}, can be studied in the exclusive decays $B \to D^*\ell\nu$ and $B \to D\ell\nu$. It can also be determined in inclusive decays where the decay products must include charm quarks. FNAL/MILC has recently compared $|V_{cb}|$ based on both determinations and there is again some tension between inclusive and exclusive results. As can be seen in Fig. 5(L), the errors from the decay to D^* are somewhat smaller than that from decay to D. With current errors, those decays agree with each other reasonably well and the real tension is between $B \to D^*\ell\nu$ and the inclusive value of $|V_{cb}|$.

Turning to rare B decays, FNAL/MILC has recently calculated the form factors needed for both SM and BSM decays through a FCNC [10, 11]. As mentioned above, this is a promising place to look for new physics. There is some tension between the SM prediction and recent LHCb measurements of $B^+ \to \pi^+\mu^+\mu^-$ and $B^+ \to K^+\mu^+\mu^-$. The LHCb measurement is smaller than the SM prediction in three of four fairly wide bins of q^2, the square of the momentum transfer to the muons. Figure 5(R) shows the comparison for $B^+ \to K^+\mu^+\mu^-$ where the difference is more pronounced. In Ref. [12], both processes are shown. For all four bins for the two processes, the tension is 1.7σ.

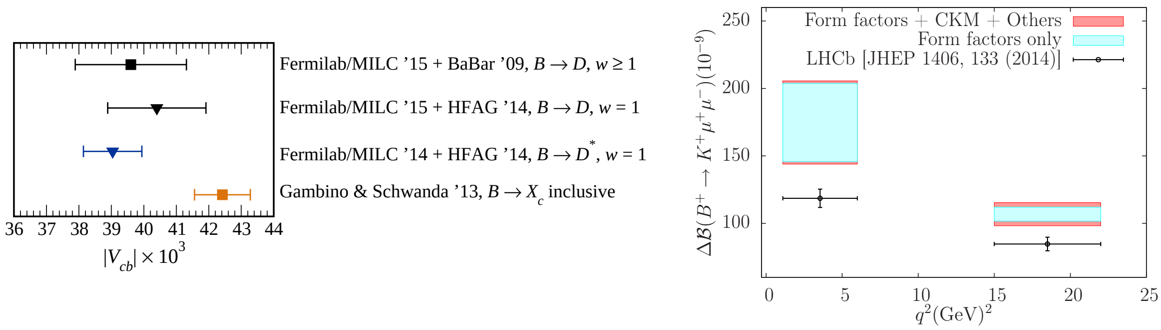

Figure 5. (L) FNAL/MILC [13] summary of results for $|V_{cb}|$, including determination from inclusive B decays to D or D^*, and inclusive decays. (R) Branching fraction for $B^+ \to K^+\mu^+\mu^-$ showing tension between SM prediction [12] and recent LHCb measurement.

4. Conclusions

Much progress has been made in lattice QCD, and more generally in lattice field theory. We concentrate here on quantities needed for the study of the CKM matrix. Calculational precision is now high enough that we can begin to look for evidence of BSM physics. We have seen a number of tensions between 1.5 and 2 standard deviations related to the CKM matrix. It will be interesting to see if reduced errors from both theory and experiment result in stronger hints (or perhaps significant evidence) of BSM physics. In the oral presentation, quark masses and α_s were also discussed. (See Ref. [3] for details.)

Acknowledgments

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References

[1] Wilson K G 1974 *Phys. Rev.* **D10** 2445–2459
[2] *Lattice 2015* URL `http://www.aics.riken.jp/sympo/lattice2015/`
[3] Aoki S *et al.* 2014 *Eur. Phys. J.* **C74** 2890 (*Preprint* `1310.8555`)
[4] Olive K A *et al.* (Particle Data Group) 2014 *Chin. Phys.* **C38** 090001
[5] Bazavov A *et al.* 2014 *Phys. Rev. Lett.* **112** 112001 (*Preprint* `1312.1228`)
[6] Bazavov A *et al.* (Fermilab Lattice, MILC) 2014 *Phys. Rev.* **D90** 074509 (*Preprint* `1407.3772`)
[7] Artuso M *et al.* (CLEO) 2005 *Phys. Rev. Lett.* **95** 251801 (*Preprint* `hep-ex/0508057`)
[8] Rosner J L and Stone S 2013 (*Preprint* `1309.1924`)
[9] Bailey J A *et al.* (Fermilab Lattice, MILC) 2015 *Phys. Rev.* **D92** 014024 (*Preprint* `1503.07839`)
[10] Bailey J A *et al.* (Fermilab Lattice, MILC) 2015 *Phys. Rev. Lett.* **115** 152002 (*Preprint* `1507.01618`)
[11] Bailey J A *et al.* 2016 *Phys. Rev.* **D93** 025026 (*Preprint* `1509.06235`)
[12] Du D, *et al.* 2016 *Phys. Rev.* **D93** 034005 (*Preprint* `1510.02349`)
[13] Bailey J A *et al.* (MILC) 2015 *Phys. Rev.* **D92** 034506 (*Preprint* `1503.07237`)