Non-zero helicity of a cyclonic vortex over localized heat source

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Abstract. Experimental and numerical study of the steady-state cyclonic vortex from isolated heat source in a rotating fluid layer is described. The structure of laboratory cyclonic vortex is similar to the typical structure of tropical cyclones from observational data and numerical modelling including secondary flows in the boundary layer. Differential characteristics of the flow were studied by numerical simulation using CFD software FlowVision. It was found that helicity in a described system has non-zero value. Physical interpretation of helicity distribution is provided.

1. Introduction

Despite decades of research the problem of tropical cyclogenesis is unsolved and attracts close attention from many scientific groups. The complexity of the problem forces researchers to study tropical cyclogenesis step by step seeing as a main goal the theory that would describe all stages of the tropical cyclone formation. Laboratory model of hurricane-like vortex was proposed and studied in [1-3]. Where rotating layer of fluid with the localized heater in the bottom was considered. The important differences of this experimental approach were more viscous fluids (in comparison with water) and using of a shallow layer. Later, series of the experiments [4] were done for the same configuration using PIV system for velocity measurements. The main focus of [4] was on integral characteristics of the azimuthal flows such as angular momentum and kinetic energy. Detailed study of different constraints of the steady-state hurricane-like vortex were studied in [5]. The three main dimensional parameters that define the vortex structure for a fixed geometry – heating flux, rotation rate and viscosity were varied independently. It was shown that viscosity is one of the main parameters that define steady-state vortex structure. Increasing of kinematic viscosity may substantially suppress the cyclonic motion for fixed values of buoyancy flux and rotation rate. Strong competition between buoyancy and rotation provides the optimal ratio of the heating flux and rotation rate for achieving cyclonic vortex of maximal intensity. It was found that relatively small variation of the rotation rate for the fluids with low kinematic viscosity may remarkably change the cyclonic vortex structure and intensity.

Here we focused our attention on differential characteristics of the convective flow, mainly helicity H which is scalar production of velocity and vorticity vectors. It is known that helicity can be very important in a developed turbulent media. The flow in our experimental system is not fully turbulent but the structure of large-scale flow with high correlation of vertical vorticity and vertical velocity is very promising for helicity formation.

2. Experimental setup

Experimental model is a cylindrical vessel of diameter $D = 300$ mm, and height $H = 40$ mm (Fig. 1). The sides and bottom were made of Plexiglas with a thickness 3 mm and 20 mm respectively. There was no cover or additional heat insulation at the sidewalls. The heater is a brass cylindrical plate mounted flush with the bottom. The diameter of the plate d is 104 mm, and its thickness is 10 mm. The brass plate is heated by an electrical coil placed on the lower side of the disc. Cylindrical vessel was placed on a rotating horizontal table (Fig. 1). Silicon oils with different values of kinematic viscosity were used as working fluids. In all experiments, the depth of the fluid layer h was 30 mm and the surface of the fluid was always open. The room temperature was kept constant by air-conditioning system, and cooling of the fluid was provided mainly by the heat exchange with surrounding air on the free surface and some heat losses through sidewalls. For low values of kinematic viscosity it takes about 2 hours to obtain a steady-state regime. Temperature inside the fluid layer was measured at mid-height ($z = 15$ mm), near the periphery (about 3 cm from the sidewall) by copper-constantan thermocouple. It was used for the estimation of the mean temperature of the fluid. The velocity field measurements were made with a 2D particle image velocimetry (PIV) system Polis. The system included a dual pulsed Nd-YaG laser, a control unit, a digital CCD camera (11 megapixels), placed in a rotating frame, and a computer. The synchronization of the operation of the laser and the CCD camera, the measurement, and the processing of the results were performed using the software package Actual Flow. Cylindrical vessel works as a lens and narrow horizontal light sheet from the periphery to the center but all area of our interest in the central part of the vessel was illuminated. Also we need to note that due to strong optical distortions we did not make PIV measurements in a close proximity to the heater at height less than 2 mm. Iterative PIV algorithms [6] and decreasing of the size of the interrogation windows from 32×32 to 16×16 pixels provided a dynamic range of approximately 500 (the ratio of the maximum and minimum resolvable particle displacement). The PIV velocity measurements were accurate to within 5%, estimated from calibration experiments in solid body rotation and long time series.

Along with the dimensional parameters (heating flux, rotation rate and kinematic viscosity) we use the set of the non-dimensional parameters which are commonly used for similar problems and can help for comparison our results with results obtained by other researchers. These are the flux Grashof number Gr_f, non-dimensional rotation velocity Re, Ekman number E and Prandtl number Pr :

$$Gr_f = \frac{g\beta h^4 q}{c\rho\kappa\nu^2} \tag{1}$$

$$Re = \frac{\Omega h^2}{\nu} \tag{2}$$

$$E = \frac{\nu}{\Omega h^2} \tag{3}$$

$$Pr = \frac{\nu}{\kappa} \tag{4}$$

where g is the gravitational acceleration, h is the layer depth, β is the coefficient of thermal expansion, c is the thermal capacity, ρ is the density, ν is the coefficient of kinematic viscosity and κ is the thermal diffusivity, q is a heat flux ($q = P/S_h$, P is the power of the heater and S_h is the heater's surface area). The value of non-dimensional rotation velocity Re is equal to the inverse value of Ekman number E but because of different physical meaning of these parameters it is convenient to use them both.
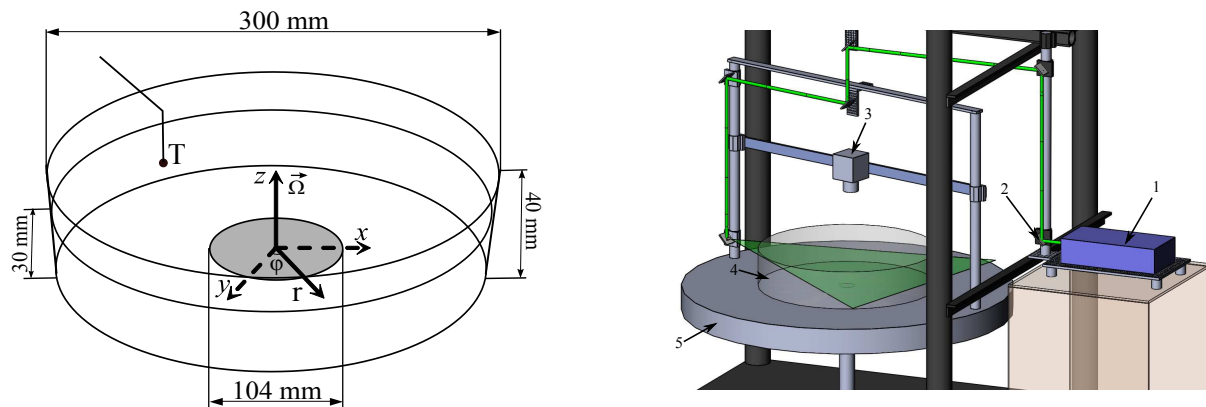
Figure 1. Experimental model, dimensions and location of the coordinate system. T - thermocouple for control of the mean temperature (left); Experimental setup: 1 - dual pulsed laser for PIV, 2 - laser sheet system, 3 - CCD camera, 4 - experimental model, 5 - rotating table (right).
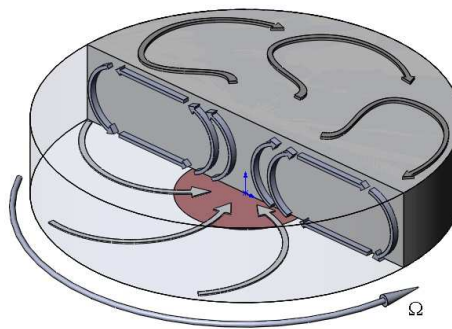

Figure 2. Scheme of large-scale circulation

3. General structure of the flow

The heat flux in the central part of the bottom initiates the intensive upward motion above the heater. Warm fluid cools at the free surface and moves toward the periphery where the cooled fluid moves downward along the side wall. After some time, large-scale advective flow occupies the whole vessel (Fig. 2, vertical cross-section). Experimental measurements of velocity fields in a non-rotating layer, in a vertical cross-section over the heating area showed that instantaneous fields are irregular and asymmetric. Along with the main updraft in the center there are less intensive but pronounced upgoing convective flows close to the periphery of the heater.

The large-scale advective flow in the lower part of the layer leads to the formation of boundary layers with potentially unstable temperature stratification above the heater and makes possible the generation of the secondary convective flows. Structure and specifics of secondary flows over the heater in the case of non-rotating cylindrical layer are described in details in [7].

The structure of the steady-state azimuthal flows (in a rotating frame) for different values of Grashof number is shown in Fig. 3. Positive (negative) values of velocity describe cyclonic (anticyclonic) motion. Distribution of azimuthal velocity is qualitatively similar to the one of a mature tropical cyclone [8]. The cyclonic vortex formation in the laboratory system can be described by following scenario. Large-scale radial circulation leads to the angular momentum transport and the angular momentum exchange on the solid boundaries. Convergent flow in the
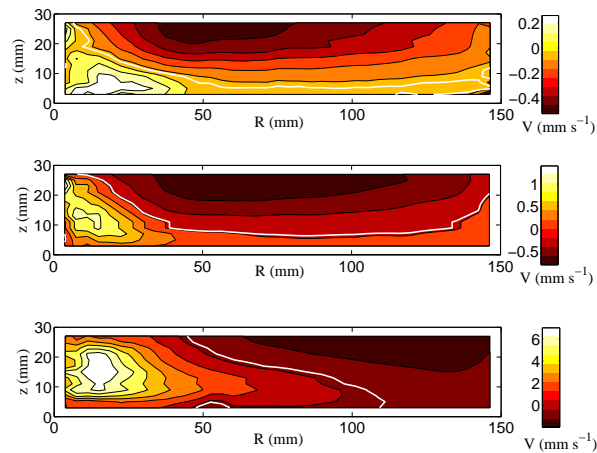
Figure 3. Mean azimuthal velocity fields for: (a) $Gr_f = 2.3 \cdot 10^5$, (b) $Gr_f = 7 \cdot 10^5$, (c) $Gr_f = 1.5 \cdot 10^7$, $\Omega = 0.069$ rad s^{-1}. Positive values describe cyclonic motion, negative - anticyclonic motion, thick solid white line shows the border between cyclonic and anticyclonic flows. Please note that for better representation of the flow structure the colour scale for each case is different.

lower layer brings the fluid parcels with large values of angular momentum from the periphery to the center and produces cyclonic motion (Fig. 2, lower horizontal cross-section). In the upper layer situation is the opposite - divergent flow takes the fluid with low values of angular momentum to the periphery resulting in anticyclonic motion (Fig. 2, upper horizontal cross-section). Friction in the viscous boundary layers leads to the sink of angular momentum in the part of the bottom occupied by cyclonic flow and produces source of angular momentum on the sidewalls when anticyclonic flow comes to the periphery. Zero net angular momentum flux on the solid boundaries is the necessary condition for the steady-state regime. Increasing of the heating flux produces more intensive radial circulation and cyclonic vortex (Fig. 3(b,c)) and pushed anticyclonic flow closer to the sidewalls.

4. Numerical simulation

Numerical simulations by computational fluid dynamics (CFD) software becomes powerful tool for studying of different hydrodynamic processes. For the numerical simulation presented here, we used the CFD package FlowVision. The fluid is assumed to be Newtonian and the flow is considered to be incompressible and laminar. The numerical finite volume code is used to solve the Boussinesq equations for thermal convection. Impermeable and no-slip velocity conditions are applied at the side wall and bottom. The upper boundary was stress-free. The bottom has a localized heat source in the central part defined through a heat flux. Geometry and physical properties of the working fluid were chosen similar to the experiment. In Table 1 values of non-dimensional parameters for presented numerical simulation are showed.

Table 1.

	$Gr_f \cdot 10^6$	Re	E	Pr
FlowVision	4.6	29	0.035	61
Experiment	4.5	27	0.036	60

There are two main mechanisms that can lead to the existence of non-zero helicity in a described system. Fig. 4 shows mean flow structure in a vertical cross-section. As a first mechanism we assume strong correlation between upward flow and vertical vorticity in a central area, over the heater. As a second mechanism we consider strong shear of radial and azimuthal velocities on the periphery which produces helicity. Fig. 5 shows mean helicity in the vertical cross-section (left figure) and mean helicity after azimuthal integration (right figure). As we assumed there is dominance of positive helicity in a central part and weak but integrally substantial negative helicity on the periphery in the upper layer. Integral helicity value is non-zero. As a next step we plan to study helicity distribution and its sources in detail.
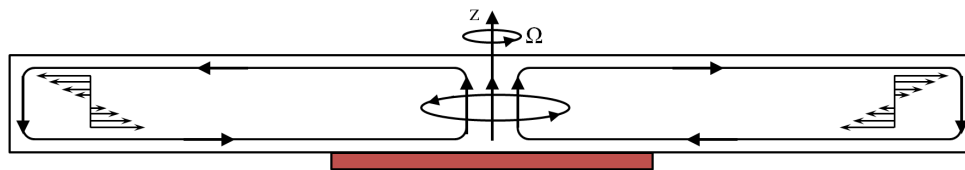

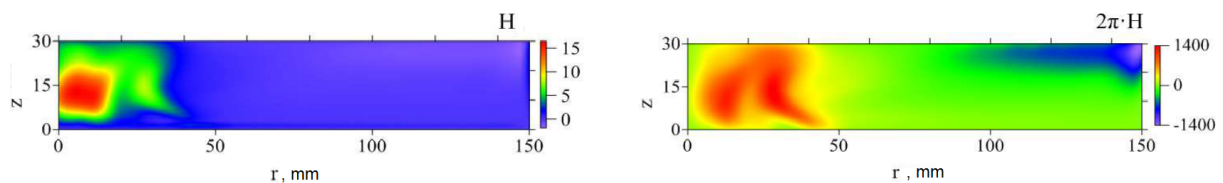**Figure 4.** Mean flow structure in a vertical cross-section.

Figure 5. $Gr_f = 4.6 \cdot 10^6$, on the left - mean helicity, on the right - mean helicity after azimuthal integration.

5. Conclusions

Helicity is an invariant of the Euler equations of inviscid fluid flow. It is resulted from strong correlation between velocity and vorticity and its conservation imposes some restrictions on topology of the flow. It comes out that realization of the flow with substantial value of helicity in laboratory conditions is a complex problem. Experimental and numerical study of convection from isolated heat source in a rotating fluid layer showed that laboratory cyclonic vortex is similar to the typical tropical cyclone. Strong correlation between vertical velocity and cyclonic and anticyclonic vorticity makes this system very promising for helicity production. This paper presented new results concerning helicity distribution and its integral value in a laboratory system. Differential characteristics of the flow were studied by numerical simulation using CFD software FlowVision. There are two main mechanisms that produce helicity in a described system. As a first mechanism we assume strong correlation between upward flow and vertical vorticity in a central area, over the heater. As a second mechanism we consider strong shear of radial and azimuthal velocities on the periphery which also produces helicity. It was found out that there is dominance of positive helicity in a central part and weak but integrally substantial negative helicity on the periphery in the upper layer. The main result is that integral helicity of the described flow is non-zero.

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