

Multiphase systems: Wave processes, state dynamics, and mathematical models

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Abstract. Phenomena that accompany the processes of shock wave loading and bubble cavitation are briefly reviewed. The mechanisms of spalling due to shock wave interaction with the free surface, formation of “laser radiation” by bubble clusters, and bubble systems in volcanic conduits (problems of their transformation to a gas-particle system and destruction) are considered.

1. Introduction

High-velocity hydrodynamics of multiphase media as an individual research field appeared and started its intense development actually in the second half of the last century. It started from solving problems of practical significance, which were based on the fact of drastic enhancement of medium compressibility at the instant when a gaseous component was formed in the liquid carrier phase. Almost simultaneously with experimental investigations of the dynamics of interaction of gas bubbles and bubble screens with shock waves, two mathematical models were developed, which later became the classical tools. The first model was developed by Rakhmatulin [1] in the form of a system of conservation laws for each phase, which are related through force interaction and joint deformation. The second one was the Iordanskii-Kogarko-van Wijngaarden (IKW) gas-dynamic model [2-4], where the bubble medium is considered as a homogeneous phase with averaged characteristics. Multiphase media with unique properties and wave processes have recently turned out to be “on demand” in solving many problems in interdisciplinary fields: fluid mechanics and solid state physics, Earth sciences, acoustics, and medicine [5]. Here we can mention a large number of physical formulations aimed at solving problems of the so-called “acoustic laser,” where active and passive bubble media are used as working media. Such media are capable of absorbing incident disturbances, amplify them, and re-emit the energy in a specified direction. We mean here free bubble systems shaped as a sphere, torus, or cord-type clusters [6-8]. In spherical clusters, significant retardation of the shock wave (SW) refracted into this cluster as compared to the external SW initiating the process in the cluster is often used. This effect allows one to control the focusing process, including the SW focal point in the cluster, its enhancement, and directed secondary radiation on the interface with the ambient liquid, by means of changing the gas phase concentration. Excitation of the bubble torus by the external SW leads not only to SW amplification in this cluster, but also to its focusing onto the axis of symmetry; as a result, a nonlinear process is observed at a certain distance from the torus plane: a series of Mach disks is formed. A cord-shaped cluster is something like a waveguide with focusing onto the axis of



symmetry and amplification of the wave propagating along the axis of symmetry. Naturally, these effects can become much more pronounced if a chemically active mixture is used as a gas phase. As was shown in [9], here is often no need to consider Todes' bimolecular kinetics or calculate thermodynamic parameters of the chemical mixture. It is sufficient to use the model of an instantaneous adiabatic explosion in a constant volume at the instant of reaching the temperature of gas ignition in the bubble, according to which the new pressure in the bubble is determined on the basis of data on the heat generated in the explosion and the ratio of specific heats of the explosion products becomes different. Investigations in a different direction, namely, shock-induced cavitation, which was developed and formulated in [10], will be also considered in the presentation. It is based on the idea of the multiphase character of cavitating media and the adequacy of the IKW model to the processes in such media, regardless which phase of loading is considered (shock wave or expansion wave (EW)).

2. Mathematical model of a multiphase medium

2.1. Mathematical IKW model

$$\begin{aligned} \frac{d\rho}{dt} + \rho \cdot \operatorname{div} \mathbf{v} &= 0, & \frac{dv}{dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + \frac{1}{\rho} \frac{\partial}{\partial z} \left(\mu \frac{\partial v}{\partial z} \right), \\ p &= p_0 + \frac{\rho_0 c_0^2}{n} \left[\left(\frac{\rho}{\rho_0(1-K)} \right)^n - 1 \right], & K &= K_0 \left(\frac{R}{R_0} \right)^3, & \frac{4}{3} \pi R^3 p_s &= \frac{m_g}{M} kT, \end{aligned} \quad (1)$$

$$R \frac{d^2 R}{dt^2} + \frac{3}{2} \left(\frac{dR}{dt} \right)^2 = \frac{p_s - p}{\rho_l} - \frac{4\mu}{\rho_l} \frac{\dot{R}}{R}.$$

These equations are the mass and momentum conservation laws, equation of state in form of Taite taking into account the bubbles, equation of gas state, and Raleigh equation.

2.2. Kinetics

$$\mu = \mu_0 \exp\{E_\mu^* K_\mu (C_0 - C) / kT\}, \quad J = J_0 \exp\{-G[(p_{ch} / \Delta p)^2 - 1]\}, \quad (2)$$

$$\frac{dm_g}{dt} = 4\pi R^2 \rho D \frac{\partial C}{\partial r} \Big|_{r=R}.$$

Here, μ is the viscosity of magmatic melt, J is the nucleation frequency, D is the diffusion coefficient.

3. Low-depth underwater explosion, spalling effects

One of the classical examples of the development of intense bubble cavitation near the free surface is the reflection of an SW generated by an underwater explosion (UE) from this surface. Figure 1 shows an axisymmetric formulation of this problem; the charge weight is 1.2 g, and the charge is located at a depth of 5 cm. A typical frame of high-speed filming ($t = 64 \mu\text{s}$, a trace of the SW front is visible) shows that the cavitation region structure differs from the expected one. The calculation is performed for conditions as close to the experimental conditions as possible and adequately reflecting available data on the state of microroughness in settled water: $R_0 = 0.5 \mu\text{m}$ and volume fraction of the gas phase $k = 10^{-11}$, (IKW model). Obviously, high-speed cameras capture only resolvable objects during the process (here $R \geq 100 \mu\text{m}$). Imposing and satisfying this condition in calculations of the dynamics of

the cavitation region structure, we found that it is this “resolution” factor that is responsible for the resultant shape of this region.

Figure 2 shows a typical frame of high-speed filming of one of the later instants of cavitation region evolution, when spalling processes begin (figure 2, ‘O’). A plane formulation was studied, where the charge (TNT, cylindrical charge, $d = 6.4$ mm) was located perpendicular to the slot walls. Destruction of the foam phase into spalling structures in the case of SW loading of a liquid layer has not been explained for a long time.

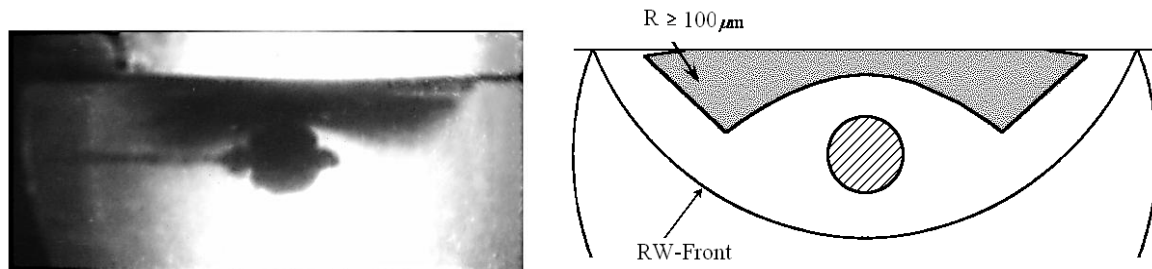


Figure 1. Underwater explosion: experiment $Q_{\text{sph}} = 1.2$ g, $h = 5$ cm; the numerical result is predicted by the IKW model.

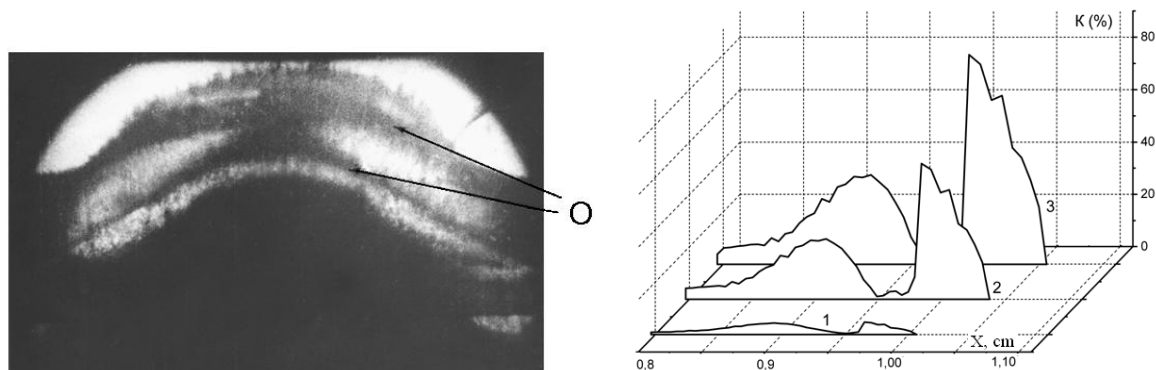


Figure 2. Plane formulation, charge $Q_{\text{cyl}} = 1.2$ g, $h = 10$ cm; the structure of the cavitation regions is described by the IKW model.

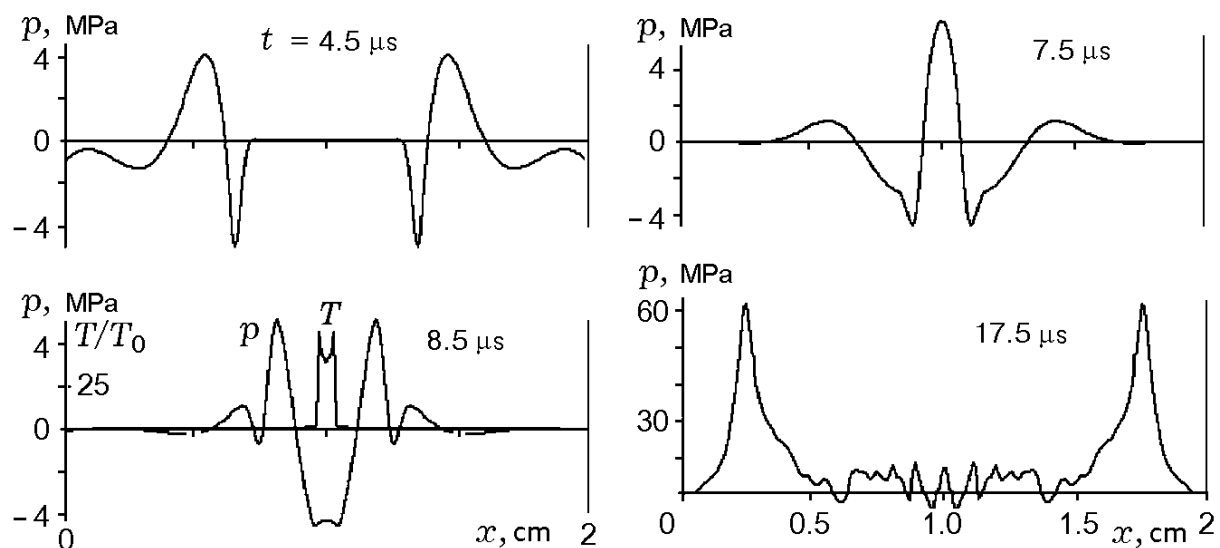


Figure 3. Model of detonation initiation in the case of a collision of weak expansion waves within a short distance of 2 cm .

An explanation was given when an unexpected fact was discovered: after its reflection from the free surface, the SW transforms into an expansion wave with an oscillating structure if SW reflection occurs in a medium with microroughness elements, which serve as cavitation nuclei behind the EW front. The calculated results in figure 2 clearly demonstrate the trend to separation of the region into individual spalling layers. This property of EWs can serve as a basis for one possible mechanism of initiation of explosive processes in readily ignited liquefied gases due to sudden depressurization of the gas holder shell, which is accompanied by generation of expansion waves. The latter can initiate cavitation processes with formation of bubbles with the chemically active mixture inside. It turned out that a collision of positive phases of even weak EWs propagating in such a medium can initiate the formation of bubble detonation waves in a bubble system, which can lead to serious consequences (figure 3). A special situation is observed for the equation of state – the model of instantaneous detonation and Todes' bimolecular model proposed in the 1930s. In the first case, when the initiation temperature is reached, the pressure in the corresponding bubbles instantaneously increases to a value determined from the transition condition $p_{\text{expl}} = \rho_{\text{det}} (\gamma_{\text{det}} - 1) \cdot Q_{\text{expl}}$.

4. SW interaction with bubble systems in the HST

We consider a horizontally located hydrodynamic shock tube (HST) with a coaxially aligned spherical bubble cluster (figure 4). An SW propagates from left to right; when the SW reaches the spherical cluster, it is refracted into the cluster. As a result, another SW is formed in the cluster; the parameters of this SW are determined by the volume fraction of bubbles in the cluster and by the parameters of the SW refracted into the cluster (figure 4): $R_{\text{cl}} = 4.5$ cm, $P_b = 3$ MPa, $k_0 = 0.01$, $R_{ST} = 15$ cm, $R_{\text{bub}} = 0.1$ cm.

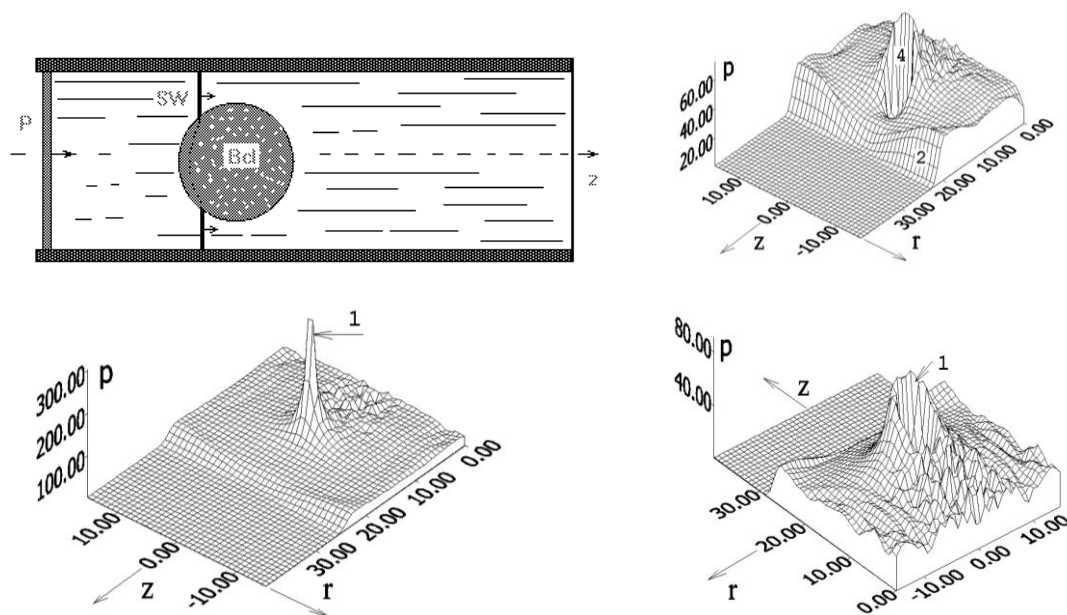


Figure 4. Interaction between the SW and the bubble cluster; dynamics of the SW field structure: SW focusing (4) in the cluster, focus (1) and SW (2) – result outside the cluster [6].

The figure demonstrates essentially nonlinear focusing of the SW (4) with a strong gradient of pressure along the front inside the cluster on its interface with the ambient medium. The HST with jumps of the cross-sectional area along the axis (figure 5) filled by a liquid with an active mixture inside gas bubbles offers a certain approach to solving new problems of wave amplification. The pattern shown in figure 5 contains two cross sections in the SW propagation direction, where the SW enters the annular channel from the left and forms a bubble detonation wave (BDW). The annular channel length is sufficient for BDW formation. In section 2, the BDW becomes expanded and

reflected from wall 2-3. Section 3 is filled with a pure liquid. The right part of figure 5 illustrates the instant of focusing of the wave with the amplitude of about 15 MPa onto the z axis; the wave amplitude in the vicinity of the axis is from 50 to 60 Pa. Domain 2, where the main wave field is formed ahead of the entrance to domain 3, is located directly under the camera. The final result is shown in the previous frame: about 50 MPa in channel 3.

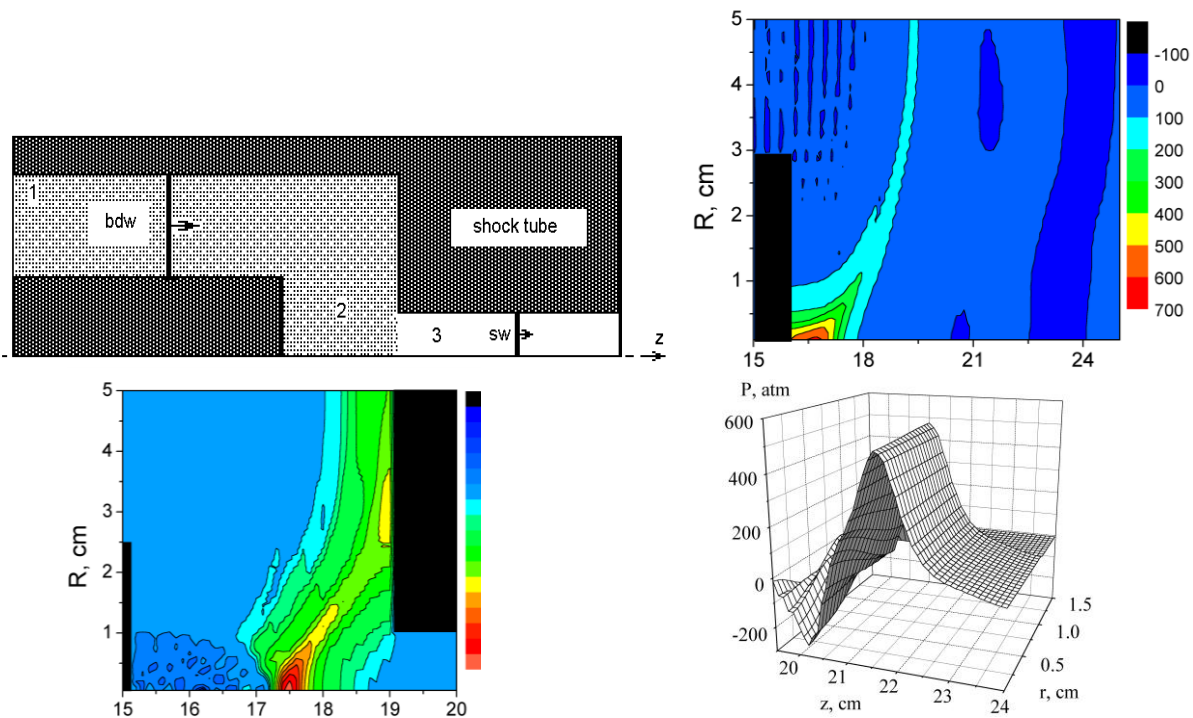


Figure 5. Hydrodynamic shock tube with internal jumps of the cross sections, the z axis is the axis of symmetry. The calculation is performed under the instantaneous detonation condition at $k_0 = 0.01$, $R_0 = 0.1$ cm, and $P_{bd} = 15$ MPa ($t = 100$ μ s) [11].

5. Dynamics of the magma flow structure behind the front of explosive decompression waves

Many investigations of the last decades are aimed at finding natural mechanisms of formation of discontinuities in the flow of intensely cavitating magma in volcano conduits, which actually pose the problem of a cyclic behavior. Its solution has to be sought first of all in the dynamics of the magma flow structure and its state. We consider a 1-km vertical column of the magma connected by its bottom with the explosive chamber of the volcano (pressure of 170 MPa); on the top, the conduit is covered by a plug separating the magma melt and the atmosphere. The magma melt is saturated with the gas (5-6%) and microcrystalites. At $t = 0$ s, the plug disappears, and a decompression wave starts to form on the free surface and propagate downward.

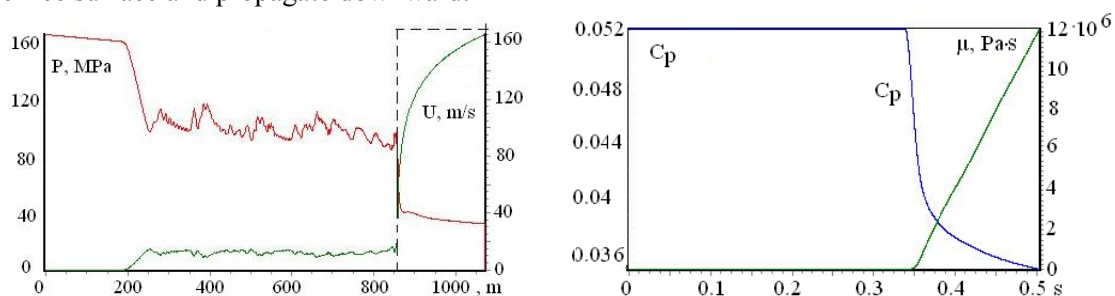


Figure 6. Anomalous zone $P-U$ (200 m) with a sharp jump in velocity and drastic reduction of the gas phase $C_p(t)$.

Cavitation processes develop vigorously behind the front of this decompression wave. Depending on the density of cavitation nuclei, an anomalous zone with a jump of the mass velocity and the basic parameters determining the state of this zone starts to form near the free surface (figure 6). In a short time, the inflow of the diffusion gas ($C_p(t)$) drastically decreases, and the main parameters reach their asymptotic values: the anomalous zone is ready to separate along the line of the mass velocity jump [12].

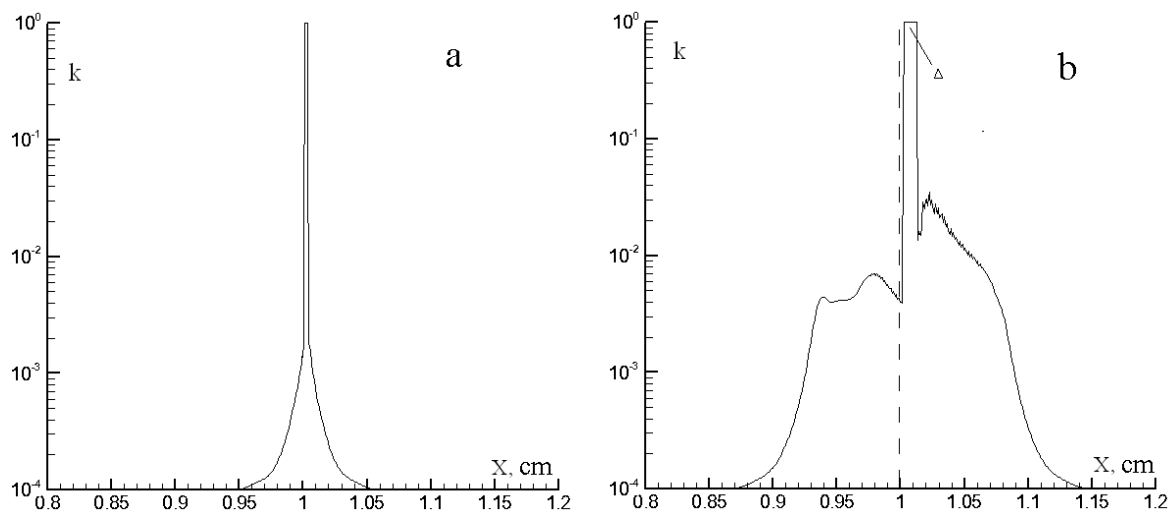


Figure 7. a) $t = 0.2 \mu\text{s}$, initial stage of the discontinuity; b) $t = 2 \mu\text{s}$, the discontinuity is already formed.

The process of formation of a discontinuity and its subsequent dynamics in a layer of a bubble liquid in the region of the mass velocity jump is calculated for the first time [13] within the framework of a general formulation of the problem.

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