

Investigation of the two-layer fluid flows with evaporation at interface on the basis of the exact solutions of the 3D problems of convection

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Abstract. New physical experiments in the Institute of Thermophysics SB RAS allow one to investigate structure of the flows of liquid layers being under action of the co-current gas flux. The flow topology is determined by four main mechanisms: natural and thermocapillary convection, tangential stresses induced by the flow of gas and mass transfer due to evaporation at the interface. Mathematical modeling of the fluid flows in an infinite channel of the rectangular cross section is carried out on the basis of a solution of special type of the convection equations. The effects of thermodiffusion and diffusive thermal conductivity in the gas phase and evaporation at the thermocapillary interface are taken into consideration. Numerical investigations are performed for the liquid-gas (ethanol-nitrogen) system under normal and low gravity.

1. Introduction

The present consideration is devoted to modeling of the convective fluid flows with evaporation using the new exact solutions of the convection equations. Obtaining the exact solutions of the governing equations has become more relevant because of the effective opportunity to model the real fluid flows [1, 2]. The solution of special type of the Boussinesq approximation of the Navier-Stokes equations can be called a generalization of the well-known Ostroumov-Birikh solutions [3, 4] for three dimensional case with respect to evaporation at interface. The group nature of the Birikh solutions has been analyzed in [5]. In [6] the three dimensional exact solution has been constructed to describe the two-layer fluid flows in an infinite channel in the non-axis-symmetrical case.

The fluid flows with an interface being under action of an adjacent gas flux are the subject of many experimental and theoretical investigations in the last decade [1, 2]. A closer interest in such problems is explained by need of study of the phenomena in the liquid media and features of the flow structure caused by the gas flow and gas flow related evaporation effects. The physical experiments are an important motivation for improvement of theoretical understanding of evaporative convection. One of the first examples of the solutions of the problems of two-layer flows with mass transfer at the "liquid-liquid" interface has been obtained in [7]. In [8, 9] the exact solutions, described the two-layer flows with evaporation, have been constructed in the case of a given specific gas flow rate [8] and under closed flow conditions in each phase [9].



Stability of the convective flows with evaporation and the effects of the intensity of the thermal load on the channel walls on the stability characteristics have been investigated in the problem statement without taking into account the Soret effect [10].

2. New exact solutions of the two-layer fluid flows with evaporation at interface

The stationary solution of the Boussinesq approximation of the Navier-Stokes equations

$$(\mathbf{v}_i \cdot \nabla) \mathbf{v}_i = -\frac{1}{\rho_i} \nabla p_i + \nu_i \Delta \mathbf{v}_i - \mathbf{g}(\beta_i T_i + \gamma C), \quad (1)$$

$$\operatorname{div} \mathbf{v}_i = 0, \quad (2)$$

$$\mathbf{v}_i \cdot \nabla T_i = \chi_i (\Delta T_i + \delta \Delta C), \quad (3)$$

$$\mathbf{v}_2 \cdot \nabla C = D(\Delta C + \alpha \Delta T_2) \quad (4)$$

are constructed. The equations (1)-(3) without marked terms are used to model the flows of the liquid in the lower layer (figure 1). The equations (1)-(4) are considered to model the flows in the upper layer so that the Dufour and Soret effects or the effects of diffusive thermal conductivity and thermodiffusion [11-14] in the gas phase are taken into account. Two layers are separated by the thermocapillary interface. By construction of the exact solution of the equations (1)-(4) the interface will be given by the relation $x = 0$. The Cartesian coordinate system is chosen so that the gravity acceleration vector \mathbf{g} ($g = |\mathbf{g}|$) is directed opposite to the Ox axis (figure 1).

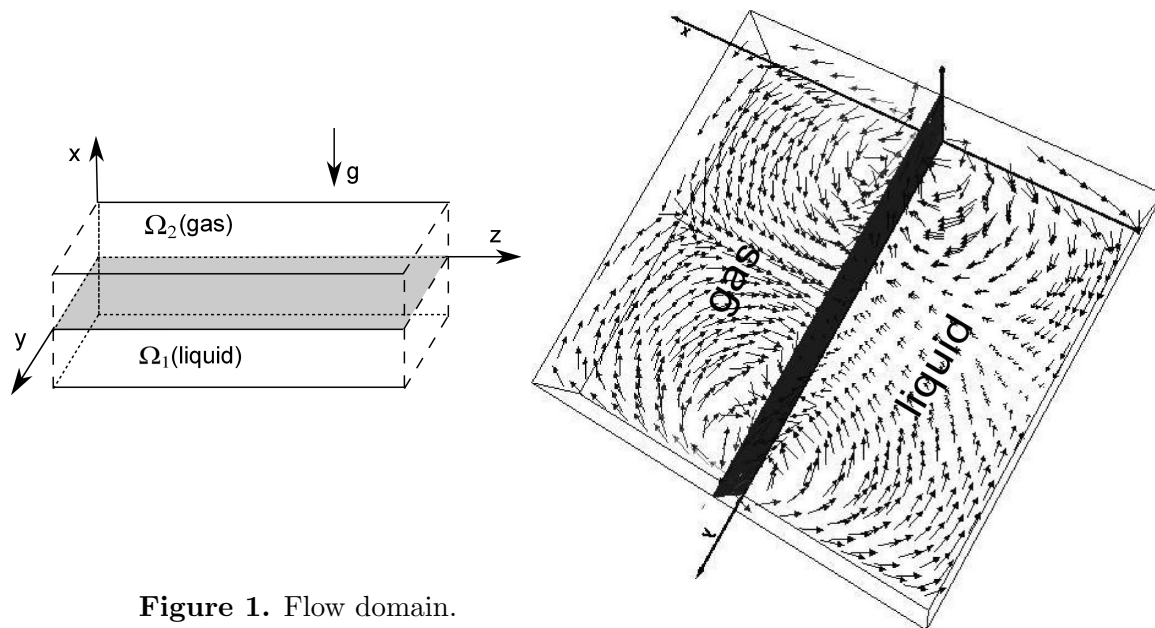


Figure 1. Flow domain.

Figure 2. Flow picture: velocity field;
 $Gr = 470$, $\tilde{T} = 0.5$.

The fluids (the liquid and the gas-vapor mixture) fulfill the infinite horizontal layers Ω_1 and Ω_2 :

$$\Omega_1 = \{(x, y, z) : -x_0 < x < 0, 0 < y < y^0, -\infty < z < \infty\},$$

$$\Omega_2 = \{(x, y, z) : 0 < x < x^0, 0 < y < y^0, -\infty < z < \infty\}.$$

We assume that the boundaries of the domains Ω_1 , Ω_2 defined by $x = -x_0$, $x = x^0$, $y = 0$, $y = y^0$ are the fixed impermeable boundaries.

The exact solution is characterized by dependence of the components of the liquid $\mathbf{v}_1 = (u_1, v_1, w_1)$ and gas velocity $\mathbf{v}_2 = (u_2, v_2, w_2)$ on the transverse coordinates (x, y) . The functions of temperature T_1, T_2 , pressure (deviation of pressure from the hydrostatic one) p_1, p_2 and vapor concentration C have the terms $\Theta_1, \Theta_2, q_1, q_2, \Phi$ similarly depending on the transverse coordinates (x, y) :

$$u_i = u_i(x, y), v_i = v_i(x, y), w_i = w_i(x, y), \quad (5)$$

$$p_1 = -A\rho_1\beta_1 g x z + q_1(x, y), \quad p_2 = -A\rho_2\beta_2 g x z + B\rho_2\gamma g x z + q_2(x, y), \quad (6)$$

$$T_1 = -Az + \Theta_1(x, y), \quad T_2 = -Az + \Theta_2(x, y), \quad (7)$$

$$C = Bz + \Phi(x, y). \quad (8)$$

The following notations are introduced: ρ_i is the density, ν_i is the kinematic viscosity coefficient, χ_i is the thermal diffusivity coefficient, β_i is the thermal expansion coefficient, γ is the concentration coefficient of density, the coefficients α and δ characterize the Soret and Dufour effects in the gas-vapor layer. The coefficients A and B determine the longitudinal temperature and concentration gradients along the interface. Subscript i is responsible for belonging to the liquid Ω_1 ($i = 1$) or gas-vapor phase Ω_2 ($i = 2$) (see figure 1).

The kinematic

$$u_1 = u_2 = 0 \quad (9)$$

and dynamic conditions (projection on the vectors tangent to the interface) should be fulfilled on the interface $x = 0$:

$$\rho_1\nu_1(u_{1y} + v_{1x}) - \rho_2\nu_2(u_{2y} + v_{2x}) = -\sigma_T\Theta_{1y}, \quad (10)$$

$$\rho_1\nu_1w_{3x} - \rho_2\nu_2w_{2x} = A\sigma_T. \quad (11)$$

Here $\sigma = \sigma_0 - \sigma_T(T - T_0)$ is the surface tension ($\sigma_T = \text{const} > 0$). The dynamic boundary condition in projection on the normal vector \mathbf{n} that coincides with the unit vector of the Ox axis is [6]:

$$-q_1 + q_2 + 2(\rho_1\nu_1u_{1x} - \rho_2\nu_2u_{2x}) = 0. \quad (12)$$

Conditions of continuity of tangential velocities and temperature are assumed to be fulfilled on the thermocapillary interface $x = 0$:

$$v_1 = v_2, \quad w_1 = w_2, \quad T_1 = T_2. \quad (13)$$

At $x = 0$ the heat transfer condition with respect to the diffusive mass flux due to evaporation and the mass balance equation are formulated with respect to the Dufour and Soret effects:

$$\kappa_1T_{1x} - \kappa_2T_{2x} - \delta\kappa_2C_x = -\lambda M, \quad M = -D\rho_2(C_x + \alpha T_{2x}). \quad (14)$$

The linearized form of an equation for saturated vapor concentration at interface is introduced [8-10]:

$$C|_{x=0} = C_*(1 + \varepsilon(T_2 - T_0)). \quad (15)$$

Here C_* is the saturated vapor concentration at $T_2 = T_0$ (T_0 is equal to 20° in [7], [9]), $\varepsilon = \lambda\mu/(R^*T_0^2)$, λ is the latent heat of evaporation, μ is the molar mass of the evaporating liquid, R^* is the universal gas constant. The relation (15) is a consequence of the Clapeyron – Clausius equation and the Mendeleev – Clapeyron equation for an ideal gas.

On the fixed impermeable walls of the channel the no-slip conditions for velocity fields

$$x = -x_0, \quad x = x_0, \quad y = 0, \quad y = y^0 : \quad \mathbf{v}_i = 0 \quad (16)$$

and the conditions of thermal insulating of the lateral walls

$$x = -x_0 : T_{1x} = 0; x = x^0 : T_{2x} = 0; y = 0 : T_{1y} = 0, T_{2y} = 0; y = y^0 : T_{1y} = 0, T_{2y} = 0 \quad (17)$$

are prescribed. The case of absence of vapor flux on the upper and lateral rigid boundaries is studied in the present statement:

$$x = x^0 : C_x = 0; y = 0, y = y^0 : C_y = 0. \quad (18)$$

The analytical technique to model three-dimensional convection of the immiscible fluids in the stationary case with evaporation at the interface is complemented by the numerical investigations so that construction of the three-dimensional flows in a channel of the rectangular cross-section with the interface will be completed. Constructing of the stationary solutions of type (5) - (8) for the two-layer convective fluid flows is reduced to sequential solving of the corresponding two-dimensional problems for the functions $w_1(x, y)$ and $w_2(x, y)$, $\Theta_1(x, y)$ and $\Theta_2(x, y)$, and $\Phi(x, y)$ and for the stream functions $\psi_1(x, y)$, $\psi_2(x, y)$ and vorticity and $\omega_1(x, y)$, $\omega_2(x, y)$ instead of the transverse components u_i, v_i of the velocity vectors ($u_i = (\psi_i)_y, v_i = -(\psi_i)_x, i = 1, 2$). The two-dimensional problems are computed in the domains $\tilde{\Omega}_i$ ($\tilde{\Omega}_1 = \{(x, y) : -x_0 < x < 0, 0 < y < y^0\}$, $\tilde{\Omega}_2 = \{(x, y) : 0 < x < x^0, 0 < y < y^0\}$).

2.1. General scheme of solution of the coupled problem

- (i) We assume that the velocity components u_i, v_i are found and will proceed from a given state.
- (ii) With given u_i, v_i we solve numerically the corresponding problems to find the third components w_i ($i = 1, 2$) of the velocity vectors.
- (iii) To find the unknown terms Θ_i ($i = 1, 2$) in the temperature functions (7) and Φ in the concentration function (8) we solve numerically the equations followed from the equations (3), (4) with the boundary conditions which are the consequence of the relations (13)-(18) on interface and fixed boundaries.
- (iv) With found Θ_i and Φ we solve numerically the systems of the equations for the stream functions and vorticity ψ_i, ω_i ($i = 1, 2$) with boundary conditions followed from the relations (9)-(13) and (16) on interface and fixed boundaries.
- (v) Transition to the step (i) should be carried out. The components u_i, v_i ($i = 1, 2$) of the velocity vectors will be recalculated. Iteration process is organized similarly to [6] with use of the convergence criteria.

The numerical procedure is based on the longitudinal transverse finite difference scheme known as the method of alternating directions [6, 15]. Note that the conditions of continuity of temperature (13) and of the heat transfer at interface (14) are used to find the interface temperature and, consequently, the temperature distributions in both phases. The form of the exact solution (7), (8) dictated by the convection equations (1)-(4) will lead to the dependence between A and B due to the interface conditions (15). Continuity of the tangential velocities (see (13)) and dynamic condition (11) help to find the values of w_1 and w_2 at interface and the third velocity components in both phases. The kinematic condition (9), the consequences of the no-slip conditions (16) in terms of ψ_1, ψ_2 are used as the boundary conditions to find the stream functions. The dynamic conditions (10), (12) written in terms of the stream function and vorticity allows us to compute the interface values of vorticity ω_1 and ω_2 and, consequently, the vorticity functions in both domains $\tilde{\Omega}_1$ and $\tilde{\Omega}_2$.

3. Numerical results

Numerical investigations are carried out in order to describe the possible flow topology for the liquid-gas system like "ethanol-nitrogen" with following physicochemical properties [16]: $\rho_1 = 0.79 \cdot 10^3$ (kg/m³), $\rho_2 = 1.2$ (kg/m³), $\nu_1 = 0.15 \cdot 10^{-5}$ (m²/s), $\nu_2 = 0.15 \cdot 10^{-4}$ (m²/s), $\beta_1 = 0.108 \cdot 10^{-2}$ (K⁻¹), $\beta_2 = 0.367 \cdot 10^{-2}$ (K⁻¹), $\gamma = -0.62$, $\kappa_1 = 0.1672$ (W/(m K)), $\kappa_2 = 0.2717 \cdot 10^{-1}$ (W/(m K)), $\sigma_T = 0.8 \cdot 10^{-4}$ (N/(m K)), $D = 0.135 \cdot 10^{-4}$ (m²/s), $\varepsilon = 0.01$ (K⁻¹), $C_* = 0.1$.

We investigate the flow topology computed with following values of the Grashoff number $Gr = \{47000, 470\}$, which correspond to the conditions of normal ($g = g_0 = 9.81 \text{ m/s}^2$) and low gravity ($10^{-2}g_0 \text{ m/s}^2$) and to the characteristic temperature drop $T_* = 10 \text{ K}$. Here $Gr = \beta_1 T_* g l^3 / \nu_1^2$, l is the characteristic length ($l = 10^{-2} \text{ m}$). The non-dimensional longitudinal temperature gradient \tilde{T} corresponds to the dimensional one A : $\tilde{T} = A l / T_*$. The non-dimensional Soret $\bar{\alpha}$ and Dufour $\bar{\delta}$ parameters are determined as follows: $\bar{\alpha} = \alpha T_*$, $\bar{\delta} = \delta / T_*$.

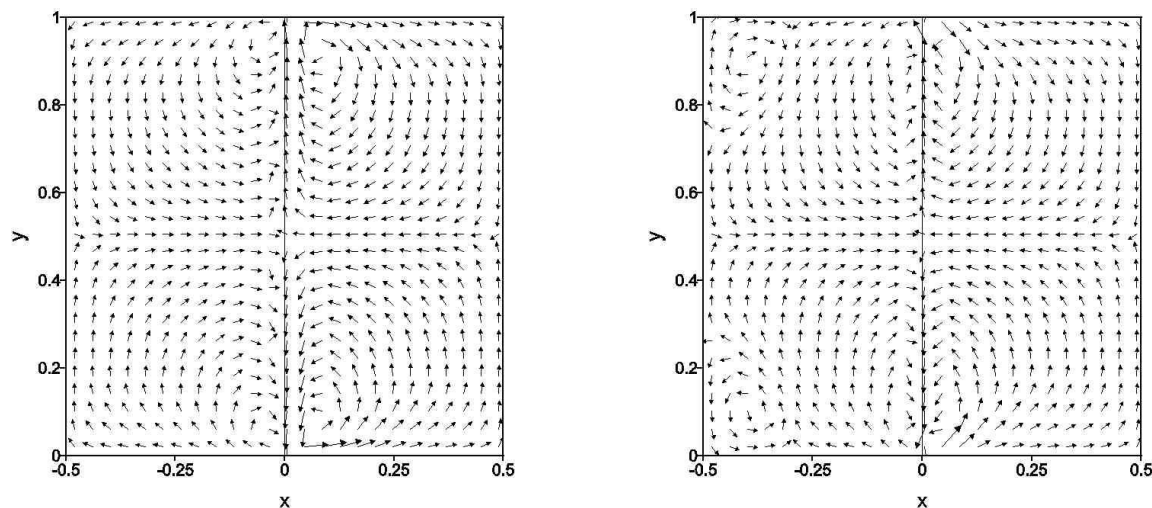


Figure 3. 2D-projection of velocity field (2D cross-section picture): $Gr = 470$, $\tilde{T} = 0.5$ (left); $Gr = 47000$, $\tilde{T} = 0.1$ (right).

Figure 2 presents the possible three dimensional flow pictures under conditions of low gravity with following values of the parameters: $Gr = 470$, $\tilde{T} = 0.5$, $\bar{\alpha} = 0.001$, $\bar{\delta} = 0.00001$ (we look from "above" from the direction of z -axis on the flow structure). The translational character of the liquid flow can be characterized as a dominant one in the case of flow structure presented in figure 2. In order to visualize a rotational character of the flows we multiply the first and second velocity components of the liquid by factor 10. Flow of the gas-vapor mixture in the upper layer is characterized by clearly marked symmetric two-vortex structure with the centers located close to the interface in the "corners". Two slight rotating vortexes transfer the liquid particles in the z -direction (upward motion). To describe the flow topology more correctly we present also the flow projection in the cross-sections. The 2D cross-section picture (see figure 3, left picture) confirms a rotational nature of the flow. The flow of the liquid in figure 2 can be characterized as a rotational motion of a smaller intensity than in the gas layer.

The intensity of the liquid flow depends on intensity of the gravitation field or interface temperature regime. More intensive rotational flow has been observed by normal gravity ($Gr = 47000$). Figure 3 (right picture) demonstrates more complicated character of the fluid flow. Two "corner" vortex structures appear by weaker action of the longitudinal temperature gradient along the interface (here $\tilde{T} = 0.1$; the same values of the Soret and Dufour parameters have been used: $\bar{\alpha} = 0.001$, $\bar{\delta} = 0.00001$). It has been found that in this case the flows have

a combined translational and rotational character. The motion becomes more complicated and we can observe the reverse flows near the solid "bottom" wall.

4. Conclusions

In this paper a stationary coupled problem of the gravitational and thermocapillary convection with respect to the diffusive mass transfer at the interface due to evaporation is studied. In the problem of convective fluid flows a new exact solution is constructed in the three-dimensional case. This solution is the analogue of the Ostroumov–Birikh solution of the convection equations. In gas-vapor phase the Dufour and Soret effects are taken into account additionally. It describes a joint flow of an evaporating viscous heat-conducting liquid and gas-vapor mixture in an infinite horizontal channel of the rectangular cross section. Reduction of the common problem to several two-dimensional problems can be carried out so that the real three-dimensional flows are described by the two-dimensional statements. The flows of both fluids (of a liquid and gas-vapor mixture) are computed. They are characterized as translational motion and progressively rotational flow and can be realized in the various forms. The numerical investigations allow one to analyse the possible flow structure with respect to the intensity of the gravitation field and longitudinal temperature gradient created on the interface.

Acknowledgments

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References

- [1] Lyulin Y and Kabov O 2014 Evaporative convection in a horizontal liquid layer under shear-stress gas flow *Int. J. Heat Mass Transfer* **70** 599–609
- [2] Goncharova O N, Rezanova E V, Lyulin Yu V and Kabov O A 2015 Modeling of two-layer liquid-gas flow with account for evaporation, *Thermophysics and Aeromechanics* **22(5)** 655–661
- [3] Ostroumov G A 1952 *Free convection under the conditions of an internal problem* (Moscow-Leningrad: Gostekhizdat Press)
- [4] Birikh R V 1969 Thermocapillary convection in a horizontal layer of liquid *Journal of Applied Mechanics and Technical Physics* **3** 43–45
- [5] Pukhnachov V V 2000 Group-theoretical nature of the Birikh's solution and its generalizations (Krasnoyarsk: Book of Proc. Symmetry and differential equations) 180–183
- [6] Goncharova O N, Kabov O A and Pukhnachov V V Solutions of special type describing the three dimensional thermocapillary flows with an interface *Int. J. Heat Mass Transfer* **55** 715–725
- [7] Shliomis M I and Yakushin V I 1972 Convection in a two-layers binary system with an evaporation, *Collected papers: Uchenye zapiski Permskogo Gosuniversiteta, seriya Gidrodinamika* **4** 129–140
- [8] Goncharova O N, Hennenberg M, Rezanova E V and Kabov O A 2013 Modeling of the convective fluid flows with evaporation in the two-layer systems *Interfacial phenomena and heat transfer* **1(3)** 317–338
- [9] Goncharova O N and Rezanova E V 2014 Example of an exact solution of the stationary problem of two-layer flows with evaporation at the interface *Journal of Applied Mechanics and Technical Physics* **55(2)** 247–257
- [10] Bekezhanova V B and Goncharova O N 2016 Stability of the exact solutions describing the two-layer flows with evaporation at interface (accepted for publication) *Fluid Dynamics Research Journal*
- [11] De Groot S R 1951 *Thermodynamics of irreversible processes* (Amsterdam: North-Holland publishing Co.; New York: Interscience Publishers Inc)
- [12] De Groot S R and Mazur P 1984 *Thermodynamics of irreversible processes* (London: Dover 1984)
- [13] Prigogine I and Dufay R 1954 *Chemical thermodynamics* (London, New York: Longmans, Green)
- [14] Gebhart B, Jaluria Y, Mahajan R L and Sammakia B 1988 *Bouyancy-induced flows and transport* (Berlin, Heidelberg, New York, London, Paris, Tokyo: Springer Verlag)
- [15] Roache P J 1976 *Computational Fluid Dynamics* (Albuquerque: Hermosa Publishers)
- [16] Weast C R C 1979 *Handbook of Chemistry and Physics* (Boca Raton, Florida: CRC Press Inc)