

Microconvection in vertical channel at given heat flux

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Abstract. A problem on stability of the viscous heat-conducting liquid flow in the vertical channel at given heat flux on the permeable solid walls is studied. The two-dimensional flow is described by an exact invariant solution of the microconvection equations. The investigation of the exact solution allows one to find out the extent of influence of the thermal load, gravity and the system geometry on the flow structure. Stability of the solution is investigated in the framework of the linear theory. The spectrum of the spatial characteristic perturbations is analyzed in the space of problem parameters. Typical forms of the hydrodynamic and thermal disturbances are presented and dependence of characteristics of the arising structures on the thermal load and gravity is established. Convective cells, hydrothermal rolls and polygonal structures can appear in the channel. By weak gravity the hydrothermal rolls are not formed. Changing heat flux and disturbance wave length lead to deformation of the cells and complication of the spatial form of the structures.

1. Introduction

A complex of problems related with modeling of flows arises by prediction of the dynamics of different technological nonisothermal processes in which liquids are used as working media (e.g. crystal growing, thermal coating, fluidic cooling). One of the problems is the search of possibilities to control the flow through the external effects or choice of liquids with different thermophysical properties. By mathematical modeling we can obtain a priori evaluations of all characteristics. Fulfillment of the estimates allows one to realize stable regimes both in laboratory experiments and in industrial plants. For this purpose the problem on stability of convective regimes is studied. Mechanisms and character of possible crisis phenomena are investigated.

Studies in space materials science are motivated by opportunity to use long-term weightlessness in work processes. Under microgravity there is no the Archimedes force, which causes the density stratification in the field of gravity, natural convection is weak and does not lead to the mixing of liquid layers with various temperature. It opens up crucially new possibilities for obtaining qualitatively new (advantaged) materials, improvement of the existing ones and modification of the fluidic technologies.

The mathematical model option has the most important significance in the theoretical investigations. As a result of elaborations of the Oberbeck–Boussinesq approximation of the Navier–Stokes equations the models of microconvection [1, 2] were obtained. In [3, 4] the results of investigations of gravitational and non-gravitational convection were generalized.



Mathematical models of convection in microgravity and microscales and the results of the liquid motions analysis in the framework of these models were presented in [5, 6].

Dissimilar internal and external factors influence the origin and the characteristics of the convective flows. Interaction of the factors can lead to the growth of liquid motion intensity and the motion can become unstable. The most important condition of correct work of the experimental or industrial equipment using liquid system is the stable state of the working fluid. Thus, it is necessary to ensure the stability of the basic state (equilibrium or motion) of the working liquid medium. For example, in the processes of the crystal growing from molten mass the appearing cellular perturbations worsen quality and properties of the obtained pattern. Therefore, efficiency of the technological process can be increased by means of stabilization of convective flow of the liquid molten mass [7, 8, 9, 10].

By mathematical modeling a special attention is paid to construction and investigation of the exact solutions describing the convective flows. The importance of exact solutions is explained by opportunities of a rapid, accurate and effective analysis of the physical factors, defining the main flow structure. In the case of a multiparameter problem we can examine influence of the separate physical factors on the flow characteristics and criteria of its stability.

In the present paper the exact invariant solution of the microconvection equations describing the non-isothermal flow in a vertical layer is investigated. The properties of the spatial normal perturbations of the main flow and the influence of the gravity and external thermal load on their structure are studied.

2. Microconvection equations and exact invariant solution

We consider non-isothermal flow of viscous heat-conducting liquid in the vertical channel $\Omega = \{|\zeta| < 1, -\infty < \xi < \infty, -\infty < \vartheta < \infty\}$ with immovable solid walls $\zeta = \pm 1$.

2.1. Governing equations and basic parameters

For description of the flow the microconvection equations of the isothermally incompressible liquid [1] are used. In the non-dimensional form the equations are [6]

$$\begin{aligned} \mathbf{w}_\tau + \mathbf{w} \nabla \mathbf{w} + \varepsilon \operatorname{rot} \mathbf{w} \times \nabla \theta + \varepsilon^2 \operatorname{div}(\nabla \theta \otimes \nabla \theta - |\nabla \theta|^2 \mathbf{I}) = \\ = (1 + \varepsilon \theta)(-\nabla q + \Delta \mathbf{w}) \operatorname{Pr} - \varepsilon \boldsymbol{\eta}(\tau) \operatorname{Pr} \theta, \end{aligned} \quad (1)$$

$$\operatorname{div} \mathbf{w} = 0, \quad (2)$$

$$\theta_\tau + \mathbf{w} \cdot \nabla \theta + \varepsilon |\nabla \theta|^2 = (1 + \varepsilon \theta) \Delta \theta, \quad (3)$$

where \mathbf{w} , q are the modified velocity and the analogue of modified pressure, θ is the temperature, $\varepsilon = \beta \theta_*$ is the Boussinesq parameter (β , θ_* are the heat expansion coefficient and temperature drop, respectively), $\operatorname{Pr} = \nu/\chi$ is the Prandtl number (ν , χ are the viscosity and thermal diffusivity coefficients, respectively), $\boldsymbol{\eta} = a^3 \mathbf{g}(t)/(\nu \chi)$ is the vector parameter of microconvection (a , \mathbf{g} are the layer thickness and the gravitation vector, respectively), \otimes denotes the tensor product. There is multiplier $\varepsilon \boldsymbol{\eta}(\tau) \operatorname{Pr}$ in equation (1) which defines vector of the Grashof numbers $\mathbf{Gr} = \varepsilon \boldsymbol{\eta}(\tau) \operatorname{Pr} = \beta \theta_* a^3 \mathbf{g}(t)/\chi^2$. If $|\mathbf{g}| = g$ and g is the gravity acceleration then $\eta = a^3 g/(\nu \chi)$ and $\operatorname{Gr} = \beta \theta_* g a^3/\chi^2$. The microconvection parameter η characterizes the relation of the velocities orders originated by the volumetric expansion and buoyancy forces.

No-slip condition and heat flux are imposed at solid walls ($\zeta = 1$)

$$\mathbf{w} + \varepsilon \nabla \theta = 0, \quad \partial \theta / \partial n = D(\zeta, \tau), \quad \zeta = (\zeta, \xi, \vartheta), \quad (4)$$

where D is the non-dimensional heat flux.

To close the problem statement the condition of zero mass flow of the liquid through any cross section of the layer $\xi = \text{const}$ is imposed for this solution.

2.2. Stationary solution

System (1)–(3) in plane stationary case admits invariant solution [11]

$$\mathbf{w} = (u(\zeta), v(\zeta), 0), \quad \theta = \theta(\zeta), \quad q = \varphi\xi + r(\zeta), \quad (5)$$

where $\varphi = \text{const}$, $r(\zeta)$ are to be defined.

Substitution of (5) into the governing equations, taking into account (4) and condition of zero mass flow, leads to the following relations

$$\varphi = 1 - \frac{p_1}{2p_2\text{Pe}}, \quad r(\zeta) = \text{const}, \quad \theta(\zeta) = 1 - \text{Pe} \zeta, \quad u(\zeta) = \text{const} = u_0,$$

$$v(\zeta) = \frac{p_1}{4p_2\text{Pe}} \zeta^2 - \frac{p_3}{2\text{Pe}^2} \zeta + \frac{p_1}{4p_2\text{Pe}} - \frac{p_4}{2\text{Pe}^2} + \frac{1 + \varepsilon - \text{Pe} \zeta}{\text{Pe}^2} [\ln(1 + \varepsilon - \text{Pe} \zeta) - 1],$$

where $\text{Pe} = u_0 a / \chi$ is the Peclet number and p_j constants ($j = 1, \dots, 4$) can be written as

$$p_1 = (\ln f_2^* - \ln f_1^*) \left[f_1^* \ln f_1^* + f_2^* \ln f_2^* + \frac{1 + \varepsilon}{\text{Pe}} (f_1^* \ln f_1^* - f_2^* \ln f_2^*) \right] + 4\text{Pe},$$

$$p_2 = 1 + \varepsilon + (\ln f_2^* - \ln f_1^*) \left(\frac{\text{Pe}}{2} - \frac{(1 + \varepsilon)^2}{2\text{Pe}} \right),$$

$$p_3 = f_1^* \ln f_1^* - f_2^* \ln f_2^* + 2\text{Pe}, \quad p_4 = f_1^* \ln f_1^* + f_2^* \ln f_2^* - 2(1 + \varepsilon),$$

$$f_1^* = 1 + \varepsilon - \text{Pe}, \quad f_2^* = 1 + \varepsilon + \text{Pe}.$$

Function q is defined up to a constant, we can assume $r \equiv 0$ without loss of generality. Furthermore, form of solution (4) restricts values of heat fluxes at the channel walls $D(-1) = D(1) = D$ and then $u_0 = -\varepsilon D / \eta$.

Thus, exact solution (5) describes a composite flow which is the superposition free-convective flow with a transverse uniform flux caused by seepage through permeable boundaries of the vertical channel. Flow consists of two counter fluxes. Convective circulation is in channel. Liquid rises near the hot wall and drop down near the cool one. Temperature is changed linearly. Equality of heat fluxes at walls is provided.

3. Stability of the microconvective flow

If the microconvection parameter $\eta < 1$, then we have a microconvective flow. Small values η can be achieved both due to the smallness of linear thickness of the channel a (convection in microscales) and due to the smallness of the gravity acceleration g (convection in microgravity). Stability of the microconvective flow in the vertical channel with permeable walls is investigated in the framework of the linear theory. Influence of the thermal load at boundaries channel and gravity on the typical form of the perturbations is studied.

3.1. Problem for the small disturbances

Let us suppose $\tilde{w}(\zeta, \tau) = \mathbf{w}(\zeta, \tau) + \mathbf{W}(\zeta, \tau)$, $\tilde{q}(\zeta, \tau) = q(\zeta, \tau) + Q(\zeta, \tau)$, $\tilde{\theta} = \theta(\zeta, \tau) + T(\zeta, \tau)$, where $\mathbf{W} = (U, V, W)$, Q, T are the small non-stationary disturbances, $\mathbf{w}(\zeta, \tau)$, $q(\zeta, \tau)$, $\theta(\zeta, \tau)$ are basic solutions (5). Now functions $\tilde{w}, \tilde{q}, \tilde{\theta}$ are the solutions of problem (1)–(3), (4).

We consider the normal spatial perturbations

$$(\mathbf{W}, Q, T) = (\mathbf{W}(\zeta), Q(\zeta), T(\zeta)) \cdot \exp[i(\alpha_1 \xi + \alpha_2 \vartheta - C\tau)] \quad (6)$$

with wavenumbers α_1 , α_2 along the ξ and ϑ axes, respectively, and complex decrement $C = C_r + iC_i$, determining the perturbations development with time. Linearized problem for the amplitudes of disturbances (6) has the following form

$$\begin{aligned} -1 < \zeta < 1: \quad & i(\alpha_1 v - C)U + \text{Pe} U' - [i\alpha_1 \varepsilon v_\zeta + \varepsilon^2 \theta_\zeta (\alpha_1^2 + \alpha_2^2)]T = \\ & = (1 + \varepsilon \theta)[-Q' + U'' - (\alpha_1^2 + \alpha_2^2)U]\text{Pr}, \end{aligned} \quad (7)$$

$$\begin{aligned} i(\alpha_1 v - C)V + (\text{Pe} + \varepsilon \theta_\zeta)V' + (v_\zeta - i\alpha_1 \varepsilon \theta_\zeta)U + (\varepsilon v_\zeta - i\alpha_1 \varepsilon^2 \theta_\zeta)T' + \\ + i\alpha_1 \varepsilon^2 \theta_\zeta T = (1 + \varepsilon \theta)[-i\alpha_1 Q + V'' - (\alpha_1^2 + \alpha_2^2)V]\text{Pr} + \frac{\text{Gr} T}{1 + \varepsilon \theta}, \end{aligned} \quad (8)$$

$$\begin{aligned} i(\alpha_1 v - C)W + (\text{Pe} + \varepsilon \theta_\zeta)W' - i\alpha_2 \varepsilon \theta_\zeta U - i\alpha_2 \varepsilon^2 \theta_\zeta T' = \\ = (1 + \varepsilon \theta)[-i\alpha_2 Q + W'' - (\alpha_1^2 + \alpha_2^2)W]\text{Pr}, \end{aligned} \quad (9)$$

$$U' + i\alpha_1 V + i\alpha_2 W = 0, \quad (10)$$

$$i(\alpha_1 v - C)T + (\text{Pe} + 2\varepsilon \theta_\zeta)T' + \theta_\zeta U = (1 + \varepsilon \theta)[T'' - (\alpha_1^2 + \alpha_2^2)T] + \varepsilon \theta_\zeta T, \quad (11)$$

$$|\zeta| = 1: \quad U + \varepsilon T' = 0, \quad V = 0, \quad W = 0, \quad T = 0. \quad (12)$$

where prime denotes the differentiation with respect to ζ . The obtained problem (7)–(12) is the spectral problem with respect to complex decrement C and defines spatial perturbations depending on α_1 , α_2 , ε , Pe , Pr and Gr parameters.

3.2. Analogue of the Squire transformation

The transformations reducing problem (7)–(12) to the equivalent 2D problem with additional parameter characterizing the spatial perturbations are found. Actually, if (8) and (9) are multiplied by $i\alpha_1$ and $i\alpha_2$, respectively, and the obtained products are summed then the result is

$$\begin{aligned} i(\alpha_1 v - C)\tilde{Z} + (\text{Pe} + \varepsilon \theta_\zeta)\tilde{Z}' + (i\alpha_1 v_\zeta + \alpha^2 \varepsilon \theta_\zeta)U + (i\alpha_1 \varepsilon v_\zeta + \alpha^2 \varepsilon^2 \theta_\zeta)T' - \\ - \varepsilon^2 \alpha^2 \theta_\zeta T = (1 + \varepsilon \theta)[\alpha^2 Q + \tilde{Z}'' - \alpha^2 \tilde{Z}]\text{Pr} + i\alpha_1 \frac{\text{Gr} T}{1 + \varepsilon \theta}, \end{aligned} \quad (13)$$

where

$$\tilde{Z} = i\alpha_1 V + i\alpha_2 W, \quad \alpha^2 = \alpha_1^2 + \alpha_2^2. \quad (14)$$

The parameter $\lambda = \alpha_1/\alpha$ is introduced to characterize the spatial perturbations and $\lambda \in [0, 1]$. Value $\lambda = 0$ corresponds to perturbations with $\alpha_1 = 0$, which are periodical with respect to ϑ and do not depend on longitudinal coordinate ξ . The disturbances are not plane one because $V \neq 0$ in this case. In the stationary regime liquid particles move along helix line whose axes are parallel to the direction of the basic motion (so called “spiral” perturbations). Other limiting case $\lambda = 1$ corresponds to plane perturbations with $\alpha_2 = 0$.

The following transformations are supplemented

$$\tilde{U} = i\alpha U, \quad \tilde{Q} = i\alpha Q, \quad \tilde{T} = i\alpha T, \quad (15)$$

then (13) takes form

$$\begin{aligned} i(\alpha \lambda v - C)\tilde{Z} + (\text{Pe} + \varepsilon \theta_\zeta)\tilde{Z}' + (\lambda v_\zeta - i\alpha \varepsilon \theta_\zeta)\tilde{U} + (\varepsilon \lambda v_\zeta - i\alpha \varepsilon^2 \theta_\zeta)\tilde{T}' + \\ + i\alpha \varepsilon^2 \theta_\zeta \tilde{T} = (1 + \varepsilon \theta) \left[-i\alpha \tilde{Q} + \tilde{Z}'' - \alpha^2 \tilde{Z} \right] \text{Pr} + \frac{\lambda \text{Gr} \tilde{T}}{1 + \varepsilon \theta}. \end{aligned} \quad (16)$$

Equations (7), (10), (11) are rewritten

$$i(\alpha\lambda v - C)\tilde{U} + \text{Pe}\tilde{U}' - [i\alpha\varepsilon\lambda v_\zeta + \varepsilon^2\theta_\zeta\alpha^2]\tilde{T} = (1 + \varepsilon\theta)[- \tilde{Q}' + \tilde{U}'' - \alpha^2\tilde{U}]\text{Pr}, \quad (17)$$

$$\tilde{U}' + i\alpha\tilde{Z} = 0, \quad (18)$$

$$i(\alpha\lambda v - C)\tilde{T} + (\text{Pe} + 2\varepsilon\theta_\zeta)\tilde{T}' + \theta_\zeta\tilde{U} = (1 + \varepsilon\theta)[\tilde{T}'' - \alpha^2\tilde{T}] + \varepsilon\theta_\zeta\tilde{T}. \quad (19)$$

It is obviously that structure of the equations for new functions \tilde{U} , \tilde{Z} , \tilde{Q} , \tilde{T} is the same that one for U , V , Q , T in the case of plane problem. In addition \tilde{U} , \tilde{Z} , \tilde{T} functions defined by (14) and (15) satisfy the boundary conditions which are equivalent to (12):

$$|\zeta| = 1 : \quad \tilde{U} + \varepsilon\tilde{T}' = 0, \quad \tilde{Z} = 0, \quad \tilde{T} = 0. \quad (20)$$

Thus, obtained transformation (14), (15) is the analogue of the Squire transformation [12] reducing the problem on the stability about three-dimensional perturbations to the corresponding two-dimensional problem. The parameter λ is to be fixed one for calculation of the characteristics of the spatial perturbations through solutions of the plane problem.

3.3. Stability of the shortwave “spiral” perturbations

Stability of the basic flow as concerning shortwave “spiral” ($\lambda = 0$) perturbations is proven analytically. Actually, let us consider amplitude equations (16) – (19) under $\lambda = 0$ and $\alpha \rightarrow \infty$. Terms containing C , the highest derivative and the highest powers of α are remained in the equations, then for \tilde{Z} function we have the following boundary value problem

$$\tilde{Z}'' = \left(\alpha^2 - \frac{iC}{\text{Pr}(1 + \varepsilon\theta)} \right) \tilde{Z} = 0, \quad \tilde{Z}(-1) = \tilde{Z}(1) = 0. \quad (21)$$

Equation (21) is multiplied by complex conjugate value \tilde{Z}^* and product is integrated over the segment $[-1, 1]$. Taking into account the boundary conditions we obtain

$$\int_{-1}^1 |\tilde{Z}'|^2 d\zeta = \frac{iC}{\text{Pr}} \int_{-1}^1 \frac{|\tilde{Z}|^2}{1 + \varepsilon\theta} d\zeta - \alpha^2 \int_{-1}^1 |\tilde{Z}|^2 d\zeta.$$

The latter is rewritten in the form

$$\frac{iC}{\text{Pr}} \int_{-1}^1 \frac{|\tilde{Z}|^2}{1 + \varepsilon\theta} d\zeta = \int_{-1}^1 |\tilde{Z}'|^2 d\zeta + \alpha^2 \int_{-1}^1 |\tilde{Z}|^2 d\zeta. \quad (22)$$

Decrement C is the pure imaginary number with the negative imaginary part from (22). The result is right in virtue of the positivity of the all subintegral terms and does not depend on temperature function θ for the basic flow. Thus, all shortwave “spiral” perturbations fade out monotonically with time.

4. Typical forms of perturbations

Calculations of the spectrum of perturbations for the melt of silicon dioxide was performed by the method of stepwise integration with orthogonalization [13]. In fact, for the specific model medium the variations of λ and α correspond to the changes of configuration of perturbations waves, Pe connects with the transversal velocity or/and channel thickness, Gr relates to the gravity action or/and channel thickness. Taking into account dependence of the transversal

velocity u_0 on heat flux value D (see subsection 2.1) the contribution of the given heat flux into the change of the typical form of the perturbations is also characterized by the Peclet number Pe .

We assume the silicon dioxide melt fills the vertical channel. Walls of the channel are permeable and constant temperature drop θ_* between them are kept, on the walls the constant heat flux are held. The following values of physical parameter for chosen model medium are used: $\nu = 2.65 \cdot 10^{-3} \text{ cm}^2/\text{s}$, $\chi = 0.49 \text{ cm}^2/\text{s}$, $\beta = 0.75 \cdot 10^{-5} \text{ }^\circ\text{K}^{-1}$, $k = 0.042 \text{ W}/(\text{cm}\cdot\text{K})$. Calculations were performed under $Pr = 5.41 \cdot 10^{-3}$ and $\varepsilon = 7.5 \cdot 10^{-5}$ in the all cases.

In Fig. 1–6 the typical hydrodynamic and thermal perturbations under different values of heat flux on the channel boundaries, gravity acceleration, wave length and λ parameter are shown. Values of the Peclet number in captions to the figures correspond to different values of the heat flux: $Pe_1 = 8.93 \cdot 10^{-5}$ for $5 \text{ W}/\text{cm}^2$, $Pe_2 = 35.71 \cdot 10^{-5}$ for $20 \text{ W}/\text{cm}^2$, $Pe_3 = 178.57 \cdot 10^{-5}$ for $100 \text{ W}/\text{cm}^2$.

Analysis of the eigenfunctions reveals the perturbations are the convective cells, hydrothermal rolls or deformed structures of the complex spatial form. Type of patterns depends on the problem parameters. Spatial sizes depend on wave length of disturbances ($\lambda_w = 2\pi/\alpha$). The formation of the oscillatory regimes is possible in the considered system. Arising structures oscillate periodically ($C_r \neq 0$) and vanish with time ($C_i < 0$). Existence of the oscillatory regimes is possible in virtue of properties of the spectral problem (7)–(12) and related with its non-self-conjugation.

Fig. 1, 2 present the disturbances which are formed in the channel with thickness $a = 0.1 \text{ cm}$ under gravity acceleration $g = 2g_0 \cdot 10^{-3}$, $g_0 = 981 \text{ cm}/\text{s}^2$. The hydrothermal rolls arise due to the interaction of the perturbations with basic flow in the channel under weak thermal load (Fig. 1a, 2a). Liquid particles rise up along the roll axis to the hot wall direction and go down to the cold wall direction. With increasing of the value of heat flux (and, consequently, the transversal velocity) the spatial form of the structures is complicated (Fig. 1b, 2b). Under essential thermal load the transition to the polygonal cells is observed. In the case the form of disturbances is entirely defined by growing transversal velocity (Fig. 1c, 2c).

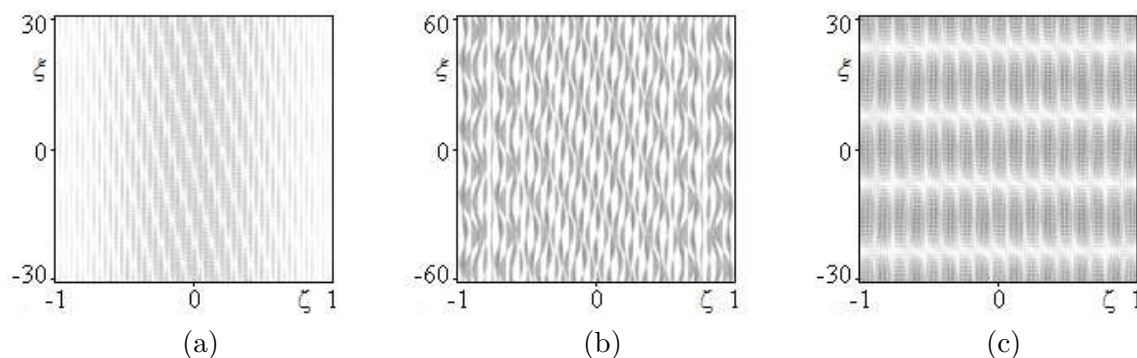


Figure 1. Distribution of the hydrodynamic perturbations in the vertical layer under $\eta = 1.51$, $Gr = 61.27 \cdot 10^{-8}$: (a) — Pe_1 , $\lambda = 1/\sqrt{2}$, $\alpha_1 = 0.2$; (b) — Pe_2 , $\lambda = 0.2$, $\alpha_1 = 0.2$; (c) — Pe_3 , $\lambda = 1/\sqrt{2}$, $\alpha_1 = 0.2$.

The hydrodynamic and thermal perturbations presented in Fig. 3, 4 were obtained for $a = 0.1 \text{ cm}$, $g = g_0 \cdot 10^{-4} \text{ cm}/\text{s}^2$. With attenuation of the gravitational action the hydrothermal rolls is changed by large-scale convective cells (Fig. 3a), which break down by growing the thermal load. The large-scale cells are changed by patterns of the complex spatial form (Fig. 3b). Further increasing the heat flux at boundaries (and, consequently, the transversal velocity) leads

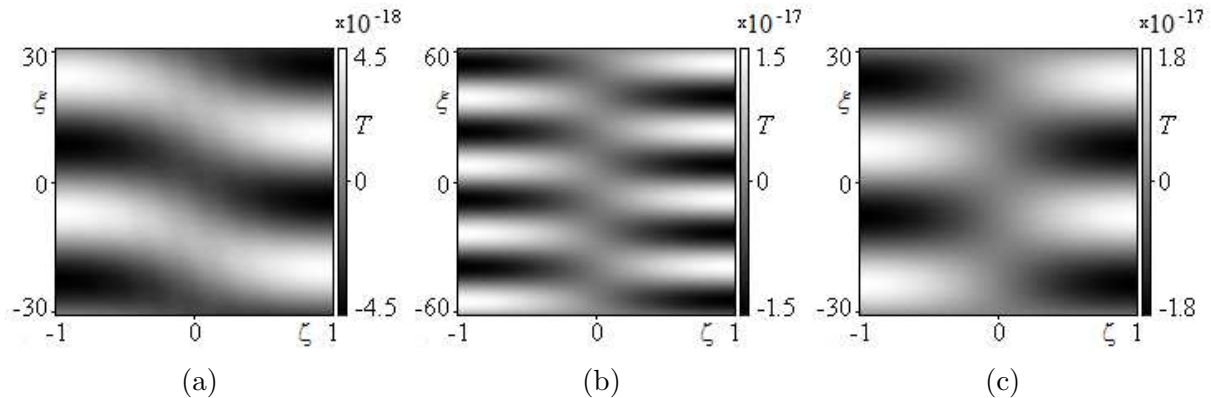


Figure 2. Distribution of the thermal perturbations in the vertical layer under $\eta = 1.51$, $Gr = 61.27 \cdot 10^{-8}$: (a) — Pe_1 , $\lambda = 1/\sqrt{2}$, $\alpha_1 = 0.2$; (b) — Pe_2 , $\lambda = 0.2$, $\alpha_1 = 0.2$; (c) — Pe_3 , $\lambda = 1/\sqrt{2}$, $\alpha_1 = 0.2$.

to formation of the convective cells (Fig. 3c). Temperature perturbations result in appearance of the thermal rolls (Fig. 4a-c).

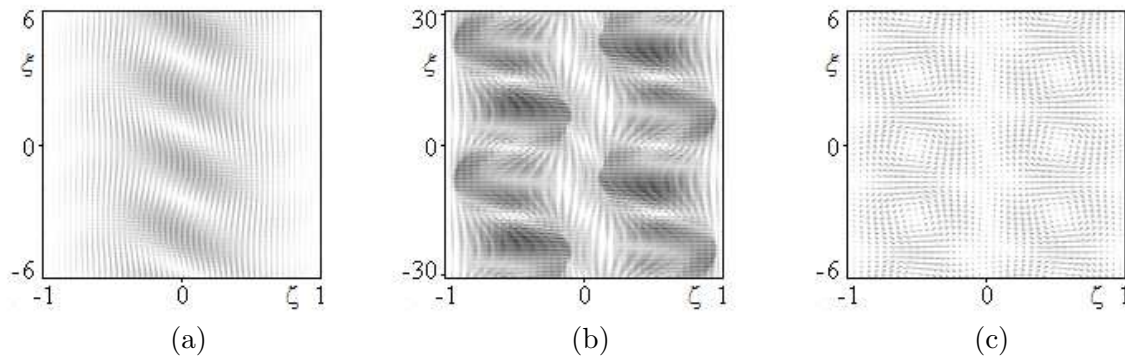


Figure 3. Distribution of the hydrodynamic perturbations in the vertical layer under $\eta = 0.0755$, $Gr = 3.06 \cdot 10^{-8}$: (a) — Pe_1 , $\lambda = 1$, $\alpha_1 = 1$; (b) — Pe_2 , $\lambda = 0.2$, $\alpha_1 = 0.2$; (c) — Pe_3 , $\lambda = 1$, $\alpha_1 = 1$.

The hydrodynamic and thermal perturbations presented in Fig. 5a, 6a are obtained for $a = 0.1$ cm, $g = g_0 \cdot 10^{-4}$, in Fig. 5b-c, 6b-c, for $a = 1$ cm, $g = g_0 \cdot 10^{-6}$. Decrease of the wave numbers causes the size reduction of the patterns. At the weak gravity the thermal rolls are changed by thermal “spots” (Fig. 6). The presence of the significant transversal velocity prevents the forming hydrothermal rolls.

It should be noted that the hydrothermal rolls are also destructed at weakening gravity force. Changing of thermal load and configuration of the perturbations wave can lead to deformation of the convective cells and more complex spatial form of the arising structures. Defining typical form of the perturbations and crisis mechanisms will allow one to solve the problem on suppression of the disturbances in the melt under obtaining ultra-pure high-quality crystals.

Acknowledgments

This work was supported by the Russian Foundation for Basic Research (project No. 14-01-00067). The first author also acknowledges the Government of the Russian Federation, for

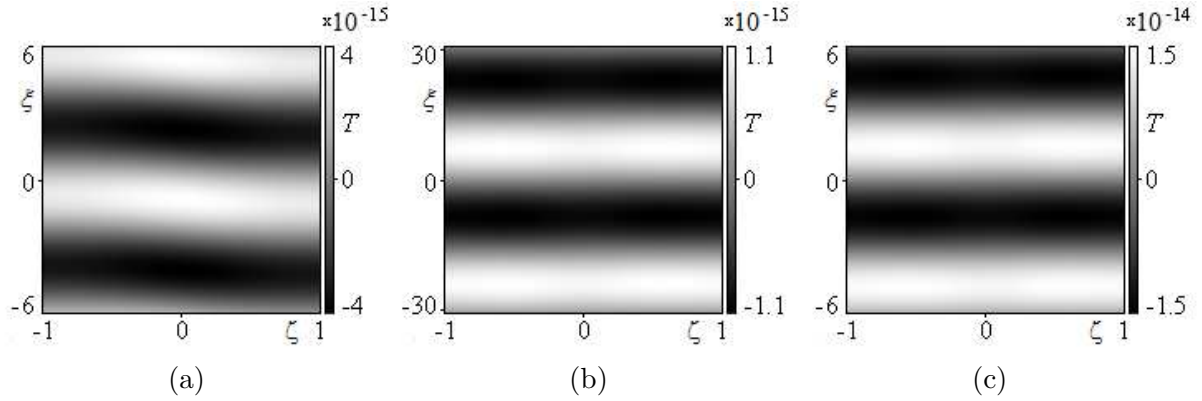


Figure 4. Distribution of the thermal perturbations in the vertical layer under $\eta = 0.0755$, $Gr = 3.06 \cdot 10^{-8}$: (a) — Pe_1 , $\lambda = 1$, $\alpha_1 = 1$; (b) — Pe_2 , $\lambda = 0.2$, $\alpha_1 = 0.2$; (c) — Pe_3 , $\lambda = 1$, $\alpha_1 = 1$.

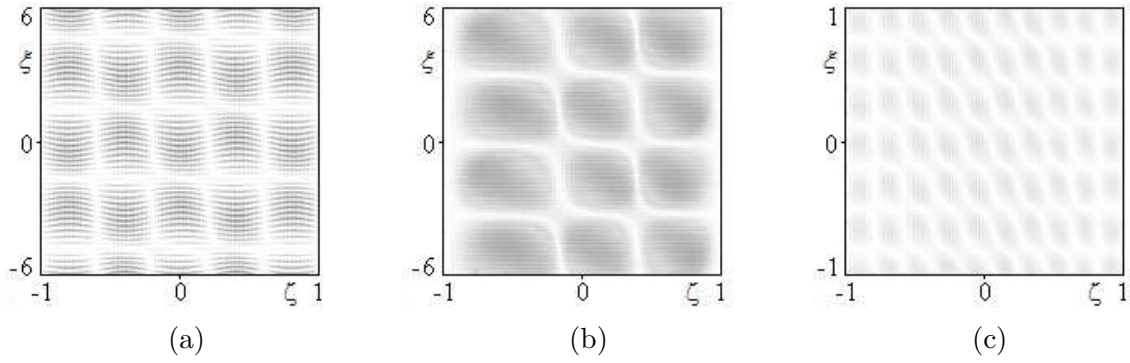


Figure 5. Distribution of the hydrodynamic perturbations in the vertical layer under $Pe = 178.57 \cdot 10^{-5}$: (a) — $\eta = 0.0755$, $Gr = 3.06 \cdot 10^{-8}$, $\lambda = 0.2$, $\alpha_1 = 1$; (b) — $\eta = 0.755$, $Gr = 30.63 \cdot 10^{-8}$, $\lambda = 0.2$, $\alpha_1 = 1$; (c) — $\eta = 0.755$, $Gr = 30.63 \cdot 10^{-8}$, $\lambda = 0.2$, $\alpha_1 = 10$.

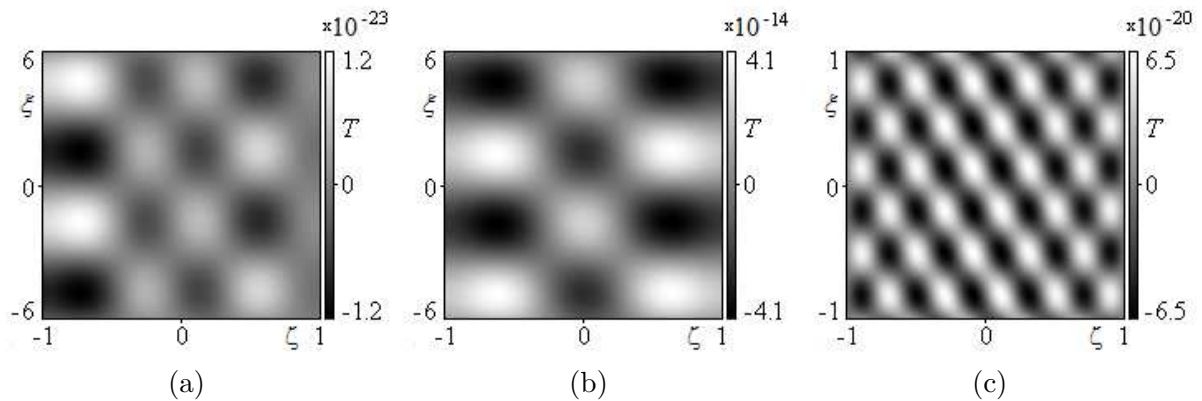


Figure 6. Distribution of the thermal perturbations in the vertical layer under $Pe = 178.57 \cdot 10^{-5}$: (a) — $\eta = 0.0755$, $Gr = 3.06 \cdot 10^{-8}$, $\lambda = 0.2$, $\alpha_1 = 1$; (b) — $\eta = 0.755$, $Gr = 30.63 \cdot 10^{-8}$, $\lambda = 0.2$, $\alpha_1 = 1$; (c) — $\eta = 0.755$, $Gr = 30.63 \cdot 10^{-8}$, $\lambda = 0.2$, $\alpha_1 = 10$.

partial support, grant for studying under the supervision of leading scientists in the Siberian Federal University (project No. 14.Y26.31.0006).

References

- [1] Pukhnachev V V 1992 *Modeling in Mechanics* **6(23)** 47-56 (in Russian)
- [2] Perera P S, Sekerka R F 1997 *Phys. Fluids* **9(2)** 376-391
- [3] Polezhaev V I, Bello M S, Verezub N A, Dubovik K G, Lebedev A P, Nikitin S A, Pavlovskii D S, Fedyushkin A I 1991 *Convective Processes in Microgravity* (Moscow: Nauka)
- [4] Polezhaev V I 2006 *Fluid Dynamics* **41(5)** 736-754
- [5] Monti R 2001 *Physics of Fluids in Microgravity* (London: Taylor & Francis)
- [6] Andreev V K, Gaponenko Yu A, Goncharova O N, Pukhnachev V V 2012 *Mathematical Models of Convection* (Berlin/Boston: Walter de Gruyter)
- [7] Müller G 1988 *Convection and Inhomogeneities in Crystal Growth from the Melt* (Berlin: Springer-Verlag)
- [8] Lebedev A P, Polezhaev V I 1990 *Uspekhi Mekhaniki* **1** 3-51 (in Russian)
- [9] Bessonov O A, Brailovskaya V A, Polezhaev V I 1997 *Fluid Dynamics* **32(3)** 379-386
- [10] Nikitin N V, Nikitin S A, Polezhaev V I 2003 *Uspekhi Mekhaniki* **4** 63-105 (in Russian)
- [11] Andreev V K, Kaptsov O V, Pukhnachev V V, Rodionov A A 1998 *Applications of Group-Theoretical Methods in Hydrodynamics* (Netherlands: Kluwer Academic Publishers)
- [12] Squire H B, 1933 *Proc. Roy. Soc. A* **142** 621-628
- [13] Godunov S K, 1961 *Uspekhi Math. Nauk* **16(3)** 171-174 (in Russian)