

Optimal sensor placement for modal testing on wind turbines

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Abstract. The mechanical design of wind turbines requires a profound understanding of the dynamic behaviour. Even though highly detailed simulation models are already in use to support wind turbine design, modal testing on a real prototype is irreplaceable to identify site-specific conditions such as the stiffness of the tower foundation. Correct identification of the mode shapes of a complex mechanical structure much depends on the placement of the sensors. For operational modal analysis of a 3 MW wind turbine with a 120 m rotor on a 100 m tower developed by W2E Wind to Energy, algorithms for optimal placement of acceleration sensors are applied. The mode shapes used for the optimisation are calculated by means of a detailed flexible multibody model of the wind turbine. Among the three algorithms in this study, the genetic algorithm with weighted off-diagonal criterion yields the sensor configuration with the highest quality. The ongoing measurements on the prototype will be the basis for the development of optimised wind turbine designs.

1. Introduction

An optimised dynamic behaviour of wind turbines is a prerequisite to ensure a reliable and economic operation over the planned lifetime. During the design of a wind turbine, the modal dynamics is profoundly analysed for a wide range of operational conditions by means of numerical simulation models in order to minimise component loads and to achieve an optimised control behaviour of the overall system. To validate the achievement of the design objectives for the dynamic behaviour of a new wind turbine, extensive experimental modal analyses on a prototype under operational conditions are conducted, see for example [1, 2, 3]. Modal parameters like eigenfrequencies, damping and mode shapes of the wind turbine are determined based on measurement data of acceleration sensors distributed over the structure, see [4]. A critical aspect that arises in experimental modal analysis is to optimally place a given set of sensors on the structure in such a way that the mode shapes in the interesting frequency range can be clearly identified.

The significance of sensor placement for experimental modal analysis is briefly illustrated by the elementary example of the cantilever beam shown in Figure 1. The objective is to identify the first and third mode shape by two acceleration sensors. Sensor positions 1 and 2 are disadvantageous as the two mode shapes cannot be separated easily from each other. Sensor



positions 3 and 4 are unfavourable as well with the third mode shape being not observable as the sensors are positioned on the nodes of that mode shape. In comparison, sensor positions 5 and 6 are adequate to identify the two mode shapes.

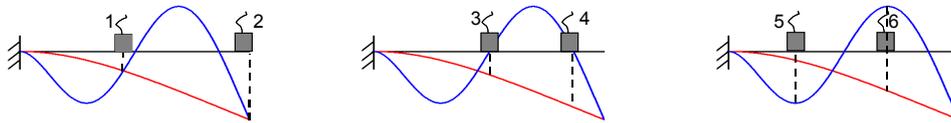


Figure 1. Comparison of sensor placements on a cantilever beam for identification of the first (red) and third (blue) mode shape.

Optimal sensor positions on a complex mechanical structure are typically found by exploiting numerical data from a finite element model, see [5, 6, 7, 8, 9]. However, for the simulation of the dynamic behaviour of wind turbines under operational conditions, multibody models are most appropriate. Due to their vastly reduced number of degrees of freedom compared to FE models, multibody models enable time-domain simulations of large overall systems. Multibody models of wind turbines are built up by elastic bodies that are described by modally reduced FE models or lumped-mass models. Nonlinearities due to large body motions are immediately included. Operational conditions are taken into account by coupling the multibody model with a wind model and the plant controller [10, 11].

In practice experimental modal analyses on wind turbines can be conducted by the method of operational modal analysis only. As a preparation of the operational modal analysis of a 3 MW wind turbine, the present contribution compares three numerical methods for finding optimal sensor positions on the wind turbine from a numerical modal analysis of the elastic multibody model of that turbine. By this, the advantage of multibody models to precisely describing the operational behaviour of the wind turbine is exploited.

The paper is organised as follows. Section 2 briefly describes the Auto Modal Assurance Criterion (AutoMAC) as a measure to determine if a set of sensor positions is able to distinguish the modes from each other. The three optimization schemes are briefly described in sections 3, 4, and 5. The optimization is conducted with the multibody model of a 3 MW wind turbine briefly described in section 6 with results presented in section 7.

2. Auto Modal Assurance Criterion AutoMAC

A modal analysis of the multibody model of the wind turbine with n_0 degrees of freedom (DoF) yields n_0 -dimensional mode shape vectors $\bar{\varphi}_i$, $i = 1, \dots, n_0$. The objective is to identify m of these mode shapes, in the following called target mode shapes, by measuring p DoF, where p is the number of sensors available. Typically, not all n_0 DoF of the numerical model can be used as measurement locations. For example rotational DoF cannot be measured by acceleration sensors, and the simulation model has DoF that are not accessible for measurement. Omitting those DoF results in a reduced set of n DoF that are considered as possible sensor locations. The objective of this study is to find p measurement DoF out of the n DoF in such a way that the m target mode shapes are identified. The target mode shape vectors φ_i , $i = 1, \dots, m$, of dimension n are collected in the (n, m) modal matrix

$$\Phi = [\varphi_1 \quad \varphi_2 \quad \dots \quad \varphi_m]. \quad (1)$$

The AutoMAC compares a set of modes with themselves and can be used to determine if the location of a set of p measurement DoF are sufficient to distinguish the modes from each

other [12]. It is defined by the (m, m) AutoMAC matrix built up with the target mode shapes from (1)

$$MAC(i, j) = \frac{(\boldsymbol{\varphi}_i^T \boldsymbol{\varphi}_j)^2}{(\boldsymbol{\varphi}_i^T \boldsymbol{\varphi}_i)(\boldsymbol{\varphi}_j^T \boldsymbol{\varphi}_j)}, \quad i, j = 1, \dots, m. \quad (2)$$

All diagonal elements of the AutoMAC matrix are equal to one since the mode shapes are correlated with themselves for the case $i = j$ while for the case $i \neq j$ the off-diagonal elements take values between 0 and 1 depending on the linear dependency between the mode shape pair $\boldsymbol{\varphi}_i$ and $\boldsymbol{\varphi}_j$. A solution that results in an AutoMAC matrix with the smallest off-diagonal elements is considered as the optimal sensor configuration.

3. Minimizing weighted off-diagonal elements

This optimisation scheme was proposed by Breitfeld [5]. Starting from a sensor set that considers all reduced n DoF of the simulation model as sensor locations, a matrix ${}^k\mathbf{B} = \text{diag}(1, \dots, 1, 0, 1, \dots, 1)$ with a zero at the k -th element is built up to describe the removal of one measurement point with index k . The AutoMAC matrix is then given by

$${}^kMAC(i, j) = \frac{(({}^k\mathbf{B} \boldsymbol{\varphi}_i)^T ({}^k\mathbf{B} \boldsymbol{\varphi}_j))^2}{(({}^k\mathbf{B} \boldsymbol{\varphi}_i)^T ({}^k\mathbf{B} \boldsymbol{\varphi}_i))(({}^k\mathbf{B} \boldsymbol{\varphi}_j)^T ({}^k\mathbf{B} \boldsymbol{\varphi}_j))}, \quad i, j = 1, \dots, m. \quad (3)$$

To measure the influence of removing point k on the AutoMAC matrix, the objective function

$$Z(k) = \sum_{i=1}^{m-1} \sum_{j=i+1}^m {}^kMAC(i, j) |i - j| \quad (4)$$

is used. It represents the weighted sum of the upper off-diagonal AutoMAC elements. Provided the mode shapes in the mode shape matrix $\boldsymbol{\Phi}$ from (1) are sorted with respect to their eigenfrequencies, the correlations between mode shapes that are wide apart by frequency have less impact on Z due to the term $|i - j|$. This weighting is important for the optimal sensor placement since mode shapes with neighbouring eigenfrequencies need to be clearly distinguished.

The procedure of removing one measurement point k by means of ${}^k\mathbf{B}$, thus evaluating the AutoMAC matrix ${}^kMAC(i, j)$ and judging the impact of the removal by evaluating $Z(k)$, is repeated for every possible measurement point. The point k with the largest value of $Z(k)$ is considered as a measurement point since his removal causes the AutoMAC off-diagonal elements to increase most. This point will be excluded from the selection process and the next iteration starts until the desired number of measurement points p is reached.

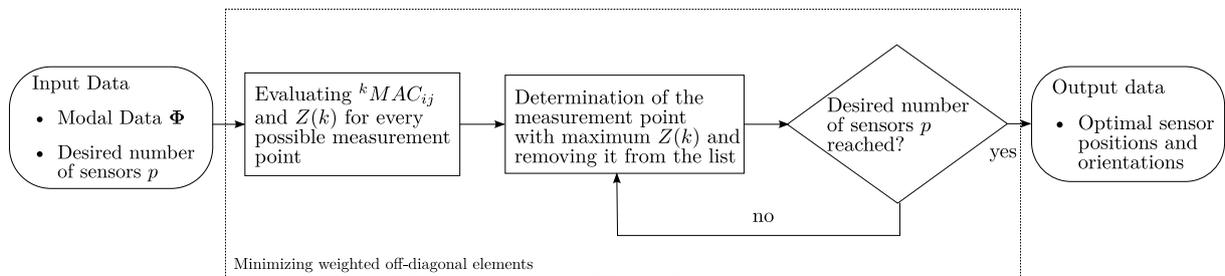


Figure 2. Algorithm for optimal sensor placements based on the minimization of off-diagonal AutoMAC elements.

4. QR decomposition

Using the QR decomposition [13] to find optimal sensor positions was first proposed by Schedlinski et al. [14]. Similar to the preceding algorithm, the QR decomposition reduces an initial sensor set comprising all n possible sensor positions to an optimal set of the desired p sensor positions. The idea of the scheme is to perform the QR decomposition on the modal matrix Φ from (1) according to

$$\Phi^T P = QR. \tag{5}$$

Here, $P \in R^{n \times n}$ is a permutation matrix, $Q \in R^{m \times m}$ an orthogonal matrix, and $R \in R^{m \times n}$ an upper triangular matrix.

Due to the definition of the QR decomposition, the element $R_{j+1,j+1}$ in R is a measure for the linear dependency between the j -th and $j+1$ -th column of the transposed modal matrix Φ^T . The higher the value of $R_{j+1,j+1}$, the more linearly dependent are those columns. In equation (5) the transposed modal matrix is considered because each column of Φ^T represents information taken from the corresponding possible measurement point. Linearly independent columns of Φ^T then correspond to linearly independent measurement signals. Based on the elements in R , the permutation matrix P is built up that arranges the diagonal elements in R in a descending order. Right-multiplying the transposed modal matrix Φ^T with P yields

$$\tilde{\Phi}^T = \Phi^T P, \tag{6}$$

where the columns of $\tilde{\Phi}^T$ are sorted by their linear dependency so that the first column represents the most linearly independent measurement point. The optimal sensor positions p are then chosen from the first p columns of $\tilde{\Phi}$. Hereby p is limited to the number of the m diagonal elements in R . If $p \geq m$, meaning that the number of desired sensor positions p exceeds the number of target mode shapes m , the first m columns of $\tilde{\Phi}^T$ are selected and the corresponding columns in Φ^T are set to zero. Next the QR decomposition is performed on the new Φ^T , and the remaining $p - m$ sensor positions are selected from the new $\tilde{\Phi}$. The scheme based on equations (5) and (6) is shown in Figure 3.

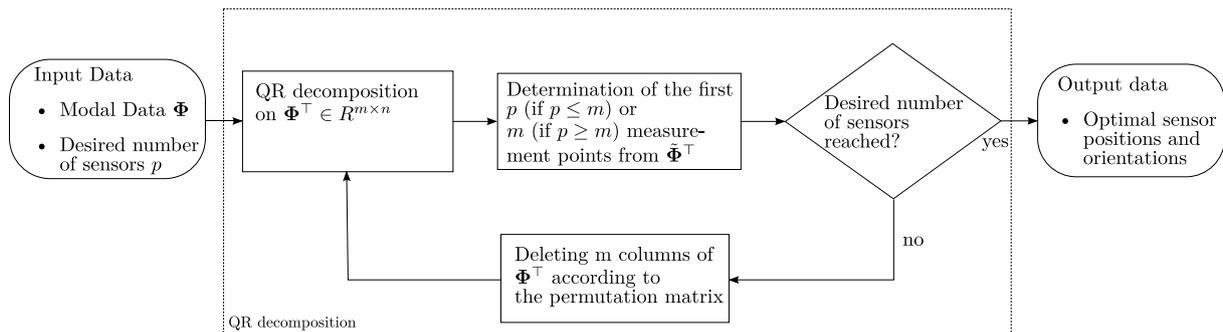


Figure 3. Algorithm for optimal sensor placements based on QR decomposition.

5. Genetic algorithm

Genetic algorithms are heuristic search algorithms based on the process of natural evolution. This process is characterised by the survival of the strongest individuals under the influence of natural selection. Hereby a combination of selection, mutation and recombination is at work to evolve those strong individuals from an initial population. Different approaches to mimic natural evolution with mathematical methods to find an optimal set of sensor positions are described in

[6, 7, 9]. In order to find optimal sensor placements with a genetic algorithm, the basic concepts coding, fitness, reproduction and recombination are briefly considered.

In a genetic algorithm one possible set of sensor locations is represented as an individual. An easy way to code an individual is to use a binary vector, so that a combination of possible sensor locations is given with

$$\mathbf{s}_p = [0 \ 1 \ 0 \ 0 \ 1 \ \dots \ 0]. \quad (7)$$

The length of \mathbf{s}_p is equal to the number of possible sensor positions n . A zero indicates that no sensor is placed on this location and a one represents a placed sensor. Thus the sum of the elements in \mathbf{s}_p is the number of desired sensors p . Another description is suggested by Anderson in [7]. Here the vector of possible sensor locations \mathbf{s}_p has the length p and contains the position numbers of the considered sensors. For the example in (7) \mathbf{s}_p then becomes

$$\mathbf{s}_p = [2 \ 5 \ \dots]. \quad (8)$$

After coding an individual by \mathbf{s}_p the corresponding fitness needs to be evaluated. The fitness represents the strength of the individual and determines its survival to the next generation. In [9] Stabb et al. define the fitness index

$$f = \frac{1}{1 + E}. \quad (9)$$

The index uses an error function E that represents the weakness of an individual. The definition of the fitness of an individual is crucial for the performance of the genetic algorithm. Therefore two different error functions E will be considered in this study.

The first error function

$$E_1 = \max(\boldsymbol{\varphi}_i^T \boldsymbol{\varphi}_j - \boldsymbol{\varphi}_{\text{ind},i}^T \boldsymbol{\varphi}_{\text{ind},j}), \quad i, j = 1, \dots, m, \quad (10)$$

evaluates the maximum error between the correlation of two mode shapes $\boldsymbol{\varphi}_i, \boldsymbol{\varphi}_j$ from the modal matrix $\boldsymbol{\Phi}$ in (1) and the correlation of the mode shapes $\boldsymbol{\varphi}_{\text{ind},i}, \boldsymbol{\varphi}_{\text{ind},j}$. The mode shapes $\boldsymbol{\varphi}_{\text{ind},i}, \boldsymbol{\varphi}_{\text{ind},j}$ are built up from the elements of $\boldsymbol{\varphi}_i, \boldsymbol{\varphi}_j$ at the sensor positions given by (8).

The second error function

$$E_2 = \sum_i^{m-1} \sum_{j=i+1}^m \text{MAC}_{\text{ind}}(i, j) e^{2-|i-j|} \quad (11)$$

calculates the weighted sum of the individuals upper off-diagonal AutoMAC elements. Similar to the valuation function (4) the correlation of neighbouring mode shapes have more impact on E_2 because of the exponential weighting.

For starting the genetic algorithm an initial population with random individuals needs to be generated. In the following, the population is coded according to (8), and the fitness of each individual is evaluated with (9). Next the natural selection criterion has to be defined that decides which individuals will pass onto the next generation. Stabb et al in [9] suggest to directly reproduce the 10% strongest individuals judged by their fitness. The remaining 90% of the new generation need to be created via crossover reproduction, mutation and direct reproduction. Crossover reproduction generates a new individual (child) by combining two randomly selected individuals (parents) from the preceding generation. For this combination a random binary crossover vector is generated. If an element of the crossover vector is 1, the corresponding

sensor position (gene) of the first parent individual is inherited to the child individual, otherwise it is inherited from the second parent individual. An example with $m = 6$ sensors reads

$$\text{parent}_1 = [2 \quad 4 \quad 7 \quad 22 \quad 25 \quad 40], \tag{12}$$

$$\text{parent}_2 = [6 \quad 8 \quad 15 \quad 30 \quad 44 \quad 50], \tag{13}$$

$$\text{crossover} = [1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0], \tag{14}$$

$$\text{child} = [2 \quad 4 \quad 15 \quad 22 \quad 44 \quad 50]. \tag{15}$$

A major part of the individuals for the new generation is generated by the above crossover reproduction. However, to ensure independence from the randomly generated initial population, the remaining individuals are produced via direct reproduction and mutation. With direct reproduction, new individuals are generated randomly similar to the generation of the initial population. With mutation a child individual is generated by altering one random gene of a parent individual.

The above process is repeated until a termination criterion is reached that defines the optimal sensor configuration. Guo et al. [6] suggest two termination criteria that stop the iteration if either of them is fulfilled. The first criterion is given as

$$|f_{\max} - f_{\text{avg}}| \leq \varepsilon, \tag{16}$$

where the difference between the maximum fitness f_{\max} and the average fitness f_{avg} of a generation needs to be smaller than a user-defined ε . The second criterion is to define a sufficient large number c so that the algorithm stops if f_{\max} did not change through the last c iterations. A schematic overview of the genetic algorithm is shown in Figure 4.

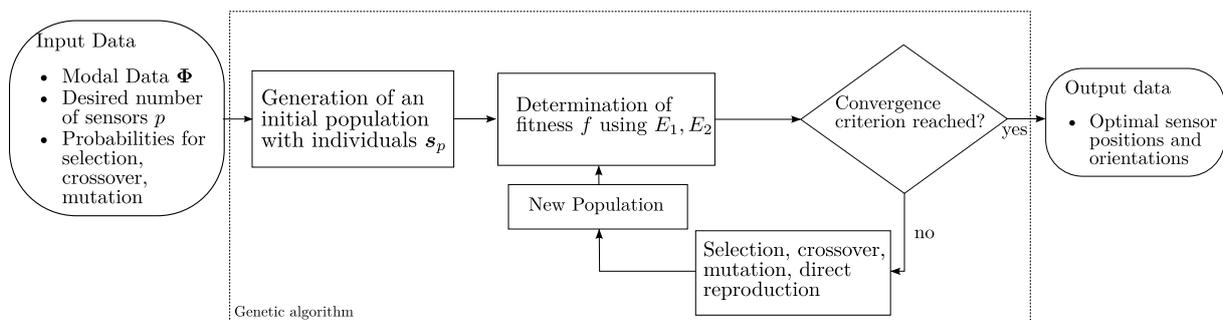


Figure 4. Algorithm for optimal sensor placements based on genetic algorithm.

6. Multibody model of a wind turbine

The optimal sensor positioning procedures are applied to the 3 MW wind turbine W2E 120/3.0fc developed by W2E Wind to Energy. A prototype of this turbine was erected near Rostock, Mecklenburg-Western Pomerania, see Figure 5. It has a rotor diameter of 120 m and is equipped with a medium speed generator.

A detailed multibody model of the wind turbine was modeled in MSC.Adams, see Figure 6. Combining a flexible tower model and flexible rotor blades modeled via lumped masses as well as a detailed drivetrain model, the turbine model comprises nearly $n_0 = 600$ degrees of freedom



Figure 5. Prototype of the W2E-120/3.0fc wind turbine.

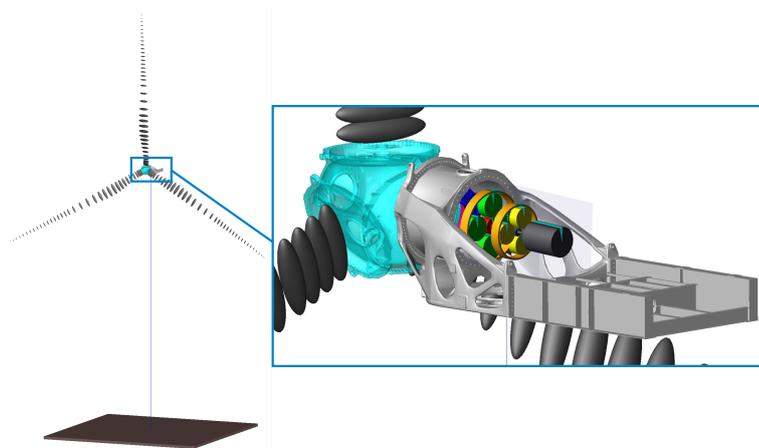


Figure 6. Multibody model of the W2E-120/3.0fc wind turbine built up in MSC.Adams.

to ensure realistic system behaviour, see [10, 11]. As part of a measuring campaign the first 19 global blade and tower mode shapes with eigenfrequencies up to 10 Hz of the 3 MW turbine are identified with $p = 19$ sensors available. The 19 target mode shapes include the first tower bending and tower torsion mode shapes as well as the first flapwise and lead-lag blade mode shapes, see Figure 7. For the problem at hand $n = 154$ of the initial $n_0 = 600$ DoF are considered as possible measurement points, omitting rotational DoF and interior DoF of the drivetrain. Additionally, the inner DoF of the tower are omitted since the first global mode shapes of tower can be identified sufficiently by measurements at the top of the tower. The major part of the reduced set of n DoF is taken by the translational DoF of the elastic rotorblades, the remaining possible sensor positions are at the nacelle, the gearbox casing and the hub.



Figure 7. Eigenmodes of the multibody wind turbine model: tower bending mode (left), flapwise blade mode (middle), lead-lag blade mode (right).

With the above conclusions the modal data $\Phi_0 \in \mathbb{R}^{n_0 \times n_0}$ provided by the multibody model is reduced to $\Phi \in \mathbb{R}^{154 \times 19}$ according to (1), comprising the 19 target mode shapes expressed through the deflection of the $n = 154$ DoF considered as possible sensor locations.

7. Results

In order to find an optimal set of sensor positions the modal data Φ were applied to the sensor positioning schemes described above. Besides Φ , the only input data required for the minimizing of the weighted off-diagonal scheme as well as for the QR decomposition is the given number of sensors, here $m = 19$. The genetic algorithm on the other hand requires additional input data. The selection of the input parameters has an impact on the performance of the scheme and needs to be defined by the user. The probabilities for mutation, selection and crossover are chosen according to the recommendations given in [6, 9]. The values c and ε and the generation size need to be adjusted in such a way that a balanced relation between generation diversity, convergence of the algorithm and computational effort is ensured. The input data used in this study for the genetic algorithm are listed in Table 1.

Table 1. User-defined input data for the genetic algorithm

Input data	Sensors []	Selection [%]	Crossover [%]	Mutation [%]	Generation size []	ε []	c []
Numeric value	19	10	80	30	100	1E-9	125

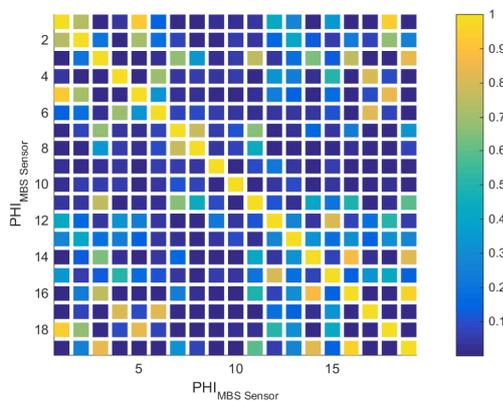


Figure 8. AutoMAC matrix generated by minimizing weighted off-diagonal elements.

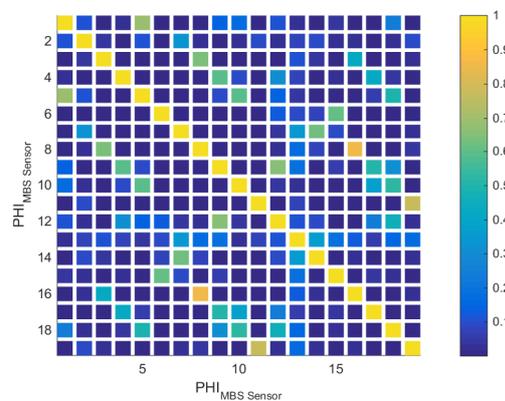


Figure 9. AutoMAC matrix generated by QR-decomposition.

The resulting sensor configurations obtained from each scheme are judged by their corresponding AutoMAC matrices, shown in Figures 8-11. The smaller the off-diagonal AutoMAC elements, the weaker is the dependency between the sensed mode shapes and therefore the higher the measurement quality.

Figure 8 shows the AutoMAC matrix obtained by the minimizing off-diagonal elements scheme. Overall, the resulting AutoMAC matrix indicates strong linear dependency between most of the target mode shapes, for example between the mode shape pairs 1-2, 1-5, 1-18 with MAC values over 0.7. Only the mode shapes 9 and 10 could be clearly identified during a measurement with the proposed sensor configuration since their MAC values show weak correlation with the other mode shapes.

The AutoMAC matrix obtained with the QR-decomposition is shown in Figure 9. Here the off-diagonal elements indicate a sensor configuration of higher quality than the one proposed by

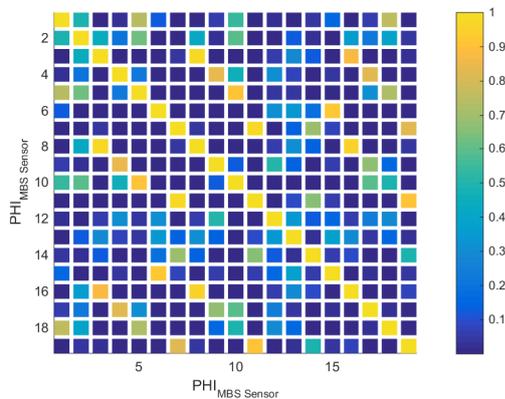


Figure 10. AutoMAC matrix generated by genetic algorithm with maximum error criteria.

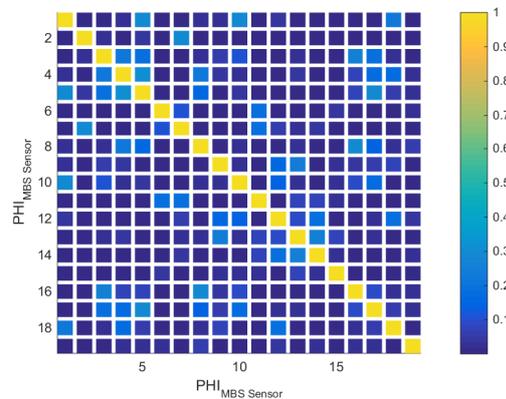


Figure 11. AutoMAC matrix generated by genetic algorithm with weighted off-diagonal criteria.

the first scheme. However, all mode shapes cannot be identified clearly since they correlate with at least one other mode shape with MAC values over 0.5. The strongest linear dependency lies between the mode pairs 8-16 and 11-19.

Next results for sensor configurations computed with the genetic algorithm are considered. The AutoMAC in Figure 10 is calculated based on the maximum error criterion while the matrix in Figure 11 is obtained with the weighted off-diagonal criterion. While the first AutoMAC matrix is equally poor as the one obtained from the scheme in Figure 8, the latter promises the best quality of the sensor positions of all considered schemes. The choice of the fitness criterion is therefore crucial to the performance of the genetic algorithm. With MAC values under 0.3, the mode shapes show little to no linear dependency between each other so that all 19 mode shapes can be clearly identified during measurement. A schematic overview of the calculated sensor positions and orientations on the wind turbine obtained by each algorithm is given in Figure 12. It is observed that the procedures with weaker performance, in particular minimizing off-diagonal elements (Figure 8) and genetic algorithm with maximum error criteria (Figure 10), tend to lump sensors in smaller areas of the structure.

The results stated above are obtained using the mode shape matrix Φ derived from the multibody model of the turbine in standstill mode. It is important to note that the dynamic characteristics of the turbine change over the operational state due to different pitch angles of the blades or centrifugal stiffening of the blades. To fully cover the impact of those effects on the sensor positioning, the above calculations need to be repeated for several operational states of the turbine. A strong advantage of multibody analyses in comparison to finite element simulations is that all operational states of the turbine can be directly simulated with one multibody model yielding Φ for each state. Depending on the impact of each operational state on the system dynamics, different sets of optimal sensor positions for the corresponding state can be calculated. This offers the possibility to switch between sensor sets during measurement to match the current operational state of the turbine and enhance the overall quality of the modal analyses. Another advantage of multibody simulations is the ability to simulate the complete system including drivetrain and controller interaction with reasonable computation time.

8. Conclusion

In preparation of an operational modal analysis of the 3 MW wind turbine W2E 120/3.0fc developed by W2E Wind to Energy, a proper set of sensor positions is determined based on

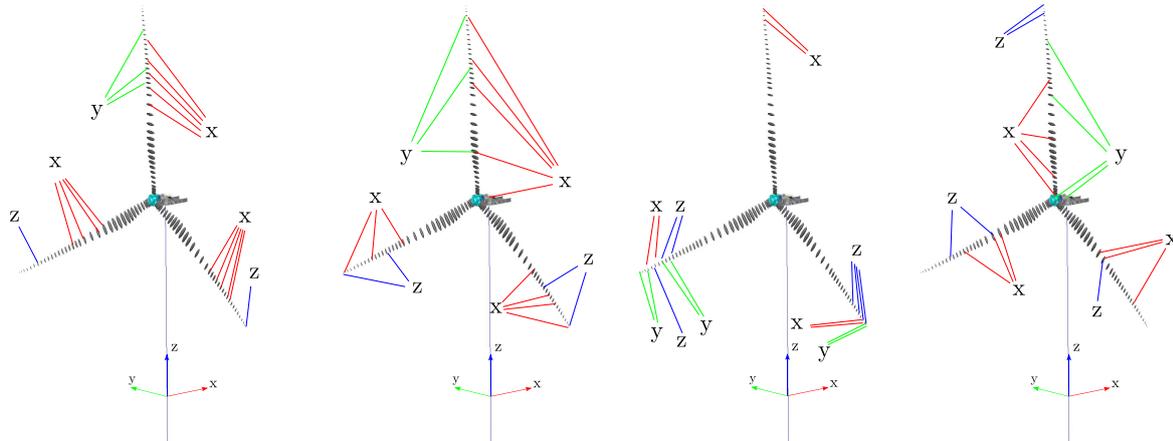


Figure 12. Schematic overview of sensor positions and orientations on the wind turbine obtained by means of (from left to right) minimizing weighted off-diagonal elements, QR-decomposition, genetic algorithm with maximum error criteria and genetic algorithm with weighted off-diagonal criterion.

a numerical modal analysis of a detailed multibody model of the turbine. In this study three different algorithms for finding optimal positions of a given number of acceleration sensors are presented. The application of the proposed schemes on modal data obtained from a multibody modal is possible without further restrictions.

Since numerical models derived by multibody analyses comprises far less DoF than a corresponding finite element model the computational effort caused by the proposed schemes is equally less, resulting in fast computations of sensor positions. Out of the three algorithms presented, the genetic algorithm with weighted off-diagonal criterion yields the sensor positions with the highest quality.

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