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PI controller design of a wind turbine: evaluation of the pole-placement method and tuning using constrained optimization

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Abstract. PI/PID controllers are the most common wind turbine controllers. Normally a first tuning is obtained using methods such as pole-placement or Ziegler-Nichols and then extensive aeroelastic simulations are used to obtain the best tuning in terms of regulation of the outputs and reduction of the loads. In the traditional tuning approaches, the properties of different open loop and closed loop transfer functions of the system are not normally considered. In this paper, an assessment of the pole-placement tuning method is presented based on robustness measures. Then a constrained optimization setup is suggested to automatically tune the wind turbine controller subject to robustness constraints. The properties of the system such as the maximum sensitivity and complementary sensitivity functions (M_s and M_t), along with some of the responses of the system, are used to investigate the controller performance and formulate the optimization problem. The cost function is the integral absolute error (IAE) of the rotational speed from a disturbance modeled as a step in wind speed. Linearized model of the DTU 10-MW reference wind turbine is obtained using HAWCStab2. Thereafter, the model is reduced with model order reduction. The trade-off curves are given to assess the tunings of the poles-placement method and a constrained optimization problem is solved to find the best tuning.

1. Introduction

Controller is an important part of the wind turbine system because it has a big influence on key wind turbine performance measures such as power quality, noise level, fatigue, and extreme loads. Controllers are also normally the first component to modify when the performance of a wind turbine is not satisfactory. Wind turbine operation has two different main regions with different objectives and control methods. In the region below the rated wind speed, namely the partial load region, the objective of the controller is to maximize power production. In this region a generator torque controller is used to change the rotational speed such that an optimal tip speed ratio is obtained. In the region above rated wind speed, namely the full load region, the task of the controller is to regulate the rotational speed and power. Normally, a PI controller is used in this region to do the regulation. Nevertheless, because the wind turbine is nonlinear and the aerodynamic gains change as the wind speed changes, a gain scheduling is required to adjusts the gains of the controller to the ever changing aerodynamic gains. Wind turbine controller tuning is generally performed with trial and error procedures, however, methods for systematic controller tuning are available at least for subsets of the controller parameters. Automatic controller tuning

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can ease and facilitate the tuning procedure especially when several rotor designs need to be evaluated as e.g. in a multidisciplinary design procedure. These methods can be divided into two main categories: pole-placement and goal-oriented optimization techniques. In this paper after an evaluation of the pole-placement method, an optimization based approach for tuning the PI controller of a wind turbine in the above rated region is presented. In the control community different methods for tuning PI controllers are available. In [17] an automatic optimum PI tuning method is presented that minimizes the time weighted integral performance criteria. In [9] a convex-concave programming is used to solve the constrained optimization problem of optimizing performance of the system subject to robustness constraints. The performance criteria are chosen to be the integrated error and the integrated absolute error and the robustness constraints are imposed by limiting maximum sensitivities. In [2] the PID design is cast into a non-convex optimization problem. Another example of tuning PI controllers is [12], where a constrained optimization is used to design PID controllers. Within the wind energy field several works have been presented on optimal tuning of the PI controllers. Hansen et al. [8] presented a method to tune the controller where the standard deviation of the blade root flapwise bending moment is minimized. Bottasso and Croce [4] and Tibaldi et al. [15] presented two methods based on goal-oriented optimization. Both investigations focus on the need to consider different objectives in the optimization to better describe the problem. All these methods are based on nonlinear time domain simulations. Tibaldi et al. [16] presented a method that aim at finding a tuning that reduces the fatigue tower subject to specific constraints. This method does not rely on time domain simulations and is based on a frequency based analysis. A similar idea of trade-off curves as in [6] is used in this work. Some robustness and performance criteria are presented and it is discussed how different measures can be achieved by changing the gains of the controller.

In this paper after describing the modeling in the section 2, the performance and robustness criteria are explained in the section 3. An analysis of the pole-placement tuning method is presented in the section 4. In section 5 a constrained optimization problem is used to find the best tuning of the PI controller with respect to some performance criteria and robustness measures.

2. Modeling

HAWCStab2 [7, 14] is used to obtain a linearized model of the DTU 10-MW reference wind turbine [3]. HAWCStab2 is an aero-servo-elastic tool for steady states computations and stability analysis of three bladed horizontal axis wind turbines. The HAWCStab2 model of the DTU 10-MW reference wind turbine is available as part of the HAWC2 model [10].

2.1. Model order reduction

HAWCStab2 returns a model with hundreds of states. The size of the model obtained directly from HAWCStab2 is too big to be used in controller analysis and design procedures. Therefore model order reduction should be used to obtain a model suitable for this application. Two methods are used to find the best size of the reduced model. The Hankel singular values [13] are used to identify the size of the reduced model, and then the residualization order reduction method [13] is used to actually reduce the model. Figure 1 shows the Hankel singular values of the model. The significance of the states on the logarithmic plot decreases smoothly and continuously. Hence, the plot does not give a clear number of states for best model order reduction, yet for states above number 40, the significances are lower than 10^{-4} and therefore probably their effect is not significant in the model. Since steady state gains and low frequencies match are important in this analysis, residualization order reduction method is used to reduce the order of the model. The reduced model can be assessed using Bode plots of the system. The Bode plots of the transfer functions from pitch angle of the blades to the rotational speed

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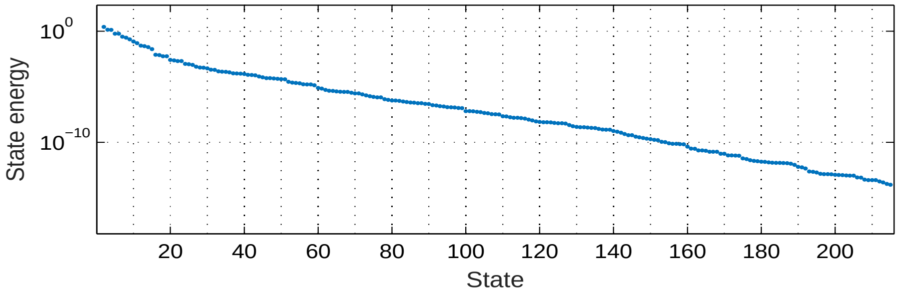

Figure 1: The first 200 Hankel singular values of the model

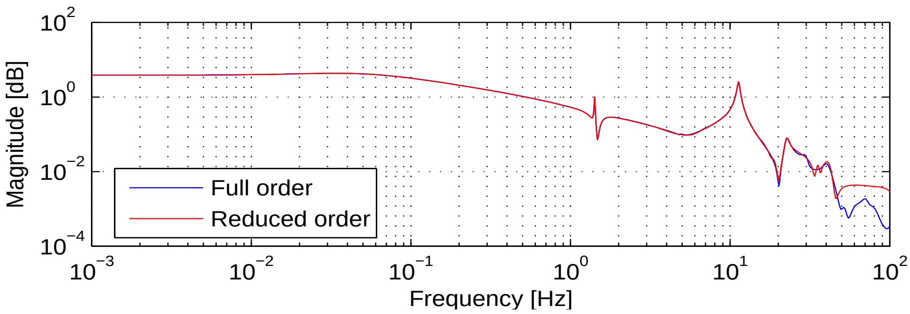

Figure 2: Bode plot of pitch to rotational speed of the reduced order models compared with the full order model

for the full order model and the reduced order model are given in Figure 2. The plot shows that the model with an order of 40 gives a satisfactory fit for frequencies below 20 Hz. At this frequency the gain of the loop transfer function becomes small and difference between the full order model and the reduced order model becomes insignificant. Besides the wind turbine model has been closed with a parametrized PI controller and the closed loop behavior of the reduced order model and the full order model has been compared for some values of PI controller gains. It has been observed that the difference between the two behaviors in terms of performance and robustness were negligible and therefore a reduced order model with 40 states was acceptable.

3. Performance and robustness measures

This section introduces the performance and robustness measures that are used to evaluate trade-offs in the tuning of the controllers. To find the trade-off curves, the open loop and closed loop transfer functions are obtained based on the parametrized PI controller and the wind turbine reduced order model. Therefore for each tuning sets, defined by values of the proportional and integral gains $k_p$and k_i, the open and closed loop transfer functions $L(j\omega)$ and $G_{cl}(j\omega)$ are found as:

$$L(j\omega) = G(j\omega)H(j\omega) \tag{1}$$

$$G_{cl}(j\omega) = \frac{L(j\omega)}{1 + L(j\omega)} \tag{2}$$

where $G(j\omega)$ is the reduced order model obtained in the previous section and $H(j\omega)$ is the frequency response of the controller. Different criteria can be used to assess performances of the closed loop system in response to disturbances and changes in the set point. In this work the response of the closed loop system to steps in the wind speed is used to calculate the

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performance of the system for disturbance rejection. Integrated absolute error (IAE) of this response is calculated for the rotational speed of the generator.

Besides the two transfer functions shown in equations (1) and (2), there are two closed loop transfer functions that are important in our analysis. They are the sensitivity function $S(j\omega)$ and the complementary sensitivity function $T(j\omega)$ found as below:

$$S(j\omega) = \frac{1}{1 + L(j\omega)} \tag{3}$$

$$T(j\omega) = \frac{L(j\omega)}{1 + L(j\omega)} \tag{4}$$

In our system configuration, the closed loop transfer function (2) and the complementary sensitivity function (4) are the same. The sensitivity and the complementary sensitivity functions are two transfer functions that reflect many interesting properties of the closed loop system. For a complete overview see [1] and [13]. The sensitivity function can be written as:

$$S = \frac{dG_{cl}/G_{cl}}{dG/G} \tag{5}$$

which shows that sensitivity function indicates how much the closed loop transfer function G_{cl} is sensitive to changes in the open loop transfer function G. Therefore, small $S(j\omega_0)$ indicates that variations in the system dynamics have small influence on the closed loop behavior of the system. Besides reciprocal of the maximum sensitivity function $1/M_s$ where $M_s = \max_\omega S(j\omega)$, is the closest distance from the loop transfer function $L(j\omega)$ to the critical point -1 on the Nyquist plot. Therefore, for our system, a small M_s indicates some variations of the system dynamics $G(j\omega)$ are acceptable and does not lead to instability of the closed loop system.

Variations in the system dynamics might lead to instability for the system, however the condition:

$$|\Delta G(j\omega)H(j\omega)| < |1 + L(j\omega)| \tag{6}$$

which can be written as:

$$\frac{|\Delta G(j\omega)|}{|G(j\omega)|} < \frac{1}{|T(j\omega)|} \tag{7}$$

guarantees that a variation of $|\Delta G(j\omega)|$ does not make the system unstable [1]. Small maximum complementary sensitivity M_t where $M_t = \max_\omega T(j\omega)$, indicates that in the above inequality the closed loop system is robust to variations in the system dynamics $|\Delta G(j\omega)|$. Therefore robustness to variations in the dynamics of the system are captured by the sensitivity and complementary sensitivity functions and one can use M_s and M_t as robustness measures of the closed loop system.

3.1. Why is robustness important?

A wind turbine is a nonlinear dynamical system and we use linearized model to approximate its behavior around an operating point to design the controller. As pitch angle is used as the gain scheduling variable, the operating point of the wind turbine is assumed to follow a specific steady state trajectory in the operating envelope of the wind turbine. In Figure 3, the solid line shows the steady state operating point of the wind turbine and the dots show the instantaneous operating point of the turbine. As it is shown in the figure, the operating point of the turbine deviates from that of the steady state which means the parameters of the model deviate from the

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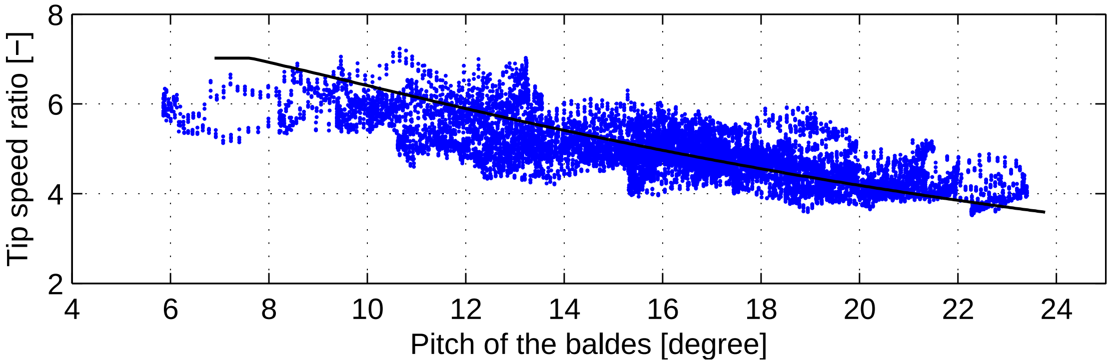

Figure 3: The steady state operating points (black-solid line), instantaneous operating points (blue dots)

ones used to design the controller. To account for these and other deviations (for more examples see [11]), it is natural to include some robustness measures into the design of the controllers. In the wind turbine control community normally gain and phase margins are used and in this paper we are using maximum sensitivity and complementary sensitivity functions. In fact the two robustness measures are closely related, see [13].

4. Analysis of the pole-placement method

Pole placement is a model based method to tune a controller. The technique consists of using a simplified linear model of the closed loop system to identify the controller parameters. The controller gains are selected such that a desired values of the poles of the model are obtained. The method can be used to tune both the PI pitch and PI torque controllers. In the case of the pitch controller the proportional and integral gains are:

$$k_P = -\frac{2\zeta_\Omega \omega_\Omega I - \partial Q_g/\partial\Omega + \partial Q/\partial\Omega}{\partial Q/\partial\theta} \tag{8}$$

$$k_I = -\frac{\omega_\Omega^2 I}{\partial Q/\partial\theta} \tag{9}$$

where ω_Ω and ζ_Ω are the desired frequency and damping ratio of the mode associated with the rotor speed regulation, I is the drivetrain inertia, $\partial Q/\partial\theta$ is the derivative of the aerodynamic torque with respect to the pitch angle or aerodynamic gain, $\partial Q/\partial\Omega$ is the derivative of the aerodynamic torque with respect to the rotational speed, here assumed to be negligible, and $\partial Q_g/\partial\Omega$ is the derivative of the generator torque with respect to the rotational speed that depends whether the wind turbine is regulated for constant torque ($\partial Q_g/\partial\Omega = 0$) or constant power ($\partial Q_g/\partial\Omega = P_r/\Omega^2$ where P_r is the rated power).

The gains that are used in this investigation, for a case where the steady-state pitch angle is zero, constant torque is selected, and ω_Ω and ζ_Ω equal 0.1Hz and 0.7 respectively, are: $k_P = 0.886$ and $k_I = 0.397$.

4.1. The simulation results

Figure 4 shows the trade-off curves obtained. The figure illustrates the IAE of the rotational speed for step changes in the wind speed. The M_t curve in the figure indicates the robustness of the system. A small M_t value, e.g. below 1.25, is desirable. Since both M_t and M_s have similar behavior for this system, only analysis with M_t are presented. The pole-placement method gives a tuning that is indicated by the black marker in Figure 4. According to the figure, the

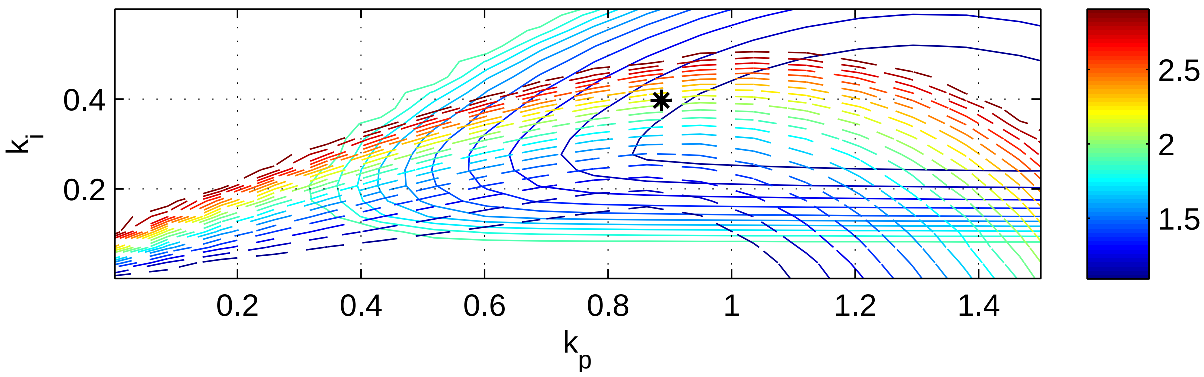

Figure 4: IAE of the rotational speed (solid), maximum complementary sensitivity M_t (dashed) and black asterisk is the tuning point of pole placement

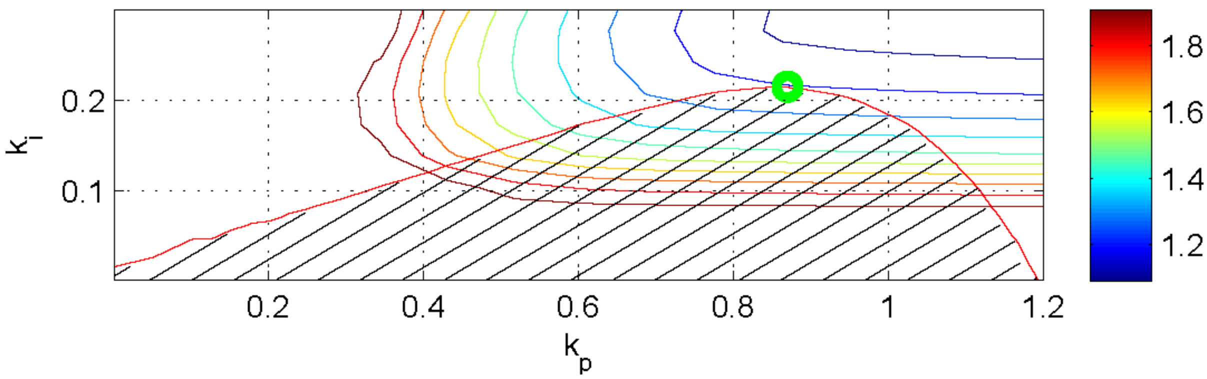

Figure 5: Contour plot of IAE overlaid with the hatched area showing $M_t \leq 1.25$

performance obtained by pole-placement is not optimal for the present cost function. Besides, for the pole-placement tuning, M_t has a value of 2.01 which indicates that robustness of the system can be improved. In fact, according to the figure, both the performance and the robustness can be improved by reducing k_i and keeping the same k_p until the best performance that can be achieved for $k_i = 0.28$. For this tuning M_t is 1.5, which indicates a better robustness level than the pole-placement tuning. However, if a better robustness measure than this point is desired, performance should be sacrificed in favor of robustness. In the next section a constrained optimization framework is introduced that automatically seeks the best performance for the system while respecting the pre-defined robustness levels.

5. Optimization setup

For the optimization setup, IAE of the rotational speed of the system is chosen as the cost function. The constraint is defined as an upper bound on M_t. The PI controller parameters, namely the proportional and the integral gains (k_p and k_i) are the design parameters of the optimization:

$$\min_{k_p,k_i} \int_0^T |e(\tau)| d\tau, \qquad \text{Subject to:} \quad M_t \leq 1.25 \tag{10}$$

in which T is the time the simulation reaches steady state. Figure 5 shows the IAE curve where the hatched area shows $M_t \leq 1.25$. As it is seen in the figure the minimum value of IAE is

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achieved when M_t is at the constraint and gets a values of 1.25 (the red contour curve). From the figure it is also clear that the problem of minimum of IAE subject to robustness constraints is equivalent to maximum of k_i subject to the same constraints, as mentioned in [1].

The optimum point is achieved for $k_p = 0.8627, k_i = 0.2077$. The optimization problem is non-convex and difficult to solve [2]. One way of solving the optimization problem is to use SWORD [5], a package especially designed to optimize PI controllers subject to robustness and noise sensitivity constraints.

6. Conclusions

A high order model was obtained using HACWStab2 and model order reduction was used to find an appropriate model for controller design. An evaluation of the pole-placement tuning method was given and it was shown that, for the example presented in this paper, it was possible to gain better performance and robustness for the system by reducing the integral gain of the controller. Then a constrained optimization problem was used to find the best tuning of the PI controller for the wind turbine. The optimum tuning of the controller was obtained by minimizing IAE of the output of the system, namely the error on the rotational speed regulation, subject to robustness measures, namely upper bound on M_t.

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