Multi-Objective Random Search Algorithm for Simultaneously Optimizing Wind Farm Layout and Number of Turbines

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Abstract. A new algorithm for multi-objective wind farm layout optimization is presented. It formulates the wind turbine locations as continuous variables and is capable of optimizing the number of turbines and their locations in the wind farm simultaneously. Two objectives are considered. One is to maximize the total power production, which is calculated by considering the wake effects using the Jensen wake model combined with the local wind distribution. The other is to minimize the total electrical cable length. This length is assumed to be the total length of the minimal spanning tree that connects all turbines and is calculated by using Prim's algorithm. Constraints on wind farm boundary and wind turbine proximity are also considered. An ideal test case shows the proposed algorithm largely outperforms a famous multi-objective genetic algorithm (NSGA-II). In the real test case based on the Horn Rev 1 wind farm, the algorithm also obtains useful Pareto frontiers and provides a wide range of Pareto optimal layouts with different numbers of turbines for a real-life wind farm developer.

1. Introduction

Among various tasks in designing a wind farm (WF), layout optimization is the one of crucial importance. This task aims to find the best solution of how many wind turbines (WTs) to install and where to install them, according to a single or multiple objective(s), and subjected to various constraints [1]. This problem is called wind farm layout optimization (WFLO) and has been studied by many researchers in the past two decades [2].

These studies can be divided into two types, depending on how the wind farm layout is modelled. One type discretizes the wind farm area into grids/cells and places turbines only at certain points of these grids/cells, thus can be called as discrete WFLO. This type of formulation was first proposed by Mosetti's seminal work on WFLO [3], in which a square area of the size 2km by 2km was divided into 10 by 10 grids with WTs placed only in the centers of these grids. By using this formulation, the proximity constraints between WTs and WF boundary constraints are automatically satisfied, and binary-coded genetic algorithm (GA) can be conveniently applied. Similar formulation has been used in many followed studies [4, 5].

The other type models the locations of turbines as continuous variables and can be called as continuous WFLO. Many studies has adopted this formulation, such as Rivas's study using simulated annealing (SA) [6], Ozturk's study using a heuristic method [7] and Feng's study using random search

(RS) [8]. While the discrete WFLO studies can optimize the wind farm layout and number of turbines at the same time, their continuous counterparts usually assume a fixed number of turbines.

Furthermore, the majority of these studies [3-8] formulate the problem as a single objective optimization problem, while the wind farm developers in real life usually face multiple objectives, which are often conflicting to each other. Recently several studies have addressed this issue using existing algorithms. Kwong et al. [9] used nondominated sorting genetic algorithm II (NSGA-II) [10] to maximize energy and minimize noise, and Chen et al. [11] applied multi-objective genetic algorithm (MOGA) to maximize the farm efficiency and minimize the cost per unit power. Both of these two studies assumed a fixed number of turbines.

In this study, we develop a new algorithm to optimize the wind farm layout and number of turbines simultaneously using the continuous WFLO formulation. It is developed based on our previous study on single-objective RS algorithm [8] and can be called as multi-objective random search (MORS). Both ideal case study using simple wind condition and real case study based on real wind condition are conducted. Problems with a fixed and unfixed number of WTs are both tested. It is also compared with NSGA-II in the ideal case study with a fixed number of WTs.

2. Problem formulation

2.1. General problem

A general multi-objective WFLO problem can be formulated as:

$$\left.\begin{array}{lll} \min & f_m(\boldsymbol{X}, N_{wt}), & m = 1,2,\dots,M, \\ \text{subject to:} & g_k(\boldsymbol{X}, N_{wt}) \geq 0, & k = 1,2,\dots,K; \\ & h_l(\boldsymbol{X}, N_{wt}) = 0, & l = 1,2,\dots,L; \\ & N_{wt}^{(L)} \leq N_{wt} \leq N_{wt}^{(U)}; & \\ & \boldsymbol{X}^{(L)} \leq \boldsymbol{X} \leq \boldsymbol{X}^{(U)}. & \end{array}\right\} \qquad (1)$$

where N_{wt} denotes the number of WTs, $\boldsymbol{X}$ is a $N_{wt} \times 2$ matrix defining their locations, f_m is the mth objective function, g_k is the kth inequality constraint function, h_l is the lth equality constraint function, $N_{wt}^{(L)}$ and $N_{wt}^{(U)}$ are the lower and upper bounds of N_{wt}, and $\boldsymbol{X}^{(L)}$ and $\boldsymbol{X}^{(U)}$ denote the lower and upper bounds for $\boldsymbol{X}$.

Common objective functions for WFLO include annual energy production (AEP), cost of energy, profit, noise emission, and so on. Typical constraints that can be modelled as inequality constraint functions include wind farm boundary and wind turbine proximity, while equality constraints usually do not show in WFLO.

Note that in its general form as described by Eq. (1), WFLO problem is a multi-objective, constrained, mixed-integer nonlinear optimization problem with an unfixed number of design variables. For modern large WFs, the number of WTs can be hundreds. Considering a large WF with $N_{wt}^{(L)} = 80$ and $N_{wt}^{(U)} = 100$, the number of proximity constraints between any two WTs is $N_{wt}(N_{wt} - 1)/2$, and the number of boundary constraints for each WT is N_{wt}. Thus the number of continuous design variables is between 160 and 200, while the number of inequality constraints, excluding other possible constraints, is between 3240 and 5050. For such a highly constrained high dimensional nonlinear optimization problem, it is usually impossible to find the global optimal solution(s) with classical analytical optimization techniques, thus most of the published works in WFLO apply heuristic optimization methods, such as GA [3,4], Monte Carlo [5], SA [6] and RS [8].

For multi-objective optimization problems, there are usually a group of solutions that can be found as the Pareto optimal solutions, which are also called as the Pareto frontier. A solution is called Pareto optimal, or non-dominated, if none of its objective functions can be further improved without degrading some of the other objective functions. If a solution satisfies all constraints and bounds, it is called as feasible solution, otherwise it is called as infeasible solution [12].

In this study the constrained-dominance as defined in [10] is used to compare solutions. According to this definition: a feasible solution always dominates an infeasible solution; among two infeasible solutions, the one with smaller overall constraint violation dominates; among two feasible solutions, the one that is better for at least one objective function and equal or better for all other objectives dominates.

2.2. Wind farm modelling

Modelling the power production accurately is essential in the context of WFLO, since the most important function of a WF is to generate power. This requires the appropriate modelling of wind, wake effects and power production.

The common practice models wind with sector-wise Weibull distributions and wind rose. In general the wind condition at a given height can be described by wind speed v and wind direction θ, and its distribution can be modelled by a probability density function (PDF) $pdf(v,\theta)$. In our previous study [13], a joint distribution of wind speed and wind direction has been proposed, which can be used to obtain such kind of PDF based on real measurement data. In this study, we consider two kinds of wind cases. The first case is called ideal wind case which assumes a constant North wind at 8 m/s, while the second case is called real wind case which uses the proposed joint distribution obtained from the wind measurement in Horns Rev as shown in Figure 1.

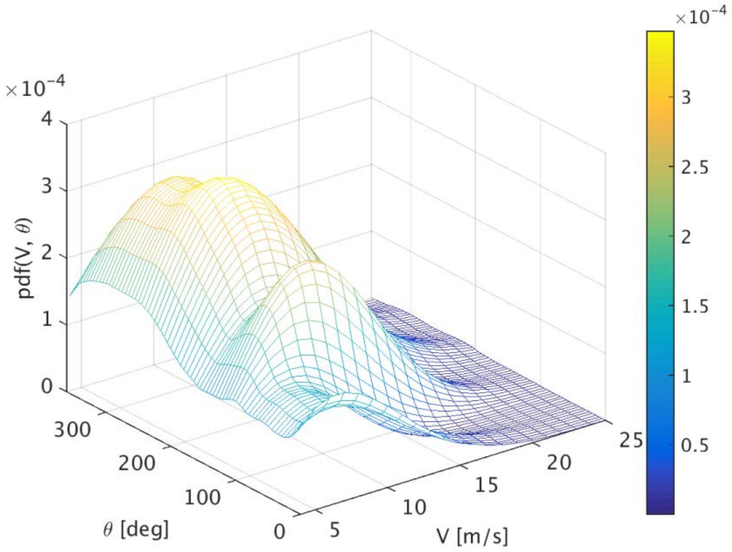

Figure 1. Joint distribution of wind speed and wind direction at Horns Rev [13]

Wake effect rises when an upwind turbine extracts energy from the wind and forms a wake that impacts the downwind turbine. It leads to reduced wind speed and increased turbulence and thus is crucial for WF modelling. Due to the nature of optimization problem, engineering wake models are commonly used in WFLO because of their low computational costs. In this study, as in most of the published WFLO studies, we use the Jensen wake model [14, 15].

For a given inflow condition (v,θ), we can first calculate the wake effects between WTs and then get the wind speed at each WT $\bar{v}_i$, then the power generated by each WT can be calculated based on the manufacturer provided power curve $P=P(v)$. Combined with the PDF of wind, we can write the expected total power produced by the WF as

$$P_{tot}=\sum_{i=1}^{N_{wt}}\iint P(\bar{v}_i)\times pdf(v\,,\theta)dvd\theta \tag{2}$$

The detailed procedure for calculating power production can be found in [8].

2.3. Objective functions

Two objectives are considered in this study, which include maximizing the expected total power (P_{tot}), and minimizing the total electrical cable length (CL). The total electrical cable length of a given WF is defined as the total length of the minimal spanning tree that connects all WTs. It can be easily calculated by Prim's algorithm [16], which is a widely used greedy algorithm to find a minimal spanning tree for a weighted undirected graph. Thus, we can write the objective functions as follows:

$$\left.\begin{aligned} f_1(\boldsymbol{X}, N_{wt}) &= -P_{tot}(\boldsymbol{X}, N_{wt}), \\ f_2(\boldsymbol{X}, N_{wt}) &= CL(\boldsymbol{X}, N_{wt}). \end{aligned}\right\} \tag{3}$$

Note that maximizing P_{tot} has been changed into minimizing $-P_{tot}$, in order to make the WFLO problem suitable for being solved by some existing algorithms, such as NSGA-II.

2.4. Constraint functions

Two kinds of constraints are considered in this study. The first kind is the boundary constraint. It requires that all the WTs are inside the WF boundary, which can be denoted as the feasible area $S_{feasible}$. This area can be usually modelled as a simple polygon, i.e., a polygon with no self-intersecting edges, and specified by a series of coordinates of its vertices. Assuming the coordinates of the ith WT are $(x_i,\ y_i)$, we can write the boundary constraints for each WT as the first N_{wt} inequality constraints with:

$$g_k = \begin{cases} 0, & \text{if } (x_i, y_i) \in S_{feasible}, \\ -\infty, & \text{if } (x_i, y_i) \notin S_{feasible}, \end{cases} \quad \text{for } i = 1, 2, \ldots, N_{wt} \text{ with } k = i. \tag{4}$$

Note that in the above boundary constraints, any WT that is outside of $S_{feasible}$ will lead to its corresponding constraint violation being infinitely large, meaning that we are totally intolerant to layouts that violate the boundary constraints.

The second kind of constraints are the WT proximity constraints, which require the distance between any two WTs is greater than or equal to a given minimal distance requirement D_{min}. Since there are $N_{wt}(N_{wt}-1)/2$ pairs of WTs, we can write the proximity constraints as the left $N_{wt}(N_{wt}-1)/2$ inequality constraints (following the N_{wt} boundary constraints) with:

$$g_k = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} - D_{min}, \quad \begin{array}{l} \text{for } i = 1, \ldots, N_{wt} - 1 \text{ with } j = i + 1, \ldots, N_{wt} \\ \text{and } k = N_{wt} + 1, \ldots, N_{wt}(N_{wt} + 1)/2. \end{array} \tag{5}$$

With the above definitions, we could easily verify whether a given inequality constraint $g_k \geq 0$ is satisfied. If it is not satisfied, the value $-g_k$ will give the degree of the constraint violation.

3. Optimization algorithms

In our previous studies, the RS algorithm has been developed for single-objective WFLO and applied in both offshore WFs [8, 13] and WFs in complex terrain [17]. In this section, we first present MORS that is extended from the single-objective RS and then give a brief introduction to NSGA-II, which we will compare with in the case studies.

3.1. MORS

Unlike the population based search methods, such as the widely used GA [10], MORS is a single solution search method. At each step, a new feasible solution is generated by adding, or removing, or moving one turbine randomly to the previous solution. This new solution is then compared with the current Pareto optimal solutions for non-dominance check, and the Pareto optimal solution set is then updated accordingly. This step is iteratively repeated until some stop conditions are met. The procedure of this algorithm is shown in the following pseudo code:

Algorithm 1: Pseudo code of MORS for WFLO

```
Initialization:
    Select an initial feasible solution $\boldsymbol{S}_0$ and evaluate its objective functions;
    Set current solution $\boldsymbol{S}_{now} = \boldsymbol{S}_0$, set initial Pareto Optimal Set (POS) $= \{\boldsymbol{S}_{now}\}$.
While stop conditions are not true:
    1. Generate a new feasible solution $\boldsymbol{S}_{new}$ by taking one of the 3 actions to $\boldsymbol{S}_{now}$:
       (a) Add a WT at a feasible random position (with probability $p_a$);
       (b) Remove a randomly chosen WT (with probability $p_r$);
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        (c) Move a randomly chosen WT to a random position (with probability $p_m$).
     2. Evaluate objective functions of the current new solution $\boldsymbol{S}_{new}$.
     3. Make non-dominance check & POS update:
        Compare $\boldsymbol{S}_{new}$ with all solutions in POS in the objective space:
        If $\boldsymbol{S}_{new}$ is dominated by one of the solutions in POS:
          Continue;
        Elseif $\boldsymbol{S}_{new}$ dominates one or several of the solutions in POS:
          Remove the solution(s) dominated by $\boldsymbol{S}_{new}$ from POS,
          Add $\boldsymbol{S}_{new}$ to POS, set $\boldsymbol{S}_{now} = \boldsymbol{S}_{new}$;
        Else:
          Add $\boldsymbol{S}_{new}$ to POS, set $\boldsymbol{S}_{now} = \boldsymbol{S}_{new}$.
        End if
  End While
  POS gives the set of Pareto optimal solutions.
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We should note that in Step 1 of Algorithm 1, i.e., generating a new feasible solution based on the current solution, we always make sure the new solution satisfies all the constraints and bound requirements. This is easily done by checking the relevant WT's position with the WF boundary and its distances with all other WTs. This relevant WT is the new WT in action (a) and the chosen WT in action (b). Also the bounds on the number of WTs are checked when adding/removing a WT. When using MORS, we can set the stop condition as when the total number of evaluations reaches a given number E_{max}. Thus, this algorithm has only three parameters to set: E_{max}, p_a and p_r (note that $p_m = 1 - p_a - p_r$) and is simple, intuitive and easy to implement.

3.2. NSGA-II

NSGA-II is a famous multi-objective genetic algorithm developed by Deb et al. in 2002 [10]. It is a fast nondominated sorting based approach with $O(MN^2)$ computational complexity (where M is the number of objective functions and N is the population size). Like all GAs, it maintains a population of solutions and evolves it by a certain number of generations.

In each generation, an offspring population of the same size N is produced from the parent population by using the usual binary tournament selection, crossover, and mutation operators. Then the combined population (parent and offspring populations) is sorted according to each solution's nondomination level. This level is evaluated based on the objective functions and constraint violations, according to the definition of constrained-dominance as introduced before (in section *2.1*). Among the solutions with the same nondomination level, a metric of crowding distance is used to sort them, which can preserve diversity of solutions in the objective space. Then the best N solutions in this combined population are selected to be the parent population for the next generation. For detailed procedure, one can refer to the original paper [10].

This algorithm has gained popularity since its birth and been applied to tackle various engineering optimization problems, such as water distributing network design [18], hydro-thermal power scheduling [19], capacitor placement in distribution circuits [20] and WFLO [9].

4. Ideal test case

We choose the widely studied Horns Rev 1 offshore WF to test the proposed algorithm. This WF is located in Denmark and has a rated capacity of 160 MW, composed of 80 Vestas V80 2MW WTs. The detailed layout and the characteristics of WT can be found in [8]. In the test case, the WF boundary as shown in the original Horns Rev 1 WF is considered and the minimal distance requirement between any two WTs is set as $D_{min} = 480\ m$, i.e., 6 times of the rotor diameter. We consider WFLO defined in Eqs. (1-5) under the ideal wind case (constant North wind at 8 m/s) as the ideal test case.

4.1. WF with a fixed number (80) of WTs

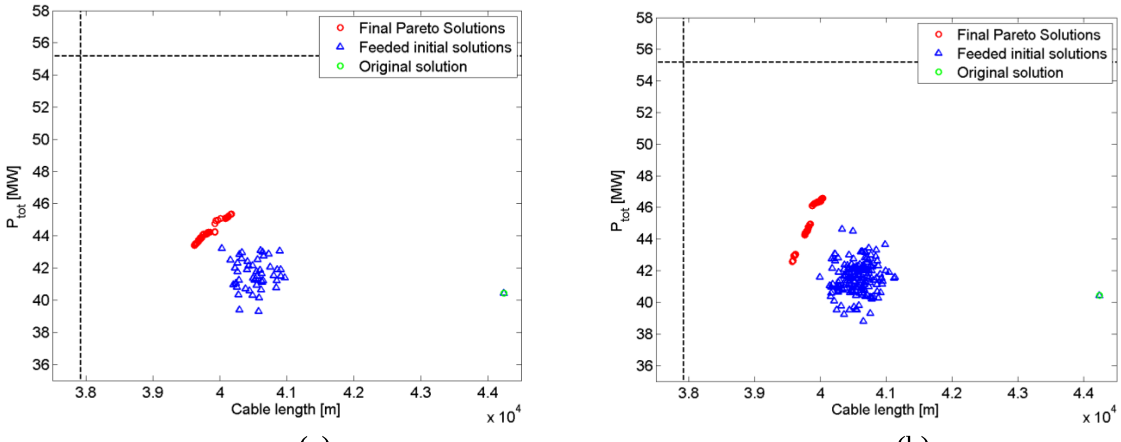
Since the real value coded NSGA-II doesn't have the ability to count the variable number of design variables, we first construct a WFLO problem with a fixed number of WTs, i.e., $N_{wt}^{(L)} = N_{wt}^{(U)} = 80$.

The NSGA-II code we use in this study is based on the matlab code written by Seshadri [21]. In this code, simulated binary crossover (SBX) and Polynomial mutation are implemented. The original code considers only unconstrained optimization problem, and we have extended it to solve constrained problem. In the initial experiments, we found that NSGA-II fails to find feasible solutions when starting from random solutions, even with a large population size and a lot of generations. This is due to the highly constrained nature of the WFLO problem. Therefore, we decide to feed in half of the initial population with feasible solutions. Those solutions are layouts obtained by applying multiple times of random move actions to the original WF layout. In each random move action, a randomly chosen WT is moved to a random position that satisfies all the constraints and bounds. The parameters for NSGA-II are shown in Table 1.

Table 1. Parameters for NSGA-II

Parameter	Value	Note
Crossover probability	0.9	
Mutation probability	1/N	(N is the number of design variables)
Distribution index for crossover operator	20	
Distribution index for mutation operator	20	
Population size	100 or 320	
Generation	500	

The final pareto solutions found by NSGA-II after 500 generation with small (100) and large (320) population sizes are shown in Figure 2. The feeded initial solutions including the original solution, i.e., the original Horns Rev 1 WF's layout, are also shown. Note also that in Figures 2 and 3, the horizontal dash line marks the upper bound for P_{tot} (the ideal total power with no wake effect, $55.20\, MW$), while the vertical dash line marks the lower bound for CL (the minimal total cable length, i.e., $D_{min} \times (N_{wt} - 1) = 37920\, m$).

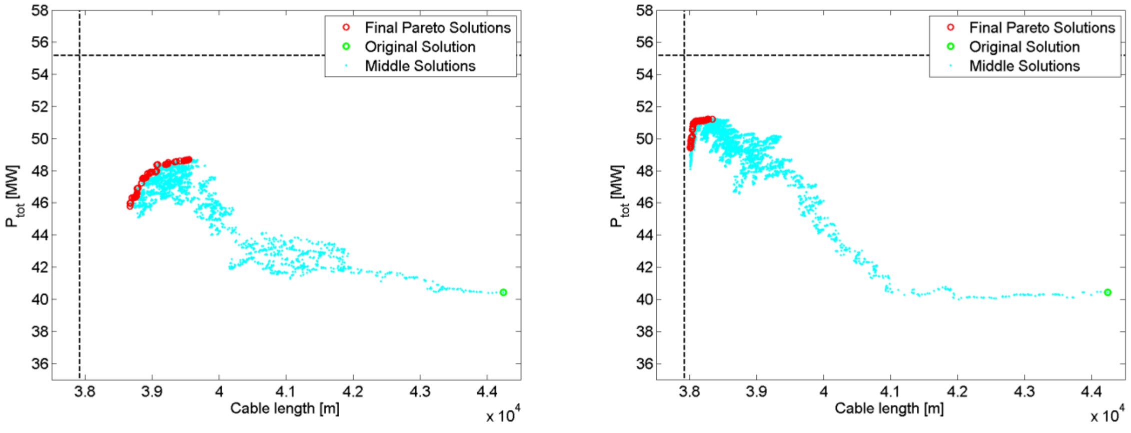


(a) (b)

Figure 2. NSGA-II results for ideal test case: 500 generations with population size: (a) 100, (b) 320

Note that among solutions of the population in the last generation, only the feasible solutions that are ranked at 1st level by non-dominance are shown, since these are the final Pareto optimal solutions found by NSGA-II. And the numbers of solutions are 94 and 128 for the results shown in Figure 2 (a) and (b) respectively.

To compare with NSGA-II, we solve the same problem with MORS, with parameters $p_a = 0.0$, $p_r = 0.0$, $p_m = 1.0$ and the maximal number of evaluations as 10000 and 160000. The results are presented in Figure 3, where the middle solutions, i.e., all the solutions that have once been included in the Pareto optimal set, are also included to show the evolving tracks of MORS.



(a) (b)

Figure 3. MORS results for ideal test case with E_{max} set as: (a) 10000, (b) 160000

To better visualize the difference between the results found by MORS and NSGA-II, we present the four sets of Pareto optimal solutions in the figure below.

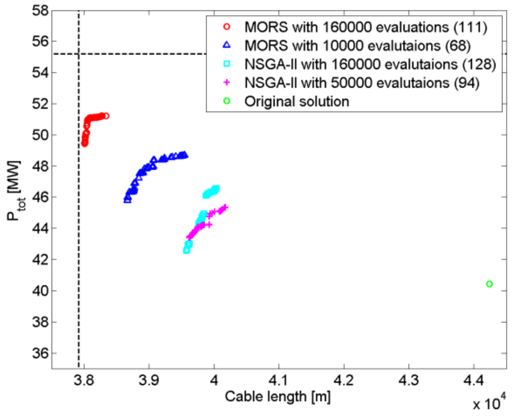


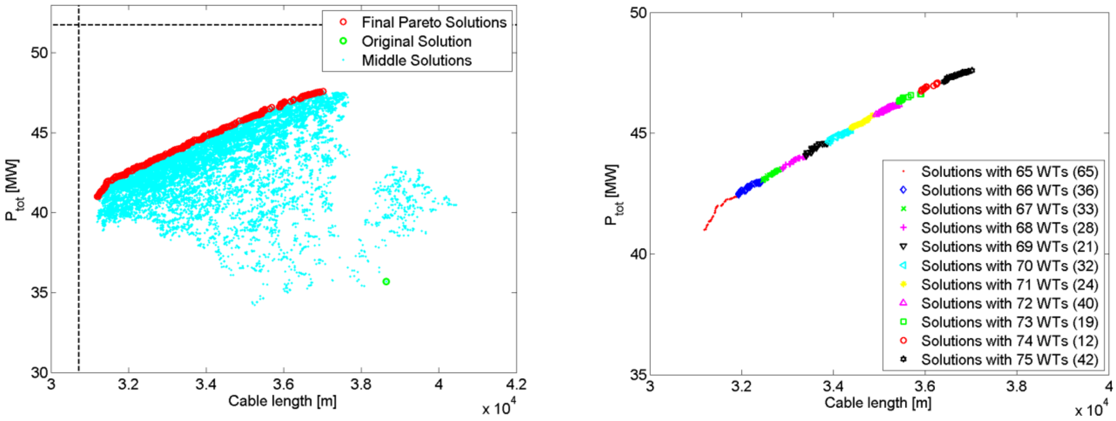
Figure 4. Comparison between different results obtained by MORS and NSGA-II (the number in brackets shows how many solutions are found in the final Pareto frontier)

We could clearly see that while both methods obtained better solutions than the original solution, MORS outperforms NSGA-II quite significantly. Even MORS with only 10000 evaluations manages to find much better Pareto optimal solutions than NSGA-II with 160000 evaluations. When increasing the maximal evaluations to 160000, MORS succeeds in finding Pareto frontier that approaching the lower bound of one of the objectives: CL. It's also worthy to note that the upper bound for P_{tot} is impossible to approach, since wake effects are quite profound for WT under wind speed around 8 m/s and there is no way to fully avoid wake effects for a WF with 80 WTs staggered in such a limited area. Thus, we might conclude that the Pareto frontier found by MORS with 160000 evaluations is actually approaching and gets quite close to the true Pareto frontier of this WFLO problem. The other thing we can note is that the number of final Pareto optimal solutions found by MORS and NSGA-II are in the same order and also spread out in a similar range in the objective space.

From the above comparison, we find that MORS works better than NSGA-II for high dimensional, highly constrained nonlinear optimization problems such as WFLO and it can find much better solutions for multi-objective WFLO. Also, MORS has the additional capacity of optimizing layout and number of WTs simutaneously. Therefore, we choose to use MORS only in the test cases below.

4.2. WF with an unfixed number (65-75) of WTs

To test the ability of dealing with an unfixed number of WTs, we consider the same problem but with $N_{wt}^{(L)} = 65$ and $N_{wt}^{(U)} = 75$, and set the initial solution as the original WF's layout with the first 70 WTs. The result obtained by MORS with $p_a = 0.1$, $p_r = 0.1$, $p_m = 0.8$ and $E_{max} = 20000$ is shown in Figure 5. Note that the dash lines in Figure 5 (a) mark the upper bound of total power and the lower bound of cable length.



(a) (b)

Figure 5. MORS result for ideal test case with an unfixed number of WTs: (a) evolving track and final solutions; (b) classification of solutions according to the number of WTs (number of solutions shown in the brackets)

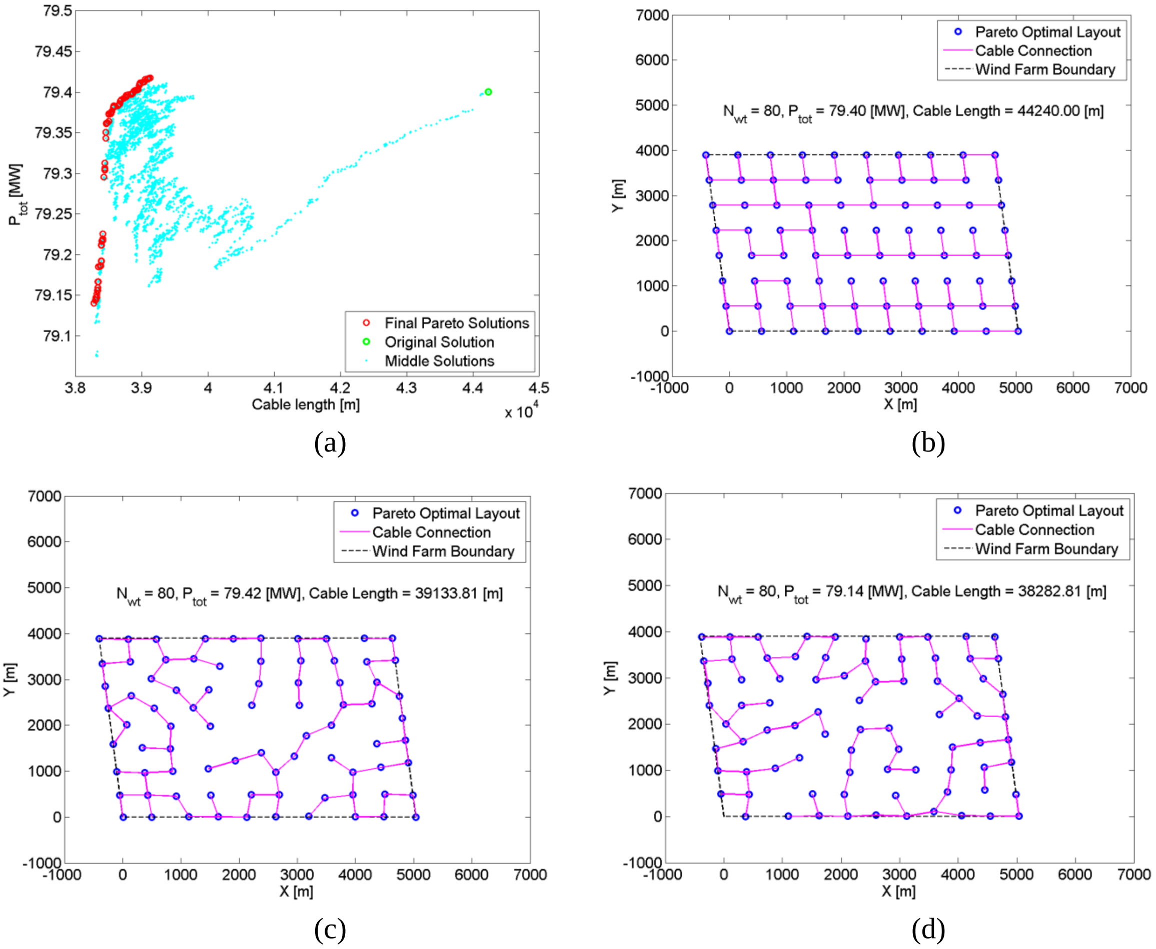
We can see that MORS is capable of finding a quite diverse Pareto frontier, which is composed of solutions with different numbers of WTs. Also for a given number of WTs, it manages to find a range of different layouts that are Pareto optimal with regards to the two objectives, i.e., power and cable length, thus provides a range of optimal options for WF developers with different preferences on the trade-off between the two objectives.

5. Real test case

Wind conditions in real WF sites are much more complicated than the ideal wind case. To test the algorithm in a realistic scenario, we consider the real wind case in Horn Rev in our real test cases. This wind case can be modelled by the two dimensional distribution as shown in Figure 1. The WFLO problem defined in Eqs. (1-5) and under the real wind conditions is hereby called the real test case.

5.1. WF with a fixed number (80) of WTs

Considering the real test case with a fixed number of WTs, i.e., $N_{wt}^{(L)} = N_{wt}^{(U)} = 80$, we run MORS with parameters $p_a = 0.0$, $p_r = 0.0$, $p_m = 1.0$ and $E_{max} = 20000$. The result we have obtained is shown in Figure 6.



(a)

(b)

(c)

(d)

Figure 6. MORS result for the real test case: (a) evolving track and final solutions; (b) original layout; (c) optimal layout with maximal P_{tot}; (d) optimal layout with minimal CL

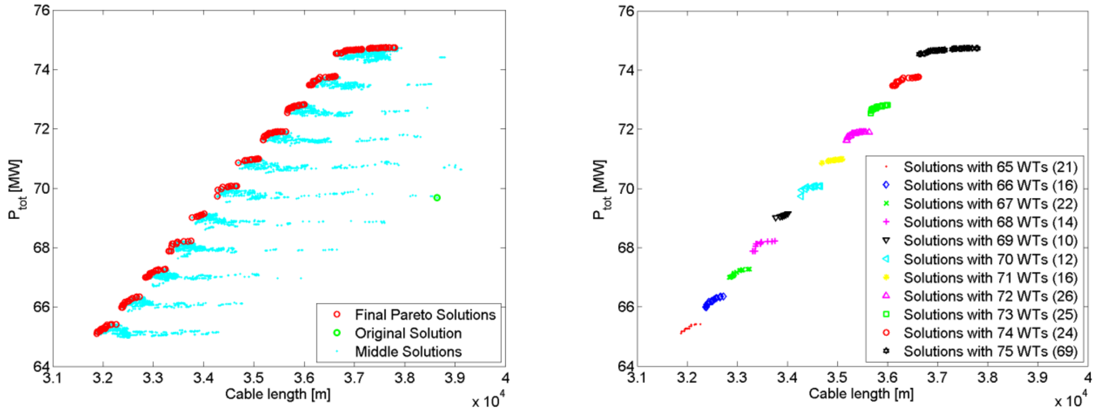
Note that in the final Pareto optimal solutions, MORS manages to reduce the cable length largely, but the achieved maximal improvement on P_{tot} is quite small. This reveals that among the two objectives of this given WFLO, cable length is easy to reduce while power is very hard to increase. This phenomenon is in line with our previous studies on the Horns Rev 1 WF [8, 13], in which we aimed to maximize P_{tot} using single-objective RS and achieved just a very small increase.

As found in [13], it matters for WFLO how many wind direction sectors are used in power calculation. Using few sectors (12 as in the common practice) will lead to impressively high but unreal improvement on power, while using a sufficiently large number of sectors (such as 360) can give seemly low but real and solid improvement. Thus, in this study, we have used $dv = 1\, m/s$ and $d\theta = 1\, deg$ when calculating P_{tot} by the integral defined in Eq. (2).

If we compare the layout optimized by MORS shown in Figure 6 (c) with the original layout shown in Figure 6 (b), we could see that the optimized layout is clearly better than the original one, as it requires a much shorter (more than 11%) cable length while maintaining the same level and even slightly higher power production.

5.2. WF with an unfixed number (65-75) of WTs

Consider the real test case with $N_{wt}^{(L)} = 65$ and $N_{wt}^{(U)} = 75$, and set the initial solution as the original WF's layout with the first 70 WTs. The result obtained by MORS with $p_a = 0.1$, $p_r = 0.1$, $p_m = 0.8$ and $E_{max} = 20000$ is shown in Figure 7.



(a) (b)

Figure 7. MORS result for the real test case with an unfixed number of WTs: (a) evolving track and final Pareto solutions; (b) classification of solutions according to the number of WTs (number of solutions shown in the brackets)

Similar with the performance for the ideal test case, MORS also finds a widely spreaded Pareto frontier in the real test case, which can be clearly divdided into 11 segment-like groups. Each of these groups acutally includes several Pareto optimal solutions with a certain number of WTs, cooresponding to one of the 11 possible values for the number of WTs $(65 \leq N_{wt} \leq 75)$. The result found by MORS can be quite useful in decision-making for WF developers, since it obtains a wide range of possible good layouts, each has a different number of WTs and a different position on the trade-off map between the two objectives: maximizing power and minimizing cable length.

6. Conclusions

We present a new algorithm called MORS in this paper, which can optimize the layout and the number of WTs simultaneously for multi-objective WFLO. As a general algorithm, it can consider multiple objectives (more than 2) and naturally deal with different constraints. In the meanwhile, it is a simple algorithm that requires only a few of parameters and it is also easy to implement and run. Without loss of generality, we consider two objectives (maximizing power and minimizing cable length) and two kinds of constraints (WF boundary and WT proximity).

After comparison with a mature and popular multi-objective evolutionary algorithm NSGA-II, it is found that MORS outperforms NSGA-II largely in the ideal test case. Furthermore, it has the additional advantage of dealing with an unfixed number of WTs. In the real test case on the Horn Rev 1 WF, MORS also shows promising performance. For the problem with a fixed number of WTs, it manages to get layout that has a much shorter cable length and a slightly higher power production. For the problem with an unfixed number of WTs, it obtains a wide range of Pareto optimal layouts with different numbers of WTs.

MORS can be a quite useful tool for WF developers. It will be further improved and tested in our future studies, which might include considering more realistic objectives (more than 2), considering other kinds of constraints such as terrain ruggedness, forbidden zones, noise emission and so on, and adding parallelism in the algorithm to harness the power of high fidelity models and high performance computing.

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