Journal of Physics: Conference Series **752** (2016) 012002 doi:10.1088/1742-6596/752/1/012002

Multiple Period States of the Superfluid Fermi Gas in an Optical Lattice

Gentaro Watanabe[1,2], Sukjin Yoon[3], Franco Dalfovo[4] and Takashi Nakatsukasa[5,6]

[1] Department of Physics and Zhejiang Institute of Modern Physics, Zhejiang University, Hangzhou, Zhejiang 310027, China
[2] Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34051, Korea
[3] School of Computational Sciences, Korea Institute for Advanced Study (KIAS), Seoul 02455, Korea
[4] INO-CNR BEC Center and Department of Physics, University of Trento, 38123 Povo, Italy
[5] Center for Computational Sciences, University of Tsukuba, Tsukuba 305-8577, Japan
[6] RIKEN Nishina Center, Wako, Saitama 351-0198, Japan

E-mail: `gentaro@zju.edu.cn`

Abstract. We study multiple period states (i.e., states whose period is a multiple of the lattice constant) of a two-component unpolarized superfluid Fermi gas in an optical lattice along the crossover between the Bardeen-Cooper-Schrieffer (BCS) and Bose-Einstein condensate (BEC) states. By solving Bogoliubov-de Gennes equations for a superfluid flow with finite quasimomentum, we find that, in the BCS side of the crossover, the multiple period states can be energetically favorable compared to the normal Bloch states and their survival time against dynamical instability drastically increases, suggesting that these states can be accessible in current experiments. This is in sharp contrast to the situation in BECs.

1. Introduction: nonlinear phenomena in superfluid Fermi gases in an optical lattice

Interplay between the nonlinearity due to the emergence of the superfluid order parameter and the periodicity of the lattice is very intriguing because these two are essential elements in condensed matter. Ultracold atomic gases in optical lattices enable us to study the subtle interplay of these effects deeply and directly (see, e.g., [1] for review) because of their high controllability of both the lattice geometry and the interatomic interaction (characterized by the s-wave scattering length a_s) (see, e.g., [2, 3]). Especially, by changing the interatomic interaction in superfluid Fermi gases using a Feshbach resonance, one can go along the crossover from the weakly coupled Bardeen-Cooper-Schrieffer (BCS) state to the Bose-Einstein condensate (BEC) state of tightly bound bosonic dimers [4, 5, 6, 7, 8], which allows us to study Bose and Fermi superfluids from a unified perspective.

Emergence of multiple period states, namely a class of stationary states whose period does not coincide with that of the external potential, but is a multiple of it, is a typical nonlinear phenomenon. For BECs in a periodic potential, it was found that multiple period states appear due to nonlinearity of the interaction term of the Gross-Pitaevskii (GP) equation [9]. However, these multiple period states in BECs are energetically unfavorable compared to the normal Bloch states whose period is equal to the lattice constant, and the lowest multiple period states are dynamically unstable [9].

Nonlinear phenomena in Fermi superfluids in a periodic potential [10, 11, 12] can be more important compared to those in Bose superfluids because of the wide implications for various systems in condensed matter physics and nuclear physics such as superconducting electrons in solids and superfluid neutrons in neutron stars (e.g., [13, 14, 15]). However, unlike the case of Bose gases, the study of nonlinear phenomena of superfluid Fermi gases is at a very infant stage (see, e.g., [16, 17]) and little has been studied about multiple period states in superfluid Fermi gases. Therefore, even a fundamental problem whether multiple period states exist along the BCS-BEC crossover was still open.

Under such circumstances, we have studied multiple period states in superfluid Fermi gases in [18]. In the present article, we summarize main results of this work (see also [1]).

2. Setup of the problem and formalism

We consider ultracold superfluid Fermi gases in the BCS-BEC crossover in a three-dimensional (3D) system flowing through a 1D optical lattice,

$$V_{\rm ext}(\mathbf{r}) = V_{\rm ext}(z) \equiv sE_R \sin^2 q_{\rm B} z, \tag{1}$$

where s is the dimensionless parameter characterizing the lattice strength, $E_R \equiv \hbar^2 q_B^2/2m$ is the recoil energy, $q_B \equiv \pi/d$ is the Bragg wave vector, and d is the lattice constant. We assume that the system is uniform in the transverse directions. We approach this problem by numerical simulations based on the Bogoliubov-de Gennes (BdG) equations [19, 8] for a supercell containing a multiple number of unit cells,

$$\begin{pmatrix} H'(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -H'(\mathbf{r}) \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \epsilon_i \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} \tag{2}$$

with $H'(\mathbf{r}) = -\dfrac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) - \mu$ and μ being the chemical potential. Here, $v_i(\mathbf{r})$ and $u_i(\mathbf{r})$ are the quasiparticle amplitudes, associated with the probability of occupation and unoccupation of a paired state denoted by an index i, and ϵ_i is the corresponding eigenenergy. The quasiparticle amplitudes $v_i(\mathbf{r})$ and $u_i(\mathbf{r})$ satisfy the normalization condition, $\int d\mathbf{r}\, [u_i^*(\mathbf{r})u_j(\mathbf{r}) + v_i^*(\mathbf{r})v_j(\mathbf{r})] = \delta_{i,j}$. Δ is the order parameter (or the pairing field), which reduces to the pairing gap in the single quasiparticle spectrum in the region of $\mu > 0$ for the uniform system. The pairing field $\Delta(\mathbf{r})$ and the chemical potential μ in equation (2) are self-consistently determined from the gap equation,

$$\Delta(\mathbf{r}) = -g \sum_i u_i(\mathbf{r}) v_i^*(\mathbf{r}) \tag{3}$$

with g being the coupling constant for the s-wave contact interaction which needs to be renormalized [20, 21, 22], and the average number density

$$n_0 = \frac{N}{\mathcal{V}} = \frac{1}{\mathcal{V}} \int n(\mathbf{r})\, d\mathbf{r} = \frac{2}{\mathcal{V}} \sum_i \int |v_i(\mathbf{r})|^2 d\mathbf{r}\,. \tag{4}$$

In the following, E_F and k_F denote the Fermi energy and wavenumber of a uniform free Fermi gas with density n_0: $E_F = \hbar^2 k_F^2/(2m)$ and $k_F = (3\pi^2 n_0)^{1/3}$. Since Δ depends on $\{u_i\}$ and $\{v_i\}$, the BdG equations (2) are nonlinear for nonzero g.

In this formalism, a stationary motion of the superfluid in the z direction, relative to the infinite periodic potential at rest, is described by solutions of equation (2) with quasimomentum

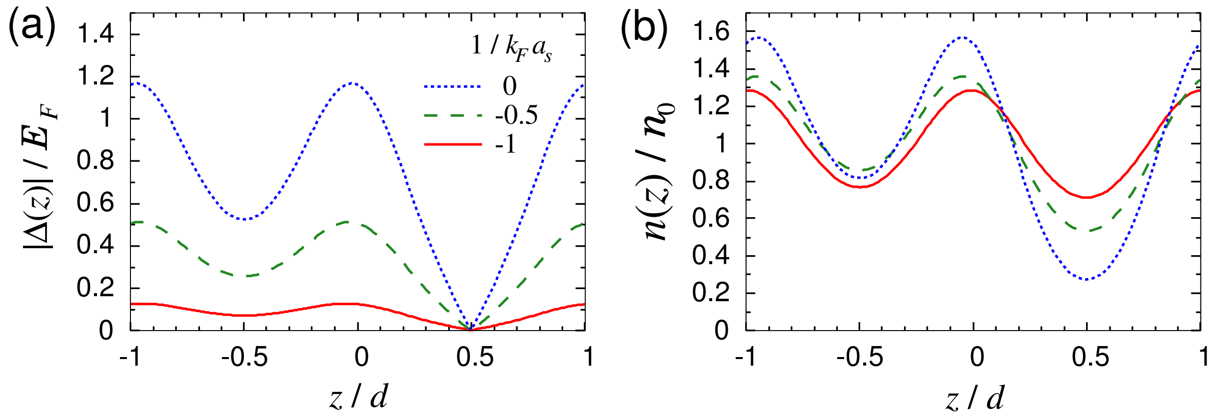

Figure 1. Profiles of (a) the magnitude of the pairing field $|\Delta(z)|$ and (b) the density $n(z)$ of the lowest period-doubled states at their first Brillouin zone edge $P = P_{\rm edge}/2 = \hbar q_B/4$ for various values of $1/k_F a_s$. Other parameters are $s = 1$ and $E_F/E_R = 0.25$. In the deep BCS side ($1/k_F a_s = -1$), unlike $|\Delta(z)|$, there is no large difference in $n(z)$ between the regions of $-1 < z/d \leq 0$ and $0 < z/d \leq 1$. However, this difference becomes more significant with increasing $1/k_F a_s$. This figure is taken from [18].

P per atom (not per pair), or the corresponding wave vector $Q = P/\hbar$, such that the quasiparticle amplitudes can be written in the Bloch form as $u_i(\mathbf{r}) = \tilde{u}_i(z)e^{iQz}e^{i\mathbf{k}\cdot\mathbf{r}}$ and $v_i(\mathbf{r}) = \tilde{v}_i(z)e^{-iQz}e^{i\mathbf{k}\cdot\mathbf{r}}$ leading to the pairing field as $\Delta(\mathbf{r}) = e^{i2Qz}\tilde{\Delta}(z)$. Here, $\tilde{\Delta}(z)$, $\tilde{u}_i(z)$, and $\tilde{v}_i(z)$ are periodic functions with period ν times d, with $\nu \in \{1, 2, 3, \cdots\}$, and the wave vector k_z of the quasiparticle lies in the first Brillouin zone for a supercell (a cell containing several primitive cells) with period ν, i.e., $|k_z| \leq q_B/\nu$.

3. Results

In figure 1, we show the magnitude of the pairing field $|\Delta(z)|$ and the number density $n(z)$ of the lowest period-doubled states (i.e., period $2d$) at their first Brillouin zone edge $P = P_{\rm edge}/2 = \hbar q_B/4$ ($P_{\rm edge} \equiv \hbar q_B/2$ denotes the quasimomentum at the first Brillouin zone edge of the normal Bloch states with period d). At this value of $P = P_{\rm edge}/2$, reflecting the fact that the current is zero, $|\Delta(z)|$ of the period-2 states has a node. The period-2 nature and the difference between the regions of $-1 < z/d \leq 0$ and $0 < z/d \leq 1$ can be clearly seen in $|\Delta(z)|$ at any value of $1/k_{\rm F}a_s$. On the other hand, especially in the BCS side ($1/k_{\rm F}a_s = -1$), there is no large difference in $n(z)$ between the two regions [see the red line in figure 1(b)]. Going to the deep BCS regime, the period-2 nature is mainly possessed by the pairing field, which decreases to zero. This reflects the fact that our period-doubled states emerge due to the superfluidity. Furthermore, we have found that the period-doubled states become energetically more stable compared to the normal Bloch states in the BCS regime (see figure 2). This is in sharp contrast to the situation of the period-doubled states in BECs and in the BEC side of the BCS-BEC crossover.

In the deep BCS regime, the period-2 states are not only energetically more stable, but also they can be long-lived although dynamically unstable. We have studied the dynamical stability of the period-2 states by solving the time-dependent BdG equations [18]. The black solid line in figure 3 shows the growth rate γ of the fastest exponentially growing mode $|\eta(t)| = |\eta(0)|\, e^{\gamma t}$ of the deviation $|\Delta(x,t)| - |\Delta_0(x)|$ from the true stationary state $\Delta_0(x)$ for the period-2 states. We

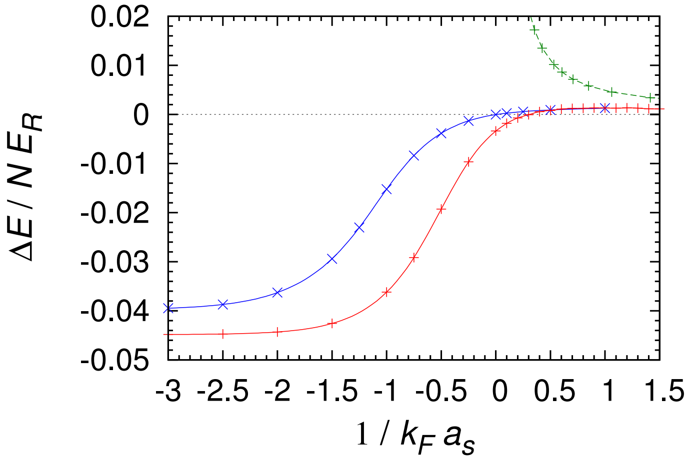

Figure 2. Difference ΔE of the energy per particle in units of E_R between the normal Bloch states and period-doubled states at $P = P_{\rm edge}/2$. Here we define $\Delta E \equiv E_2 - E_1$, where E_1 and E_2 represent the energy of the normal Bloch states and that of the period-doubled states, respectively. Parameters we have used are $s = 1, 2$ and $E_F/E_R = 0.25$. The red solid line with $+$ ($s = 1$) and blue solid line with $\times$ ($s = 2$) show the results obtained by solving the BdG equations and the green dashed line shows the results by the GP equation for corresponding parameters. Note that, in the BCS side, the energy of the period-doubled states at $P = P_{\rm edge}/2$ is lower than that of the normal Bloch states while the latter is lower than the former in the deep BEC side. This figure is taken from [18].

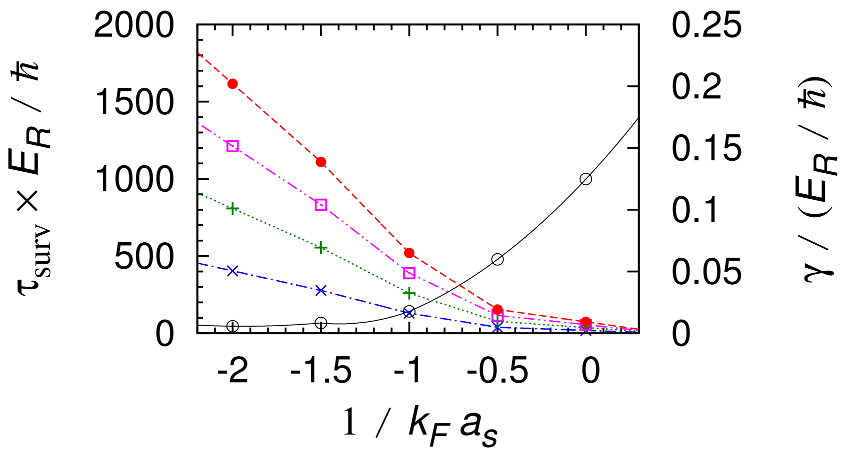

Figure 3. Growth rate γ of the fastest growing mode (black solid line) and survival time $\tau_{\rm surv}$ of the period-2 state at $P = P_{\rm edge}/2$ ($s = 1$ and $E_F/E_R = 0.25$). Blue dashed-dotted, green dotted, magenta dashed double-dotted, and red dashed lines show $\tau_{\rm surv}$ for relative amplitude $\tilde{\eta}(0)$ of the initial perturbation of 10%, 1%, 0.1%, and 0.01%, respectively. This figure is taken from [18].

see that γ is suppressed with decreasing $1/k_F a_s$, which makes the period-2 states long-lived in the BCS regime. The growth rate γ corresponds to the imaginary part of the complex eigenvalue for the fastest growing mode obtained by the linear stability analysis [23, 2], which is intrinsic property of the initial stationary state independent of the magnitude of the perturbation.

On the other hand, the actual survival time $\tau_{\rm surv}$, the timescale for which the initial state is destroyed by the large-amplitude oscillations, depends on the accuracy of their initial preparation. The survival time can be estimated by $\tilde{\eta}(0)e^{\gamma t} \sim 1$, where $\tilde{\eta}(0)$ is the relative amplitude of the initial perturbation with respect to $|\Delta_0|$ for the fastest growing mode. In figure 3, we show $\tau_{\rm surv}$ for various values of $\tilde{\eta}(0)$. This result suggests that if the initial stationary state is prepared within an accuracy of 10% or smaller, this state safely sustains for time scales of the order of $100\hbar/E_R$ or more in the BCS side, corresponding to $\tau_{\rm surv}$ of more than the order of a few milliseconds for typical experimental parameters [24]: For the recoil energy of bosonic molecules $E_{R,b} = 2\pi \times 7.3\text{kHz} \times \hbar$ used in the experiment of [24], $1\hbar/E_R = 0.0109$ msec. In the deep BCS regime ($1/k_F a_s \ll -1$), $\tau_{\rm surv}$ increases further and may become larger than the time scale of the experiments, so that the period-doubled states can be regarded as long-lived states and, in addition, they have lower energy than the normal Bloch states in a finite range of quasimomenta. Therefore, by (quasi-)adiabatically increasing the quasimomentum P of the superflow starting from the ground state at $P = 0$, multiple-period states such as the period-doubled states could be realized experimentally in the deep BCS regime.

4. Summary and conclusion

Study of nonlinear phenomena in superfluid Fermi gases is a new research frontier. Indeed, the system of superfluid Fermi gas has richer physics compared to that of a Bose gas: Feshbach resonances enable us to explore the crossover between two limiting cases of the weakly coupled Bardeen-Cooper-Schrieffer (BCS) state and the Bose-Einstein condensate (BEC) state of tightly bound bosonic dimers. Furthermore, this system could provide implications for other interesting systems such as matter in neutron stars.

Emergence of stationary states whose period is a multiple of the lattice constant, i.e., multiple period states is a typical nonlinear phenomenon. In [18], we have studied multiple period states of superfluid Fermi gases in an optical lattice. By solving Bogoliubov-de Gennes equations, we have found that, in the BCS side of the crossover, the multiple period states can be energetically favorable compared to the normal Bloch states and their survival time against dynamical instability drastically increases, suggesting that these states can be accessible in current experiments [24].

Acknowledgments

This work was supported by the Zhejiang University 100 Plan, by the Junior 1000 Talents Plan of China, by IBS through Project Code (IBS-R024-D1), by the Max Planck Society, MEST of Korea, Gyeongsangbuk-Do, Pohang City, for the support of the JRG at APCTP, by NRF of Korea through the Basic Science Research Program (Grant No. 2012R1A1A2008028), by JSPS KAKENHI (Grant No. 24105006), by ERC through the QGBE grant, and by the Provincia Autonoma di Trento. Part of calculations were performed on RICC in RIKEN.

References

[1] Watanabe G, Venkatesh B P and Dasgupta R 2016 *Entropy* **18** 118
[2] Pethick C J and Smith H 2008 *Bose Einstein Condensation in Dilute Gases* 2nd edn (Cambridge: Cambridge University Press)
[3] Bloch I, Dalibard J and Zwerger W 2008 *Rev. Mod. Phys.* **80** 885
[4] Eagles D M 1969 *Phys. Rev.* **186** 456
[5] Leggett A J 1980 in *Modern Trends in the Theory of Condensed Matter* (Berlin: Springer-Verlag) pp 13–27
[6] Nozières P and Schmitt-Rink S 1985 *J. Low Temp. Phys.* **59** 195
[7] Sá de Melo C A R, Randeria M and Engelbrecht J R 1993 *Phys. Rev. Lett.* **71** 3202
[8] Giorgini S, Pitaevskii L P and Stringari S 2008 *Rev. Mod. Phys.* **80** 1215
[9] Machholm M, Nicolin A, Pethick C J and Smith H 2004 *Phys. Rev.* A **69** 043604
[10] Watanabe G, Yoon S, Dalfovo F 2011 *Phys. Rev. Lett.* **107** 270404

[11] Watanabe G and Yoon S 2013 *JKPS* **63** 839
[12] Yu D, Yi W and Zhang W 2015 *Phys. Rev.* A **92** 033623
[13] Gezerlis A and Carlson J 2008 *Phys. Rev.* C **77** 032801(R)
[14] Gezerlis A and Carlson J 2010 *Phys. Rev.* C **81** 025803
[15] Watanabe G, Dalfovo F, Pitaevskii L P and Stringari S 2011 *Phys. Rev.* A **83** 03321
[16] Ku M J H, Ji W, Mukherjee B, Guardado-Sanchez E, Cheuk L W, Yefsah T and Zwierlein M 2014 *Phys. Rev. Lett.* **113** 065301
[17] Ku M J H, Ji W, Mukherjee B, Yefsah T and Zwierlein M 2016 *Phys. Rev. Lett.* **116** 045304
[18] Yoon S, Dalfovo F, Nakatsukasa T and Watanabe G 2016 *New J. Phys.* **18** 023011
[19] de Gennes P G 1966 *Superconductivity of Metals and Alloys* (New York: Benjamin) ch 5 p 137
[20] Randeria M 1995 *Bose-Einstein Condensation* ed A Griffin *et al* (Cambridge: Cambridge University Press) ch 15 p 355
[21] Bruun G, Castin Y, Dum R and Burnett K 1999 *Eur. Phys. J.* D **7** 433
[22] Bulgac A and Yu Y 2002 *Phys. Rev. Lett.* **88** 042504
[23] Ring P and Schuck P 1980 *The Nuclear Many-Body Problem* (New York: Springer)
[24] Miller D E, Chin J K, Stan C A, Liu Y, Setiawan W, Sanner C and Ketterle W 2007 *Phys. Rev. Lett.* **99** 070402