Journal of Physics: Conference Series **751** (2016) 012004 doi:10.1088/1742-6596/751/1/012004

Enhancing the performance of Q-cascade for separating intermediate components

S Zeng[1], D J Jiang[1], Y N Zhang[1], V D Borisevich[2], G A Sulaberidze[2], A Yu Smirnov[2]

[1] Department of Engineering Physics, Tsinghua University, Beijing 100084, China
[2] National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe highway 31, Moscow, 115409, Russia

E-mail: zengs@tsinghua.edu.cn

Abstract. It has been shown recently that Q-cascade with an expansion of the entering flow at the intermediate withdrawal point is able to obtain a concentration of an intermediate component far exceeding the concentration limit available from an end withdrawal. To enhance the applicability of this approach, it is necessary to reduce the relative total flow while maintaining the concentration of the intermediate target component unchanged. Optimization is carried out by using the technique of cascade segmentation, and using the mass numbers of the virtual components in the segments and the lengths of the segments as decision variables. The results demonstrate that the relative total flow is considerably reduced through optimization.

1. Introduction

Because the concentration is limited of an intermediate component available from an end withdrawal of a cascade, the separation has to be performed successively two or more times, with a withdrawal of the previous separation as the feed of the next to obtain its concentration to the required level. The whole separation process may involve one or more cascades, possibly, of different configurations. This definitely increases the complexity of separation, and consequently increases the cost. Therefore, it is of great interest to explore new separation cascades or new ways of separation to enrich more efficiently an intermediate component to the required concentration.

The model cascade with continuous profile, Q-cascade, has been recently investigated quite extensively. It has also been applied to obtaining an intermediate component by employing a technique which may be referred to as profile broadening [1][2]. With this technique the concentration of an intermediate component can be enriched to any level. However, as we known, for a separation task, there are a number of cascades that are able to satisfy the requirement. Taking into account practical applications, of all cascades in fulfilling the same separation task, a cascade that has a smaller relative total flow is considered to have a better separation performance and is more competitive. Here we present a way of enhancing the performance of a Q-cascade.

2. Q-cascade with profile broadening

Figure 1 illustrates a Q-cascade profile with profile broadening in the enriching section. The enriching section is split into two segments, whose lengths are $S_{\mathrm{P}}^{(1)}$ and $S_{\mathrm{P}}^{(2)}$, respectively. The location of a stage in a segment is indicated by l, and counted in the way as shown in the figure. The entering flow profiles of the two segments are $L_{\mathrm{P}}^{(1)}$ and $L_{\mathrm{P}}^{(2)}$, respectively. For the stripping section, the length is $S_{\mathrm{W}}^{(1)}$, and the profile is $L_{\mathrm{W}}^{(1)}$. F is the feed of the cascade, and P and W are, conventionally, referred to as the product and the waste. The intermediate withdrawal W_{m} takes place at the border of the first and the second segments, where is the location of the profile broadening.

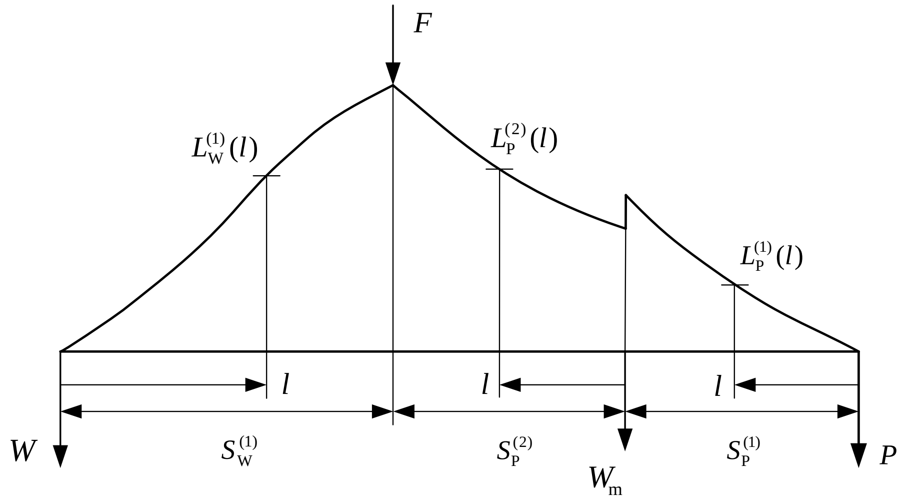

Figure 1. Illustration of the profile of a Q-cascade with flow broadening at the intermediate withdrawal.

According to the Q-cascade theory [3], for the stripping section and the enriching section, we have:

$$C_{i,\mathrm{W}}^{(n)}(l)L_{\mathrm{W}}^{(n)}(l) = 2WC_i^{\mathrm{W}} f_{i,\mathrm{W}}^{(n)}(l) \tag{1}$$

$$C_{i,\mathrm{P}}^{(n)}(l)L_{\mathrm{P}}^{(n)}(l) = 2PC_i^{\mathrm{P}} f_{i,\mathrm{P}}^{(n)}(l) + \begin{cases} 0 & (n < m) \\ 2W_{\mathrm{m}}C_i^{\mathrm{m}} f_{i,\mathrm{P}}^{n,m}(l) & (n \geq m) \end{cases} \tag{2}$$

Here, $C_{i,\mathrm{W}}^{(n)}(l)$ $(n = 1,2,\ldots,N_{\mathrm{W}})$ is the concentration of the i-th component at stage l of the n-th segment in the stripping section, and similarly, $C_{i,\mathrm{P}}^{(n)}(l)$ $(n = 1,2,\ldots,N_{\mathrm{P}})$ the concentration of the i-th component at stage l of the n-th segment in the enriching section. The component index for the isotope mixture $i = 1,2,\ldots,N_{\mathrm{C}}$, with N_{C} the number of components in the multi-component isotope mixture to be separated. N_{W} and N_{P} are the numbers of segments in the stripping and enriching sections, respectively. The intermediate withdrawal takes place at the beginning of the m-th segment. C_i^{W}, C_i^{P}, and C_i^{m} are respectively the concentrations of the i-th component in the waste, product and intermediate withdrawals. Obviously, for the cascade shown in Figure 1, $N_{\mathrm{W}} = 1$, $N_{\mathrm{P}} = 2$, and $m = 2$. In equations (1) and (2),

$$f_{i,\mathrm{W}}^{(n)}(l) = \begin{cases} (\mathrm{e}^{Q_{i,\mathrm{W}}^{(n)}l} - 1)(Q_{i,\mathrm{W}}^{(n)})^{-1} + \mathrm{e}^{Q_{i,\mathrm{W}}^{(n)}l} f_{i,\mathrm{W}}^{(n-1)}(S_{\mathrm{W}}^{(n-1)}) & (n \geq 1) \\ 0 & (n = 0) \end{cases} \tag{3}$$

$$f_{i,\mathrm{P}}^{(n)}(l) = \begin{cases} (1-\mathrm{e}^{-Q_{i,\mathrm{P}}^{(n)}l}) + \mathrm{e}^{-Q_{i,\mathrm{P}}^{(n)}l} f_{i,\mathrm{P}}^{(n-1)}(S_{\mathrm{P}}^{(n-1)}) & (n \neq m) \\ (1-\mathrm{e}^{-Q_{i,\mathrm{P}}^{(n)}l})(Q_{i,\mathrm{P}}^{(n)})^{-1} + \mathrm{e}^{-Q_{i,\mathrm{P}}^{(n)}l} f_{i,\mathrm{P}}^{(n-1)}(S_{\mathrm{P}}^{(n-1)})J_{\mathrm{L}} & (n = m) \\ (1-\mathrm{e}^{-Q_{i,\mathrm{P}}^{(n)}l})(Q_{i,\mathrm{P}}^{(n)})^{-1} & (n = 1) \end{cases} \tag{4}$$

$$f_{i,\mathrm{P}}^{n,m}(l) = \begin{cases} (1-\mathrm{e}^{-Q_{i,\mathrm{P}}^{(n)}l})(Q_{i,\mathrm{P}}^{(n)})^{-1} + \mathrm{e}^{-Q_{i,\mathrm{P}}^{(n)}l} f_{i,\mathrm{P}}^{n-1,m}(S_{\mathrm{P}}^{(n-1)}) & (n > m) \\ (1-\mathrm{e}^{-Q_{i,\mathrm{P}}^{(n)}l})(Q_{i,\mathrm{P}}^{(n)})^{-1} & (n = m) \\ 0 & (n < m) \end{cases} \tag{5}$$

Note that $n = 1,2,\ldots,N_{\mathrm{W}}$ for the stripping section, and $n = 1,2,\ldots,N_{\mathrm{P}}$ for the enriching section. In equation (4), J_{L} is the parameter, here referred to as the broadening parameter, that takes into account the flow profile broadening. At the common border of two segments, if without flow profile broadening, the profile should be continuous across the border, that is,

$$L_{\mathrm{P}}^{(n-1)}(S_{\mathrm{P}}^{(n-1)}) = L_{\mathrm{P}}^{(n)}(0) \tag{6}$$

If a broadening is carried out, for instance at the beginning of the b-th segment,

$$L_{\mathrm{P}}^{(b-1)}(S_{\mathrm{P}}^{(b-1)}) = J_{\mathrm{L}} L_{\mathrm{P}}^{(b)}(0) \tag{7}$$

We let $b = m$, which means that the location where there is the intermediate withdrawal is the location where the broadening happens. It is clear that the meaning of the broadening parameter is actually the ratio of the flows at the border of two connecting segments. The quantities $Q_{i,\mathrm{W}}^{(n)}$ and $Q_{i,\mathrm{P}}^{(n)}$ are the Q-parameters of the n-th segment in the stripping and enriching sections, respectively, and defined as

$$Q_{i,\mathrm{W}}^{(n)} = \varepsilon_0 (M_{\mathrm{W}}^{*(n)} - M_i)\,,\ Q_{i,\mathrm{P}}^{(n)} = \varepsilon_0 (M_{\mathrm{P}}^{*(n)} - M_i) \tag{8}$$

where $M_{\mathrm{W}}^{*(n)}$ and $M_{\mathrm{P}}^{*(n)}$ are the mass numbers of the virtual component in the corresponding segments, M_i is the mass number of the i-th component, satisfying $M_i < M_j$ if $i < j$, and ε_0 is the enrichment separation factor for unit mass difference. When $n = m$, we have

$$C_i^{\mathrm{m}} = C_i^{\mathrm{P}} f_{i,\mathrm{P}}^{(m-1)}(S_{\mathrm{P}}^{(m-1)}) \left[\sum_{j=1}^{N_{\mathrm{C}}} C_j^{\mathrm{P}} f_{j,\mathrm{P}}^{(m-1)}(S_{\mathrm{P}}^{(m-1)}) \right]^{-1} \tag{9}$$

The relative total flow of the cascade is defined as:

$$G_{\mathrm{RT}} = \frac{\varepsilon_0^2}{2W_{\mathrm{m}}} \left(\sum_{n=1}^{N_{\mathrm{W}}} \int_0^{S_{\mathrm{W}}^{(n)}} L_{\mathrm{W}}^{(n)}(l)\mathrm{d}l + \sum_{n=1}^{N_{\mathrm{P}}} \int_0^{S_{\mathrm{P}}^{(n)}} L_{\mathrm{P}}^{(n)}(l)\mathrm{d}l \right) \tag{10}$$

which is the cascade total flow G divided by $2W_{\mathrm{m}}/\varepsilon_0^2$, and stands for the total flow needed to obtain a unit quantity of product with the required concentration. In multi-component isotope separation, so far we have no unanimously accepted definition of value function and separative power, as in the case of a binary separation, to evaluate the performance of a separation facility. Therefore, it is reasonable to think that, to fulfill a separation task, the cascade with a minimum relative total flow has the best performance, because a less relative total flow means that a smaller number of separation units will be used. Note that in [2], the relative total flow is defined as the cascade total flow divided by $2P/\varepsilon_0^2$. We think the definition in equation (10) is more appropriate to reflect the economic consideration, as the actual product, which is the intermediate withdrawal W_{m} other than P, is directly taken into account.

3. Enhancing cascade performance

In [2], separation is carried out to separate the 4-th component of the Krypton isotopes. Table 1 gives the components of Krypton, and their mass numbers and natural concentrations.

Table 1. Krypton's components, their mass numbers and natural concentrations.

i	1	2	3	4	5	6
M_i	78	80	82	83	84	86
C_i^{F}	0.0035	0.0227	0.1156	0.1152	0.5690	0.1740

Here, we take the last separation task in [2] as the example here to demonstrate how to enhance the cascade performance. In this separation task, $C_k^{\mathrm{m}} = 0.72\ (k = 4)$, $\varepsilon_0 S_{\mathrm{W}} = 6$, $J_{\mathrm{L}} = 1.4^{-1}$, $W_{\mathrm{m}} / P = 0.1$ $(W_{\mathrm{m}} / F \approx 2.285 \times 10^{-2})$, and the mass numbers of the virtual component $M_{\mathrm{W}}^{*(1)} = M_{\mathrm{P}}^{*(2)} = 83.5$, $M_{\mathrm{P}}^{*(1)} = 82.5$. To fulfill this task, $G_{\mathrm{RT}} = 548.72$ in [2]. However, this is not the best cascade for this separation task. Without stating this clearly, people might mistake this cascade as the cascade of choice and compare it with other cascades. Then the conclusion would be misleading. In order to compete with other ways of separation, it is necessary to make G_{RT} minimum. For this purpose, we set up the following separation task:

$$\begin{aligned} &\text{given}: W_{\mathrm{m}} / F,\ J_{\mathrm{L}} \\ &\min G_{\mathrm{RT}}(M_{\mathrm{W}}^{*(1)}, M_{\mathrm{P}}^{*(1)}, M_{\mathrm{P}}^{*(2)}, \varepsilon_0 S_{\mathrm{W}}^{(1)}, \varepsilon_0 S_{\mathrm{P}}^{(1)}, \varepsilon_0 S_{\mathrm{P}}^{(2)}) \\ &\text{s.t. } C_k^{\mathrm{m}} = 0.72 \end{aligned} \tag{11}$$

4. Results and discussions

Four separation tasks are considered, and the results are presented in Table 2.

Table 2. Results for 4 separation tasks.

Task	1	2	3	4
W_{m} / F	0.02285	0.02285	0.15	0.15
J_{L}	1.4^{-1}	1.4^{-1}	1.4^{-1}	1
$M_{\mathrm{W}}^{*(1)}$	83.5	85.0000	83.4996	83.4929
$M_{\mathrm{P}}^{*(2)}$	83.5	83.7387	83.5428	83.5621
$M_{\mathrm{P}}^{*(1)}$	82.5	82.5240	82.5360	82.5554
$\varepsilon_0 S_{\mathrm{W}}^{(1)}$	6	.202688	5.70738	5.71418
$\varepsilon_0 S_{\mathrm{P}}^{(2)}$	13.4050	8.38396	6.90026	6.92507
$\varepsilon_0 S_{\mathrm{P}}^{(1)}$	2.35987	9.10625	12.0108	12.3014
G_{RT}	548.770	86.5874	82.0462	83.3437

Task 1 is the task mentioned above and computed in [2]. To allow readers to make later comparisons, in the table six digits are given for the values obtained from computation; the rest values are the specified ones. The cascade flow profiles as well as the concentration distributions of the target component C_k, $C_k^{(n)}(l)$, are plotted in Figure 2 and Figure 3, respectively, where s is defined as:

$$s = \begin{cases} l & (0 \le l \le S_{\mathrm{W}}^{(1)}, \text{in the stripping section}) \\ S_{\mathrm{W}}^{(1)} + S_{\mathrm{P}}^{(2)} - l & (0 \le l \le S_{\mathrm{P}}^{(2)}, \text{in segment 2 of the enriching section}) \\ S_{\mathrm{W}}^{(1)} + S_{\mathrm{P}}^{(2)} + S_{\mathrm{P}}^{(1)} - l & (0 \le l \le S_{\mathrm{P}}^{(1)}, \text{in segment 1 of the enriching section}) \end{cases} \tag{12}$$

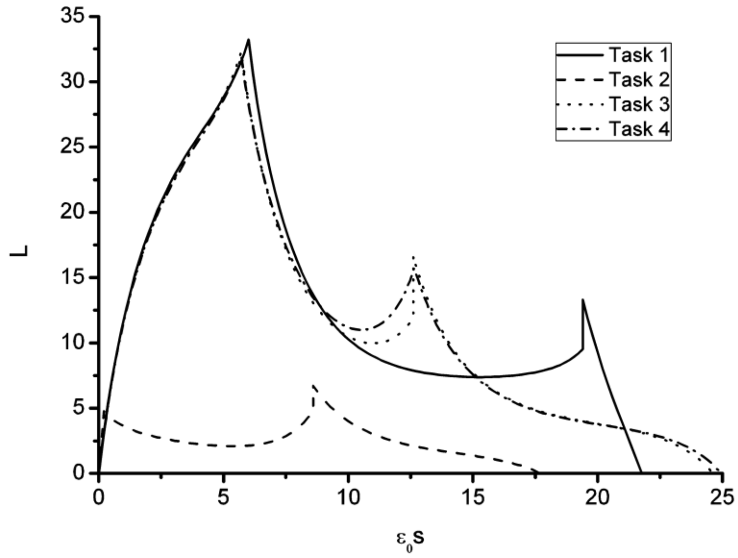

Figure 2. Flow profile distributions of the 4 tasks along cascades.

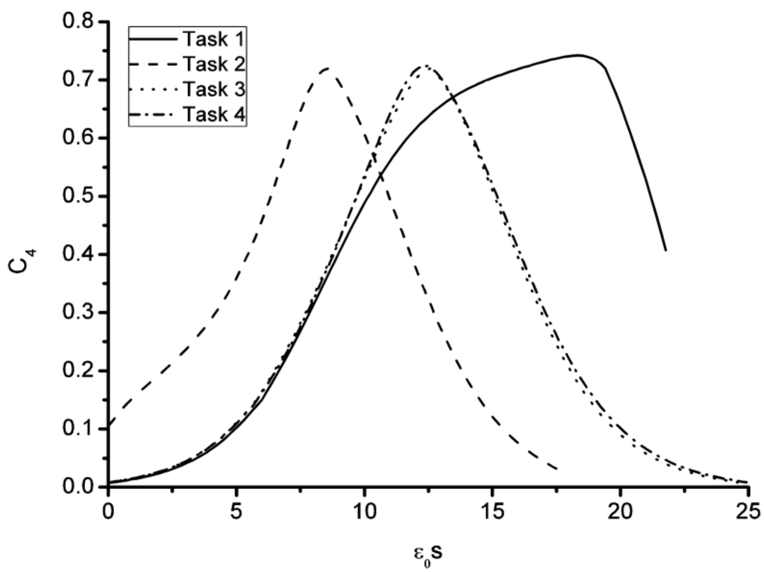

Figure 3. Concentration distributions of C_k along cascades.

The results show that taking some measures to enhance the separation performance is absolutely necessary. One can see that by employing minimization of the relative total flow, there is a reduction of at least 6 times in the relative total flow, which is very significant. In other words, to obtain the same quantity of product, the cascade can be about 6 times smaller in scale, or to use the same scale of cascade, about 6 times more product can be produced.

G_{RT} of Task 2 is larger than that of Task 3, which is surprising, because W_{m}/F of Task 2 is much smaller than that of Task 3. We see that $M_{\mathrm{W}}^{*(1)}$ has a value of 85, which is the upper bound set up for the minimization. So it is understandable that the result of Task 2 has not yet reached the real minimum. However, the larger the upper bound for $M_{\mathrm{W}}^{*(1)}$ is, the smaller $\varepsilon_0 S_{\mathrm{W}}^{(1)}$, and the smaller G_{RT}; the minimum is always obtained when $M_{\mathrm{W}}^{*(1)}$ takes its upper bound. The tendency seems to be that when $M_{\mathrm{W}}^{*(1)}$ tends to infinity, the real minimum is obtained. In this case, $\varepsilon_0 S_{\mathrm{W}}^{(1)}$ becomes zero, that is, the cascade has no stripping section. That a cascade without a stripping section can fulfill the task suggests that such a cascade has some surplus separation capability for the specified separation task, which is one of the reasons why a larger value of W_{m}/F is taken in Task 3.

It is interesting to note that the shapes of the concentration distributions in Tasks 2-4 are alike. Also note that for the 3 tasks the locations of the intermediate withdrawal are very close to or coincident with the peak locations of the concentration, which indicates that the separation capability of the cascades are made full use, whereas for Task 1 the location of the intermediate withdrawal is somewhat far away from the peak location of the concentration, which suggests that the cascade is not configured to its best separation state and some separation capability is wasted.

In Task 4, the only difference from Task 3 is $J_{\mathrm{L}} = 1$, that is, there is no cascade flow profile broadening specified. Observing Figure 2, we see clearly that the broadening is automatically incorporated into the flow profile. However, with J_{L} specified, G_{RT} is reduced slightly, which indicates that J_{L} can also be used as a parameter to further enhance the cascade performance.

5. Conclusions

The performance of Q-cascades in separating an intermediate component is enhanced significantly through optimization of the relative total flow. The optimization is minimizing the relative total flow by using the mass numbers of the virtual component in and the lengths of the cascade segments as the decision variables.

In comparison with a cascade in literature for separating the ^{83}Kr isotope, it is achieved that a cascade with a relative total flow of being approximately six times less is able to produce the same amount of the target component of the required concentration, or a cascade with about the same amount of relative total flow to produce approximately six times more product.

The flow profile broadening is automatically taken into account in the optimization. However, the benefit of the broadening can be further promoted to reduce the relative total flow by using the broadening parameter.

Acknowledgment

This research is supported by National Natural Science Foundation of China (Grant No.11575097), National Key Basic Research and Development Program of China (Grant No.2014CB744100), and the Russian Fund for Basic Research (Grant No.16-58-53058 GFEN_a).

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