

Influence of parameters of a Q-cascade with a local flow “extension” on mass-transfer of components along the cascade length

A Yu Smirnov, G A Sulaberidze

National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe highway 31, Moscow, 115409, Russia

E-mail: AYSmirnov@mephi.ru

Abstract. The presented results complement the theory of separation cascades having a local increase of working substance flow at its internal stages (cascades with the flow "extension") used to obtain isotopic products with relatively high concentrations of intermediate components. Based on the Q-cascade model theoretical dependences of the maximum possible concentration of the target component in the end product flow of the cascade, the maximum achievable concentration of the target component inside the cascade and the value of the cascade total relative flow versus the length of the cascade section between the end and intermediate product flows - $\varepsilon_0 S_E$ were obtained. Comparison of these dependences allows to approximately determine the optimal value $\varepsilon_0 S_E$ for the given value of the target component concentration in one of the cascade product flows.

1. Introduction

In the separation of multicomponent isotope mixtures enrichment of intermediate isotopes causes particular difficulties [1]. This is due to fact that in the case of multi-isotope mixtures separation, the intermediate components are produced at both ends of a cascade (conventionally designated as “heavy” and “light” ones) together with the utmost components, the components with the smallest and largest mass numbers. As a result, increase in the concentrations of the intermediate components in the light and heavy fractions of a three-flow (one ingoing and two outgoing flows) cascade is limited by the maximum achievable values [2].

There exist a number of specific methods for enrichment of intermediate components of multi-isotope mixtures designed to achieve maximum concentrations in the outgoing flows of a three-flow cascade. These methods are based on enrichment of a mixture in a target isotope by a series of sequential operations in the same cascade or by use of the so-called dual cascade, where the product flow from the first cascade serves as a feed flow of the second one [3]. Alternatively, one can use the multi-flow cascades [4-7].

In the papers [6–7] it was demonstrated the fundamental possibility of obtaining relatively high concentrations of intermediate components in the additional product flow within the cascade having an “extension” of a working substance flow at its internal stage. The basic principle of such a cascade work can be explained by the following statement: changing a gradient of a working substance flow at one of the internal stages of a cascade leads to change of the concentration gradients of all mixture components over cascade stages. It was shown that increasing the concentration of the intermediate



target component can be achieved at the internal cascade stages by selection of a certain gradient flow function. Besides, an additional product flow at one of these cascade stages enables to achieve a higher target component concentration as compared to the maximum achievable one [5-7].

However, the results of [5-7] do not answer the question of the maximum possible effect of increasing the concentration of the intermediate component in the cascade with the "extension" as well as about the influence of the parameters of "extension" (the "flow jump", etc.) on the mass transfer characteristics of the desired intermediate component in such a cascade.

The aim of this work is to study the mass transfer regularities of the intermediate target component in the cascade with the flow "extension" depending on the length of the section with the flow "extension". As the object of the study the Q-cascade [8] with the flow "extension" is considered. Analytical dependences to assess the maximum possible concentration of the target component in the end product flow of the cascade with flow "extension" and its internal stages are obtained.

2. Mathematical model

The detailed mathematical model of the Q-cascade with the flow "extension" is presented in [6-7]. In this paper its brief description is presented.

Consider the cascade, external parameters which are power flow F with concentrations of the components C_i^F , and three exit streams: an end product flow P with concentrations of the components C_i^P , an additional product flow E withdrawn from an intermediate stage of the cascade with concentrations C_i^E and a waste flow W with concentrations C_i^W (Figure 1.). The number of stages in the enriching and depleted sections of the cascade is denoted, respectively, S_P and the S_W , the number of stages between the two product flows is denoted S_E . One supposes that the flow of the working substance $L(s)$ is continuous over the length of the cascade, with the exception of the points in which the additional product is withdrawn and the cascade feed flow is injected. Also, suppose that that the flow of the working substance vanishes at the cascade ends [6, 8]. In the point where the additional product is withdrawn the flow of the working substance increases abruptly in the K -times.

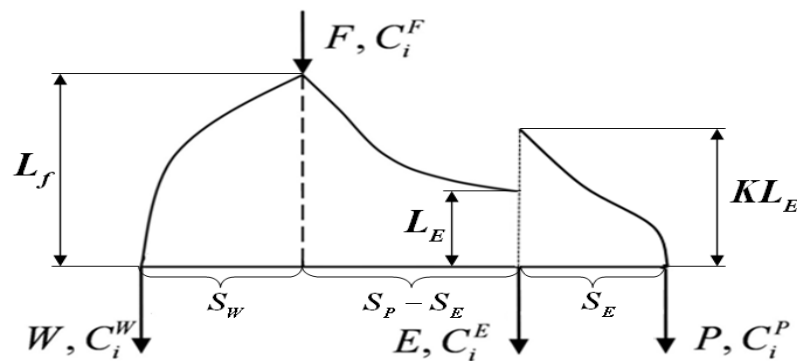


Figure 1. Schematic drawing of a cascade with extension of a flow profile.

Expressions for the calculation of external parameters and total flow of this cascade in the steady-state operation are as follows [6-7]:
for the enriching section:

$$\frac{P}{F} = \sum_{j=1}^m \left\{ \frac{C_j^F \cdot \frac{e^{Q_j^I S_W} - 1}{Q_j^I}}{\left[\frac{1 - e^{-Q_j^{II} S_E}}{K \cdot Q_j^{II} \cdot e^{Q_j^I (S_P - S_E)}} + \frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} + \frac{e^{Q_j^I S_W} - 1}{Q_j^I} \right] + \frac{E C_j^E}{P C_j^P} \cdot \left[\frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} + \frac{e^{Q_j^I S_W} - 1}{Q_j^I} \right]} \right\}, (1)$$

$$C_j^P = \frac{F}{P} \cdot \frac{C_j^F \cdot \frac{e^{Q_j^I S_W} - 1}{Q_j^I}}{\left[\frac{1 - e^{-Q_j^{II} S_E}}{K \cdot Q_j^{II} \cdot e^{Q_j^I (S_P - S_E)}} + \frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} + \frac{e^{Q_j^I S_W} - 1}{Q_j^I} \right] + \frac{E C_j^E}{P C_j^P} \cdot \left[\frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} + \frac{e^{Q_j^I S_W} - 1}{Q_j^I} \right]} \quad (2)$$

for the depleted section:

$$\frac{W}{F} = \sum_{j=1}^m \left\{ \frac{C_j^F \cdot \left\{ \left[\frac{1 - e^{-Q_j^{II} S_E}}{K \cdot Q_j^{II} \cdot e^{Q_j^I (S_P - S_E)}} + \frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} \right] + \frac{E C_j^E}{P C_j^P} \cdot \frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} \right\}}{\left[\frac{1 - e^{-Q_j^{II} S_E}}{K \cdot Q_j^{II} \cdot e^{Q_j^I (S_P - S_E)}} + \frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} + \frac{e^{Q_j^I S_W} - 1}{Q_j^I} \right] + \frac{E C_j^E}{P C_j^P} \cdot \left[\frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} + \frac{e^{Q_j^I S_W} - 1}{Q_j^I} \right]} \right\}, (3)$$

$$C_j^W = \frac{F}{W} \cdot \frac{C_j^F \cdot \left\{ \left[\frac{1 - e^{-Q_j^{II} S_E}}{K \cdot Q_j^{II} \cdot e^{Q_j^I (S_P - S_E)}} + \frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} \right] + \frac{E C_j^E}{P C_j^P} \cdot \frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} \right\}}{\left[\frac{1 - e^{-Q_j^{II} S_E}}{K \cdot Q_j^{II} \cdot e^{Q_j^I (S_P - S_E)}} + \frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} + \frac{e^{Q_j^I S_W} - 1}{Q_j^I} \right] + \frac{E C_j^E}{P C_j^P} \cdot \left[\frac{1 - e^{-Q_j^I (S_P - S_E)}}{Q_j^I} + \frac{e^{Q_j^I S_W} - 1}{Q_j^I} \right]} \quad (4)$$

Also, the following relations are taken place [6-7]:

$$C_j^E = \frac{C_j^P \cdot \frac{1 - e^{-Q_j^{II} S_E}}{Q_j^{II}}}{\sum_{i=1}^m \left(C_i^P \cdot \frac{1 - e^{-Q_i^{II} S_E}}{Q_i^{II}} \right)}, \quad (5)$$

$$\frac{C_j^E}{C_j^P} = \frac{C_n^E}{C_n^P} \cdot \frac{Q_n^{II}}{Q_i^{II}} \cdot \frac{1 - e^{-Q_i^{II} S_E}}{1 - e^{-Q_n^{II} S_E}}. \quad (6)$$

The total relative flow of the cascade can be calculated as follows:

$$\begin{aligned} \frac{\sum L}{2P} = \sum_{j=1}^m & \left\{ \left[\frac{W}{P} C_j^W \left(e^{Q_i^I S_W} - 1 - Q_i^I S_W \right) + C_j^P \left(e^{-Q_i^I (S_P - S_E)} - 1 + Q_i^I (S_P - S_E) \right) + \right. \right. \\ & \left. \left. + \frac{E}{P} C_j^E \left(e^{-Q_i^I (S_P - S_E)} - 1 + Q_i^I (S_P - S_E) \right) \right] \right\} / (Q_i^I)^2 + \\ & C_j^P \left[\frac{e^{-Q_i^{II} S_E} - 1 + Q_i^{II} S_E}{(Q_i^{II})^2} + \frac{(e^{-Q_i^{II} S_E} - 1)(1 - e^{Q_i^I (S_P - S_E)}) + Q_i^{II} S_E}{K Q_i^I Q_i^{II} e^{Q_i^I (S_P - S_E)}} \right]. \end{aligned} \quad (7)$$

where $Q_i^{I,II}$ – are constants which role will be considered below.

From the theory of the Q -cascade it is followed that the values Q_i connected with each other by conditions [8]:

$$Q_i - Q_k = \varepsilon_{ik}. \quad (8)$$

For the molecular-kinetic separation methods [8]:

$$Q_i = \varepsilon_0 (M - M_i), \quad (9)$$

where M_i – the mass number of the i -th component, ε_0 – enrichment coefficient per unit mass difference, M – the parameter which choice in the range $M_1 \leq M \leq M_m$ allows to calculate all values Q_i and, consequently, the flow distribution $L(s)$ [8].

Parameter M plays an important role: setting value M determines the flow distribution $L(s)$ and the ratio of the product flow to the waste one. All this causes that components with mass numbers $M_i < M$ enriched with the lightest component to “light” end of the cascade, and components with $M_i > M$ - enriched with the heaviest component to the “heavy” end of the cascade. Within the Q -cascade model by varying of the M one can achieve accumulation of the intermediate component in the internal stages of the cascade. For example, by setting M , so that in one section - $M_n < M$ (n -target component number), and in the other - $M_n > M$. Under this condition the flow distribution $L(s)$ is such that the n -th component will begin to enrich in opposite directions in different sections of the cascade. In this case, the n -th component will be “locked” on intermediate stages of the cascade. As a result, the concentration of n -th component in the maximum will increase, in comparison with the Q -cascade without extension. Thus, suppose that in the sections between the waste flow blade and the additional product flow the parameter M is set to $M^I > M_n$ with its corresponding constant Q_i^I ($i = \overline{1, m}$), and in the rest section M takes the value M^{II} with corresponding constants Q_i^{II} ($i = \overline{1, m}$).

3. Results and discussion

Consider the effect of some parameters of the Q -cascade with the “extension” on the mass transfer of components along its length. Let us assess the impact of the value of S_E (the length of the section located between the end and additional product flows) to mass transfer of the target intermediate component along the length of the cascade. Detailed analysis of these relationships allows to define the maximum possible value of the target component concentrations in the end product flow and the maximum concentration of a given component within the cascade.

As follows from the analysis of relations connecting the external and internal parameters of the cascade, for the study the influence of the S_E on mass transfer of the target component, the most interesting are two cases:

Case 1: $\varepsilon_0 S_P \gg 1$, $\varepsilon_0 S_P \gg \varepsilon_0 S_E$;

Case 2: $\varepsilon_0 S_P \gg 1$, $\varepsilon_0 S_P \sim \varepsilon_0 S_E$.

The first case corresponds to the “long” cascade in which the length of the end section is small compared to the entire length of the enriching section (see Fig. 1). This cascade corresponds to small influence of “extension” on the component mass transfer along its length. The second case corresponds to a cascade in which a significant part of the enriching section coincides with the section with “extension”. Note also that in both cases the condition $\varepsilon_0 S_W \gg 1$ is taken. For small values $\varepsilon_0 S_W$ and $\varepsilon_0 S_P$ (in the case of “short” cascades) the concentration distribution of the intermediate component may not maximum at internal cascade stages which makes such cascades inappropriate to obtain relatively high concentrations of intermediate components [2, 7].

After limiting transitions in (2) it is possible to obtain formulas for the maximum achievable concentration of the target component with n number at the end product flow of the cascade for the cases 1 and 2, respectively:

$$(C_n^P)_{\max} = \frac{C_n^F / \left(1 + \frac{E}{P} \frac{C_i^E}{C_i^P}\right)}{\sum_{j=1}^n C_j^F / \left(1 + \frac{E}{P} \frac{C_j^E}{C_j^P}\right)}, \quad (10)$$

$$(C_n^P)_{\max} = \frac{C_n^F / \left(\frac{Q_n^I}{KQ_n^{II}} \frac{1 - e^{-Q_n^{II} S_E}}{e^{Q_n^I S_W}} + 1 + \frac{E}{P} \frac{C_n^E}{C_n^P}\right)}{\sum_{j=1}^{n-1} C_j^F / \left(1 + \frac{E}{P} \frac{C_j^E}{C_j^P}\right) + C_n^F / \left(\frac{Q_n^I}{KQ_n^{II}} \frac{1 - e^{-Q_n^{II} S_E}}{e^{Q_n^I S_W}} + 1 + \frac{E}{P} \frac{C_n^E}{C_n^P}\right)}. \quad (11)$$

From these ratios it is evident that their use for assessment is difficult due to the fact that unknown values are E/P , C_j^E , C_j^P explicitly included in them. However, relations (10), (11) can be used for practical assessments with condition $E/P \rightarrow 0$, provided that corresponds to an absence of the additional product flow or case of its negligible smallness in relation to other external cascade flows. Using the condition $E/P \rightarrow 0$ the above relations (10) and (11) can be rewritten as:

$$(C_n^P)_{\max} = \frac{C_n^F}{\sum_{j=1}^n C_j^F}, \quad (12)$$

$$(C_n^P)_{\max} = \frac{C_n^F}{\sum_{j=1}^{n-1} C_j^F + C_n^F / \left(\frac{Q_n^I}{KQ_n^{II}} \frac{1 - e^{-Q_n^{II} S_E}}{e^{Q_n^I S_W} - 1}\right) + 1}. \quad (13)$$

It can be seen that the expression (12) coincides with the corresponding expression obtained for monotonic profile cascades [2]. Thus, in case of smallness of the section with “extension” with respect to the entire length the cascade mass transfer regularities in cascade with the flow “extension” will be

similar to those for cascade of monotonic flow profile. With extending of the section with "extension" concentration distribution of the target component will be more different from the corresponding distribution in the cascade of monotonic flow profile that will cause changes in the maximum achievable concentration of the target component in the end product flow of the cascade.

Fig. 2 presents us a graph showing the analytical dependence of the maximum concentration of the isotope ^{183}W on the length of the cascade at the section with the "extension" - $\varepsilon_0 S_E$. The following parameters were given: $\varepsilon_0 S_W = 5,94$, $K=1,5$, $M^I = 183,5$, $M^{II} = 182,5$. From the analysis of the character of the curve on Fig. 2 it follows that with increasing of the length of the section with "extension" the target component concentration in the end product decreases. Obviously, this is due to the fact that the target component begins to actively accumulate inside the cascade.

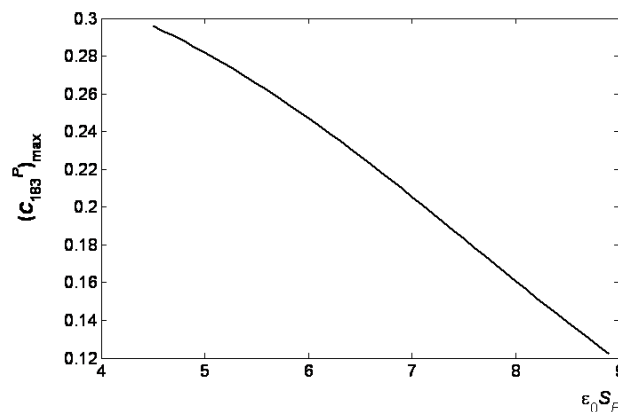


Figure 2. Analytical dependence of the limiting concentration of ^{183}W in the end product flow versus length of the utmost section of the cascade with "extension" $\varepsilon_0 S_E$.

The conclusions based on analysis of the dependence shown in Fig. 2, fully confirmed by Fig. 3, which shows the concentration dependence of ^{183}W in the end product flow of the cascade with "extension" derived from direct calculation of the cascade with different $\varepsilon_0 S_E$ which were varied from $\varepsilon_0 S_P/2$ to $\varepsilon_0 S_P$. In the calculations, the following values were given: $\varepsilon_0 S_W = 5,94$, $\varepsilon_0 S_P = 9$, $K=1,5$, $M^I = 183,5$, $M^{II} = 182,5$, $E/P = 0$.

From a comparison of Fig. 2 and 3 it is evident that both dependences consistent qualitatively and quantitatively. This fact indicates the correctness of the (12) formula.

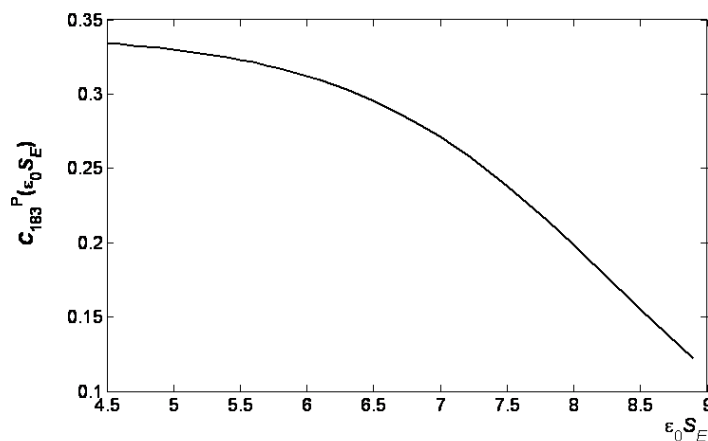


Figure 3. Dependence of the concentration of ^{183}W in the end product flow versus length of the utmost section of the cascade with "extension" $\varepsilon_0 S_E$.

On Fig. 4, 5 are shown dependences of the maximum concentration of the target component (target component concentration at the point of its maximum inside the cascade) and relative total flow of the cascade versus the length of end section $\varepsilon_0 S_E$. It can be seen that both of the curves has maximum values. However, it should be noted that the maxima of both dependencies are achieved at different values of $\varepsilon_0 S_E$. Comparison of the curves in Fig. 3-5 allows to determine approximately the optimum value $\varepsilon_0 S_E$ for the given (or approximately defined) values of the target component concentration in one (or both) of the cascade product flows. Found in this way the value $\varepsilon_0 S_E$ can then be used as an initial approximation of this parameter in the strict solution of the problem of optimization of Q-cascade with “extension”. Furthermore, approximately the results for a given value $\varepsilon_0 S_E$ of the optimal parameters of the cascade (the value of the relative total flow etc.) can be used during the estimations, for example, to determine the unit cost of production of stable isotopes in cascade with “extension”.

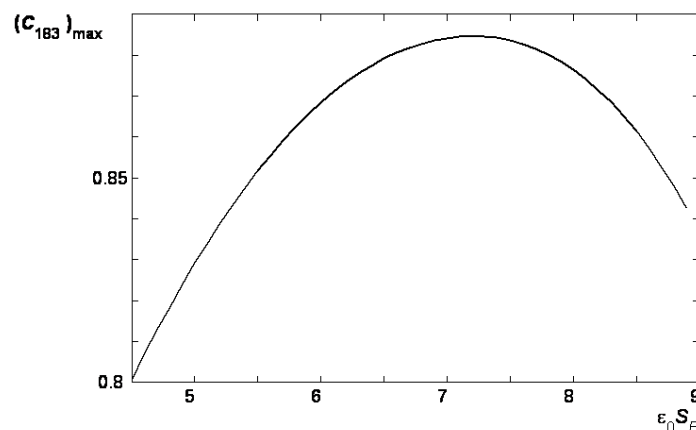


Figure 4. Dependence of the maximum concentration of ^{183}W inside the cascade versus length of the utmost section of the cascade with “extension” $\varepsilon_0 S_E$.

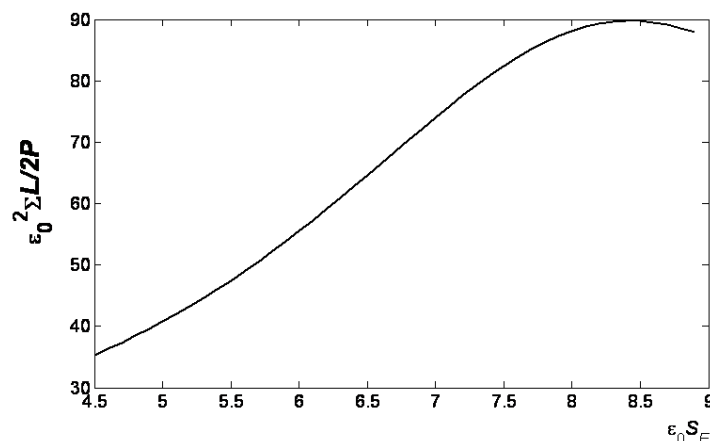


Figure 5. Dependence of the cascade relative total versus length of the utmost section of the cascade with “extension” $\varepsilon_0 S_E$.

It is quite obvious that the mass transfer regularities described above in the absence of the additional product flow will not differ from the relevant laws in cascades with the “extension” in case if the additional product will be quite small $E/P \ll 1$. Therefore, obtained relations can be used to

find the “good” initial approximations in solving problems of calculation and optimization of the parameters of cascades with “extension” with the additional product flow.

Conclusion

The presented results complement the theory of separation cascades having a local increase of working substance flow at its internal stages (cascades with the flow "extension") used to obtain isotopic products with relatively high concentrations of intermediate components. Based on the Q-cascade model theoretical dependences of the maximum possible concentration of the target component in the end product flow of the cascade, the maximum achievable concentration of the target component inside the cascade and the value of the cascade total relative flow versus the length of the cascade section between the end and intermediate product flows - $\varepsilon_0 S_E$ were obtained. Comparison of these dependences allows to approximately determine the optimal value $\varepsilon_0 S_E$ for the given value of the target component concentration in one of the cascade product flows.

Acknowledgement

This research is carried out in the framework of the joint Sino-Russian project “Separation of multicomponent isotope mixtures by the method of centrifugation to obtain new functional materials for nuclear energy, fundamental physics and medicine” supported by the Russian Fund for Basic Research (RFBR) under the contract no. 16-58-53058 GFEN_a and the National Natural Science Foundation of China (NSFC) under the contract no. 1151101155 (Sulaberidze G.A.), and the grant No.14.Y30.16.6284-MK of the President of the Russian Federation to support young scientists (Smirnov A.Yu.).

References

- [1] Sulaberidze G A, Borisevich V D 2001 *Sep. Sci. Technol.* **36** 1769–817
- [2] Minenko V P 1972 *At. Energy* **33** 814–6
- [3] Vetsko V M, Laguntsov N I, Levin E V et al 1987 *At. Energy* **63** 692-7
- [4] Sulaberidze G A, Borisevich V D, Xie Q X 2006 *Theor. Found. Chem. Eng.* **40** 5-14
- [5] Smirnov A Yu, Sulaberidze G A, Xie Q, Borisevich V D 2015 *Chem. Eng. Res. Des.* **95** 47
- [6] Smirnov A Yu, Sulaberidze G A, Borisevich V D 2010 *Theor. Found. Chem. Eng.* **44** 888-96
- [7] Smirnov A Yu, Sulaberidze G A 2013 *Theor. Found. Chem. Eng.* **47** 375-80
- [8] Zeng S, Sulaberidze G A, Jiang D, Borisevich V D, Smirnov A Yu 2012 *Sep. Sci. Technol.* **47** 1591–5