

Procedure of measuring the longitudinal emittance of electron beam

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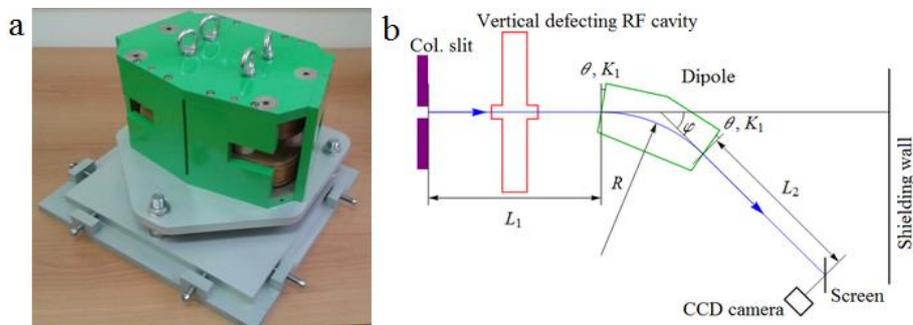
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Abstract. The procedure of measuring the longitudinal emittance of electron beam generated by RF gun and reconstruction of its longitudinal phase portrait is proposed. Measuring system consists of vertical deflecting RF cavity, horizontal bending dipole and screen. The beam spot on the screen is used to reconstruct the longitudinal phase portrait. In the proposed procedure an electromagnetic field of the vertical deflecting RF cavity can be approximated by the TM_{110} mode of pillbox cavity. This approximation allows analytically solve the motion equations of the electron motion in the vertical deflecting RF cavity. The report contains description of the vertical deflecting RF cavity and the dipole, the formulae underlying the procedure and the results of numerical simulation.

1. Introduction

The spectrometer for measuring the energy of the electron beams was designed in MSU Laboratory of electron accelerators (figure 1 (a)) for GunLab [1] and bERLinPro [2] projects. The aim of the bERLinPro project is to expand the required accelerator physics and technology knowledge which is mandatory for generation of a high current (100 mA), high brilliance (norm. emittance below 1 mm mrad) electron beam. Its predecessor is GunLab project. GunLab is an research and development platform for superconducting RF photoinjectors of bERLinPro.



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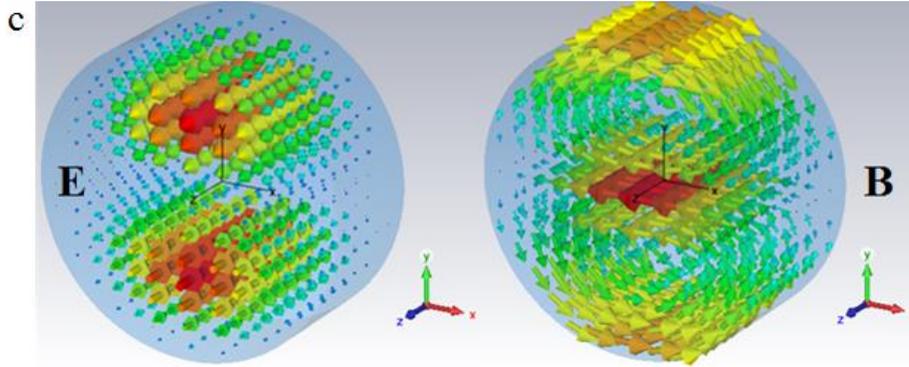


Figure 1. (a) The spectrometer dipole designed in MSU LEA; (b) the measuring system; (c) electromagnetic field of the TM_{110} mode.

The spectrometer is a dipole magnet. The main parameters of the spectrometer are listed in the Table 1. When the vertical deflecting RF cavity is placed in front of the dipole, you get a system (figure 1 (b)) that allows to reconstruct the longitudinal phase portrait and to measure the longitudinal emittance. The vertical coordinate of the electron on the screen is connected with the time of flight to the cavity and the horizontal coordinate of the electron is related to the momentum. For GunLab and bERLinPro projects the vertical deflecting RF cavity is developed in DELTA and its status can be found at [3].

Table 1. The main spectrometer parameters.

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
Gap	35 mm	L_2	1 m	φ	45°	K_1	0.246
L_1	0.8 m	R	0.25 m	θ	7.593°		

2. Theory

We will work with a pillbox cavity instead of the real vertical deflecting RF cavity. Electromagnetic field of the TM_{110} mode of the pillbox cavity can be written as [4]:

$$\mathbf{E} = \mathbf{e}_z \omega B_0 y \exp(-i(\omega t + \psi)), \mathbf{B} = i\mathbf{e}_x B_0 \exp(-i(\omega t + \psi)). \quad (1)$$

Here ω – cyclic frequency, B_0 – magnetic field amplitude. We suppose that the pillbox cavity length is a half of wavelength. The beam moves along the axis z of the pillbox cavity. The electromagnetic field of the TM_{110} mode of the pillbox cavity and coordinate system are shown on figure 1 (c).

To find the electron coordinates at the pillbox cavity output we should solve the motion equations of the electron in the electromagnetic field (1). Solutions have the form:

$$y_{\text{cav}} = y_0 - \frac{eB_0\beta\lambda^2}{2\pi^2 m_0 \gamma c} \left(\cos\psi - \frac{\pi}{2} \sin\psi \right), y'_{\text{cav}} \equiv \frac{p_{y\text{cav}}}{p_0} = \frac{p_{y0}}{p_0} + \frac{eB_0\beta\lambda}{\pi p_0} \sin\psi, \quad (2)$$

$$p_{z\text{cav}} = p_{z0} + \frac{e^2 B_0^2 \beta \lambda^2}{2\pi^2 m_0 \gamma c} \cos\psi \left(\cos\psi - \frac{\pi}{2} \sin\psi \right) - \left(2eB_0 y_0 + \frac{eB_0 p_{y0} \lambda}{2m_0 \gamma c} \right) \cos\psi. \quad (3)$$

Here λ – the wavelength, β – the relative speed, γ – the relative energy, m_0 – the electron rest mass, ψ – the field phase when the electron is flying in the cavity, p_0 – the average beam momentum, y_0 , p_{y0} , p_{z0} – the initial electron coordinate and momentum. Assume that the momentum spread of electrons in the beam is considerably less than the average beam momentum. Suppose also that at the entrance to the measuring system the beam has zero transversal size and zero divergence. After passing the

pillbox cavity beam flies rest of the drift section L_1 , the dipole and the drift section L_2 (figure 1 (b)). The electron coordinates on the screen are calculated by means of transformation matrix M of this system:

$$x_s = M_{16}(p_{z\text{cav}} - p_0)/p_0, y_s = M_{33}y_{\text{cav}} + M_{34}y'_{\text{cav}}. \quad (4)$$

We can find approximate expression for the flight phase of the electron in the cavity by substitution (2) to (4):

$$\psi = \arcsin\left(\frac{y_s}{\sqrt{A^2 + B^2}}\right) - \arcsin\left(\frac{A}{\sqrt{A^2 + B^2}}\right),$$

$$A = -\frac{eB_0\beta\lambda^2}{2\pi^2 m_0\gamma c} M_{33}, B = \frac{eB_0\beta\lambda^2}{4\pi m_0\gamma c} M_{33} + \frac{eB_0\beta\lambda}{\pi p_0} M_{34}$$

And we can find the electron momentum by substitution (3) to (4):

$$p_{z0} = \left(1 + \frac{x_s}{M_{16}}\right) p_0 - \frac{e^2 B_0^2 \beta \lambda^2}{2\pi^2 m_0 \gamma c} \cos\psi \left(\cos\psi - \frac{\pi}{2} \sin\psi\right).$$

Also we can estimate the average beam momentum and the momentum rms spread after the pillbox cavity using (2, 3):

$$p_z = p_{z0} + \frac{e^2 B_0^2 \beta \lambda^2}{4\pi^2 m_0 \gamma c} \left(1 + s \left(\cos(2\psi_0) - \frac{\pi}{2} \sin(2\psi_0)\right)\right), s = \frac{\lambda}{2\pi\Delta} \sin\left(\frac{2\pi\Delta}{\lambda}\right), \quad (5)$$

$$\sigma_{pz} = \sqrt{\sigma_{pz0}^2 + \frac{1}{2} \left((eB_0 \sigma_{y0})^2 + \left(\frac{eB_0 \sigma_{py0} \lambda}{2m_0 \gamma c} \right)^2 \right) (1 + s \cos(2\psi_0)) + C}, S = \frac{\lambda}{4\pi\Delta} \sin\left(\frac{4\pi\Delta}{\lambda}\right), \quad (6)$$

$$C = \frac{1}{2} \left(\frac{e^2 B_0^2 \beta \lambda^2}{4\pi^2 m_0 \gamma c} \right)^2 \left(\left(\frac{\pi^2}{4} + 1 \right) (1 - s^2) + (s^2 - S) \left(\pi \sin(4\psi_0) + \left(\frac{\pi^2}{4} - 1 \right) \cos(2\psi_0) \right) \right). \quad (7)$$

Here Δ is the beam length, ψ_0 – the field phase when the beam center is flying in the cavity. We take into account that the beam can has nonzero transversal rms size σ_{y0} and nonzero momentum rms spread σ_{py0} and σ_{pz0} . But we suppose that the beam has uniform distribution of electrons.

3. Comparison of theory and numerical simulation

We carried out numerical simulations of electron beam dynamic in the measuring system. The simulations were performed using ASTRA code [5]. The electron beam had the following characteristics: $\sigma_{y0} = 0.38$ mm, $\Delta = 11.5$ mm, $p_0 = 3.06$ MeV/c, $\sigma_{py0} = 1.88$ keV/c, $\sigma_{pz0} = 3.62$ keV/c, $\psi_0 = 0^\circ$. The frequency and amplitude of the electromagnetic field of the vertical deflecting RF cavity were equal to 1.3 GHz and 16.7 mT respectively.

Figure 2 (a) shows the beam spots on the screen of the measuring system. Figure 2 (b) shows the reconstructed longitudinal portraits. Portrait 1 is the true longitudinal phase portrait of the beam (rms longitudinal emittance $\varepsilon_z = 18.4$ ps MeV/c). When the beam passes the measuring system without the horizontal collimation slit, we see on the screen spot 2 and reconstruct portrait 2 (rms longitudinal emittance $\varepsilon_z = 33.3$ ps MeV/c). If the horizontal collimation slit (width 0.1 mm) is placed before the pillbox cavity, the beam spot on the screen will change to spot 3 and we reconstruct portrait 3 (rms longitudinal emittance $\varepsilon_z = 17.3$ ps MeV/c). Also we simulated the dynamics of the beam with zero

transversal size and zero divergence in the measuring system. Spot of this beam on the screen is spot 4 and its reconstructed longitudinal phase portrait is portrait 4 (rms longitudinal emittance $\epsilon_z = 20.2$ ps MeV/c).

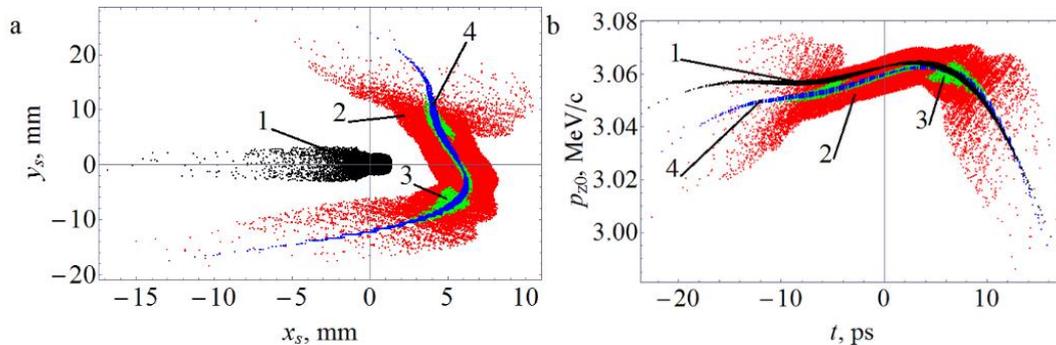


Figure 2. (a) The beam spots on the screen of measuring system; (b) the longitudinal phase portraits.

From figure 2 (b) we can see that the horizontal collimation slit at the entrance to the measuring system greatly improves the results of the longitudinal portrait reconstruction. This is because the reconstruction procedure assumes that the input electron beam has zero transversal size and zero divergence.

From figure 2 (a) it can be noted that the vertical deflecting RF cavity affects the average beam momentum. In figure 2 (a) spot 1 is the beam spot when the vertical deflecting RF cavity is turned off. It is evident that all the other beams, which were affected by the electromagnetic field of the mode TM_{110} of the vertical deflecting RF cavity, are shifted along the horizontal axis x_s . Simulations show that the average momentum and the rms momentum spread of the beam which was deflected by the cavity are equal to $p_z = 3.082$ MeV/c and $\sigma_{pz} = 7.14$ keV/c. Theoretical estimations (5-7) give the values $p_z = 3.082$ MeV/c and $\sigma_{pz} = 7.21$ keV/c.

4. Conclusion

The report describes the reconstruction procedure of the longitudinal phase portrait of the electron beam generated by RF gun. The results of applying this procedure are in a good agreement with the true longitudinal portrait and the longitudinal emittance. Furthermore, it was found that the vertical deflecting RF cavity affects the average momentum and the momentum rms spread of the beam. There are given good estimations of their behavior in this report.

References

- [1] Völker J, *et al* 2014 *Proc. IPAC2014* 630-632
- [2] Abo-Bakr M, *et al* 2014 *Proc. IPAC2014* 340-342
- [3] Ferrarotto A, Riemann B, Weis T, Kamps T, Glock H-W and Völker J 2015 *Proc. IPAC2015* 827-829
- [4] Burt G 2012 Transverse deflecting cavities *arxiv.org*
- [5] Floettmann K 2014 A space charge tracking algorithm *DESY*
http://www.desy.de/~mpyflo/Astra_manual/