

Multicriterial approach to beam dynamics optimization problem

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Abstract. The problem of optimization of particle beam dynamics in accelerating system is considered in the case when control process quality is estimated by several functionals. Multicriterial approach is used. When there are two criteria, compromise curve may be obtained. If the number of criteria is three or more, one can select some criteria to be main and impose the constraints on the remaining criteria. The optimization result is the set of efficient controls; a user has an opportunity to select the most appropriate control among them. The paper presents the results of multicriteria optimization of beam dynamics in linear accelerator LEA-15-M.

1. Introduction

The paper is devoted to multicriteria optimization of charged particle beam dynamics. The problem of beam dynamics optimization in accelerating and focusing systems is urgent and widely studied by many researchers. Saint-Petersburg State University takes part in the research in this area; in particular, the books and papers [1-9] should be mentioned.

Many beam dynamics optimization problems are conceptually multicriterial because there are several beam characteristics to be improved or restricted, and one has to introduce multiple quality criteria in order to find the best solution. Such an approach is considered in [10-11].

When several criteria are conflicting, the improvement of one of them leads to the worsening of the others. In this case multicriteria optimization is preferable. The optimization result is the set of efficient controls providing quality criteria vectors that can't be improved. The optimization problems with conflicting criteria are, in particular, treated in [10], [12-15].

2. Trajectory ensemble control problem

Let particle beam evolution be described by the following differential equations presented in [1]:

$$\frac{dx}{dt} = f(t, x, u),$$
$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x} f(t, x, u) + \rho(t, x) \operatorname{div}_x f(t, x, u) = 0,$$

with initial conditions:

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$$x(0) = x_0 \in M_0, \quad \rho(0, x) = \rho_0(x).$$

Here $t \in [0, T]$ is independent variable; T is a constant; $x \in \Omega \subset E^n$ is n -dimensional vector of particle phase coordinates; E^n is n -dimensional Euclidean space; $u = u(t)$ is r -dimensional control function; $f(t, x, u)$ is n -dimensional vector function; M_0 is an open bounded set of non-zero measure; $\rho_0(x)$ is continuously differentiable nonnegative function and $\int_{M_0} \rho_0(x_0) dx_0 = 1$.

Let us assume the initial particle phase state x_0 to be the value of random variable X_0 with probability density $\rho_0(x_0)$. Consider particle phase state $x(t, x_0, u)$ to be the value of an n -dimensional random variable X_t with probability density $\rho(t, x)$. Let us assume $f(t, x, u)$ to be sufficiently smooth function and control $u = u(t)$ to be piecewise continuous.

So for every admissible control $u(t)$ the ensemble of trajectories $x(t, x_0, u)$ emanating from M_0 is introduced. The set $M_{t,u} = \{x_t = x(t, x_0, u) : x_0 \in M_0\}$ is called trajectory ensemble cross-section.

Let us introduce beam dynamics quality criterion to be a functional of the form

$$I(u) = \int_0^T \int_{M_{t,u}} \Phi(t, x_t) \rho(t, x_t) dx_t dt + \int_{M_{T,u}} g(x_T) \rho(T, x_T) dx_T. \quad (1)$$

where $\Phi(t, x)$, $g(x)$ are smooth nonnegative functions. The integrands mentioned are chosen to correspond the physical meaning of quality functional. For example, they may be penalty functions providing the restrictions account [2], [15-18].

The problem of functional (1) minimization with respect to control vector $u(t)$ is defined to be trajectory ensemble programmed control problem. The function $u^{(0)}(t)$ providing functional (1) minimum is determined to be optimal control.

3. Formulation of multicriteria optimization problem

When treating beam dynamics optimization problems, we often use several criteria, sometimes conflicting. Let us consider the example, specifically, the problem of longitudinal beam dynamics control in travelling-wave linear accelerator. Particle phase state is described by the vector $x(t) = (\gamma(t), \varphi(t))$, where γ is reduced energy and φ is the phase. The optimization objectives are as follows: 1) to maximize bunching coefficient; 2) to provide the required energy $\hat{\gamma}$ at device exit; 3) to maximize capture coefficient k_c in the acceleration mode. The analogous problems are investigated in [1-2], [18].

We can formulate trajectory ensemble control problem to be multicriteria optimization problem following [9], [14]. Let us introduce the criteria $K_i(u)$, $i = \overline{1,3}$ corresponding to the objectives mentioned above. The particular criteria $K_i(u)$, $i = \overline{1,2}$, may be suggested in the form

$$K_1(u) = \int_0^T \int_{M_{t,u}} \Phi(\varphi_t) \rho(t, \gamma_t, \varphi_t) d\gamma_t d\varphi_t dt, \quad K_2(u) = \int_{M_{T,u}} G(\gamma_T) \rho(T, \gamma_T, \varphi_T) dx_T$$

The integrand $\Phi(\varphi)$ is penalty function defined as follows:

$$\Phi(\varphi) = g(\varphi, \varphi_{\min}(t), \varphi_{\max}(t)), \quad g(\omega, \omega_1, \omega_2) = \begin{cases} (\omega - \omega_1)^2, & \omega < \omega_1 \\ 0, & \omega \in [\omega_1, \omega_2] \\ (\omega - \omega_2)^2, & \omega > \omega_2 \end{cases}.$$

Here $\varphi_{\min}(t), \varphi_{\max}(t)$ are given functions describing the admissible phase boundaries. The integrand for the second particular criterion is $G(\gamma) = (\gamma - \hat{\gamma})^2$. The third criterion $K_3(u) = k_c$ is not represented in the integral form and is calculated numerically.

So we consider the criteria vector $K(u) = (K_1(u), K_2(u), K_3(u))$ instead of the functional (1). The problem is formulated to minimize the criteria $K_i(u)$, $i = \overline{1, 2}$ and to maximize the criterion $K_3(u)$.

Now we will transform the problem formulation to obtain the convenient visual representation of the Pareto set (i.e. the set of unimprovable criteria vectors). Let us choose the first two criteria to be main and impose the restriction on the third criterion:

$$K_3(u) \geq k_c^*,$$

where k_c^* is capture coefficient lower boundary. Using this approach we can obtain the Pareto set for two main criteria (it is the compromise curve).

4. Optimization algorithm

The algorithm of approximate compromise curve constructing is described in [9-10] in detail. In short form it can be presented as follows. Let us assume the control vector u to depend on the vector of parameters $\theta = (\theta_1, \dots, \theta_p)$ taking values in a given set $\Theta \subset E^p$, i.e. $u = u(\theta)$.

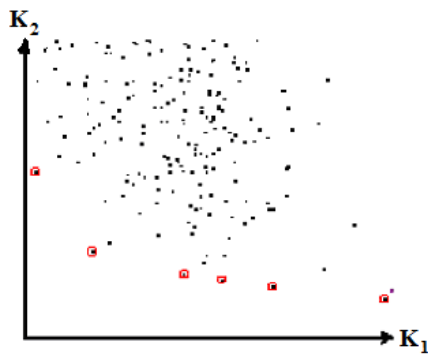


Figure 1. The points of the set V in main criteria space.

1). Simulation of N random vectors $\theta^{(i)}, i = \overline{1, N}$ uniformly distributed in Θ . The simulation methods are described in [14,19]. Control functions $u^{(i)} = u(\theta^{(i)})$, $i = \overline{1, N}$ obtaining.

2). Approximate calculation of trajectories of J model particles for every control $u^{(i)}$, $i = \overline{1, N}$. Calculation of criteria values $K_1(u^{(i)}), K_2(u^{(i)}), K_3(u^{(i)})$, $i = \overline{1, N}$ with

the use of Monte Carlo method expounded in [15,19].

3). Constructing of the set $V = \{K_1(u^{(i)}), K_2(u^{(i)}): K_3(u^{(i)}) \geq k_c^*, i = \overline{1, N}\}$ in main criteria space.

Table 1. The values of main criteria and capture coefficient.

K_1	K_2	K_3
0.0027	1.4563	0.94
0.0066	1.1284	0.90
0.0080	0.9685	0.92
0.0014	1.6221	0.90
0.0120	0.6424	0.91
0.0065	1.2440	0.90

4). Selection of unimprovable points in the set V according to the method given in [9-10], [14]. The points selected constitute the approximate compromise curve. The corresponding controls are called the *approximately efficient controls*.

5. Numerical results

The device under consideration is travelling-wave accelerator LEA-15-M [9] with initial energy $W_0 = 40$ keV, structure length $L = 0.78$ m and accelerating wavelength $\lambda = 0.1$ m. The numerical experiment was carried out for $N = 150$ random vectors of parameters and $J = 50$ model particles.

The points of the set V in main criteria space are presented at Fig. 1. The points of the approximate compromise curve are marked.

Table 1 presents the values of main criteria for the points of the approximate compromise curve and corresponding values of capture coefficient.

The controls obtained as a result of multicriteria optimization provide sufficiently high beam quality at accelerator exit. A user can select the proper control among approximately efficient controls taking into account beam and device characteristics obtained.

6. Conclusion

When treating beam dynamics optimization problems, we often use several criteria, sometimes In this paper multicriteria approach is applied to beam dynamics optimization problem. Multicriteria optimization of longitudinal beam dynamics in linear accelerator is carried out. The approximate compromise curve is obtained.

Multicriteria optimization method is especially effective in combination with directed methods based on the analytical representation of quality functional variation obtained by Ovsyannikov [1]. Any approximately efficient control may be used as initial control for the directed method.

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References

- [1] Ovsyannikov D A 1990 *Modeling and Optimization of Charged Particle Beam Dynamics* (Leningrad: Leningrad State University Press)
- [2] Vladimirova L V 1995 Optimization of interacting particle beam dynamics in linear accelerator *Journal of Computer and Systems Sciences Int.* **6** 178-83
- [3] Bondarev B I, Durkin A P et al 2001 The LIDOS.RFQ.DESIGNER development. *Proc. IEEE Particle Accelerator Conf. (Chicago, IL)* sponsors: US Department of Energy Office of Naval Research ed P Lucas and S Webber (IEEE) pp 2947-49
- [4] Ovsyannikov D A, Ovsyannikov A D, Antropov I V and Kozynchenko V A 2015 Software complex BDO-RFQ *Proc. III Int. Conf. "Stability and Control Processes" in Memory of V.I.Zubov (SCP) (5-9 October 2015 St. Petersburg)* ed L A Petrosyan and A P Zhabko (St. Petersburg: IEEE) pp 335-37 DOI: 10.1109/SCP.2015. 7342132
- [5] Ovsyannikov D A, Ovsyannikov A D, Antropov I V and Kozynchenko V A 2005 BDO-RFQ code and optimization models *2nd Int. Conf. on Physics and Control (PhysCon 2005) (24-26 August 2005 St. Petersburg)* 1513994 pp 282-88
- [6] Ovsyannikov A D, Ovsyannikov D A, Balabanov M Yu and Chung S-L 2009 On the beam dynamics optimization problem *Int. Journal of Modern Physics A* **24** (5) 941-51
- [7] Svistunov Yu A, Zuev Yu V, Ovsyannikov A D and Ovsyannikov D A 2011 The elaboration small size deuteron Accelerator for neutron generator 1 MeV energy *Vestnik St. Petersburg University, Ser. 10: Applied Mathematics, Informatics, Control Processes* **1** 49-59
- [8] Ovsyannikov D A 2012 Mathematical modeling and optimization of beam dynamics in accelerators *Proc. XXIII Russian Particle Accelerator Conf. RuPAC-2012 (24-28 September 2012 St. Petersburg)*(Geneva: JACoW <http://www.JACoW.org>) pp 68-72
- [9] Vladimirova L V 2014 Multicriteria optimization of beam dynamics *The Bulletin of Irkutsk State University. Series "Mathematics"* **7** pp 3-18
- [10] Sobol I M and Statnikov R B 2006 *Optimal Parameters Choice in the Problems with Multiple Criteria* (Moscow: Nauka)
- [11] Vladimirova L V Global extremum search on the basis of density and its mode estimation *Proc. 20th Int. Workshop: Beam Dynamics & Optimization (BDO) (30 June-4 July 2014 St. Petersburg)* (St. Petersburg: IEEE) p 186
- [12] Gubanov V A, Zaharov V V and Kovalenko A N 1988 *Introduction to System Analysis* (Leningrad: Leningrad University Publishing)
- [13] Podinovskiy V V and Nogin V D 1982 *Pareto-optimal Solutions of Multicriteria Problems*

- (Moscow: Nauka)
- [14] Vladimirova L V 2001 On application of multicriterial optimization in partial beam control problem *Proc. 11th Int. IFAC Workshop: Control Applications of Optimization* ed V Zakharov vol 1 (Oxford: Pergamon Press) pp 363-67
 - [15] Vladimirova L V 2014 Monte Carlo methods in beam dynamics optimization problem *Vestnik St. Petersburg University, Ser. 10: Applied Mathematics, Informatics, Control Processes* **1** 30-39
 - [16] Drivotin O I and Ovsyannikov D A 2006 Modeling of self-consistent distributions for longitudinally non-uniform beams. *Nuclear Instruments and Methods in Physics Research A* **558** (1) 112-118
 - [17] Vladimirova L V and Fatyanova I A 2015 Construction of regression experiment optimal plan using parallel computing. *Proc. III Int. Conf. "Stability and Control Processes" in Memory of V.I.Zubov (SCP) (5-9 October 2015 St. Petersburg)* ed L A Petrosyan and A P Zhabko (St. Petersburg: IEEE) pp 361-363 DOI: 10.1109/SCP.2015.7342140
 - [18] Rubtsova I D 2015 Optimization of iterative beam dynamic process *Proc. III Int. Conf. "Stability and Control Processes" in Memory of V.I.Zubov (SCP) (5-9 October 2015 St. Petersburg)* ed L A Petrosyan and A P Zhabko (St. Petersburg: IEEE) pp 198-200 DOI: 10.1109/SCP.2015.7342092
 - [19] Ermakov S M 1975 *Monte Carlo Methods and Related Questions* (Moscow: Nauka)