A theoretical basis of the approach for the magnetic field penetration measurement

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Abstract. An approach for the assessment of London penetration depth of superconducting films is proposed. This approach is based on the analysis of linear response of the sample to a local low-frequency alternating magnetic field generated by the measuring coil disposed near the film surface. A visual "electrical engineering" model of induced currents distribution in the superconductor taking into account the kinetic inductance was developed for a description of this response. The possibility of determining of the penetration depth from changing the inductance of the system "coil-sample" is shown in the framework of this model. The sensitivity of the proposed method for the films with different thicknesses is considered.

1. Introduction

Currently the problem of determining the temperature dependence of the magnetic field penetration depth λ is important, because it allows analyzing the microscopic properties of the superconducting condensate. Most of the existing methods for determining λ require complex expensive equipment (precise magnetometers, microwave technique, etc.) and use rather complicated theoretical models, which not always exactly correspond to the real geometry of the experiment.

The low-frequency linear induction technique examines the impact of superconducting sample on the self-inductance of the measuring coil and is the most attractive from the point of simplicity of technical realization. For thin films and plates a coil is located close to a sample surface and its axis is perpendicular to the sample plane. This type of apparatus enables measurements both in linear and in nonlinear induction techniques [1].

Thus, to establish a relation between the measured change in inductance ΔL and the magnetic field penetration depth λ, a model of the distribution of induced currents in superconducting films taking into account the geometry of the experiment is needed.

2. Model

A coil of circular cross section, placed near the surface of a thin superconductive film, is considered. Circular currents are induced in the sample when a sinusoidal alternating current $I_0(t)$ is passed through the coil. Two-dimensional model was developed to find a circular currents distribution. In framework of this model to describe an inhomogeneous current distribution in the sample, we consider the system of isolated thin coaxial turns (circuits), inductively coupled with each other and the excitation coil. The sample is divided notionally into $N=N_r{\cdot}N_z$ coaxial circuits with the centres located

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on the axis of the excitation coil. Here N_r is the number of turns in the radial direction, and N_z is the number of turns in the film thickness direction. For sufficiently thin films, with thickness not exceeding λ, we can put N_z=1 ("one-dimensional" model, 1D). "Two-dimensional" (2D) model with N_z >1, allowing to obtain the distribution of the current density in depth, is used in general case. Hereinafter, we assume a homogeneous current distribution in each turn. That is fulfilled when coil thickness is significantly less than the London penetration depth λ.

We introduce the following notation: r_k, dr and dz are radius, width and thickness of k–th circuit, $I_k(t)$ is current induced in it, L_0 is self-inductance of the coil, L_k is the full inductance of the k–th circuit (sum magnetic L_k^{mag} and kinetic L_k^{kin} inductances), M_k is the mutual inductance of the k–th circuit and the excitation coil, M_{ik} – mutual inductance of the i–th and k–th circuits coinciding with a magnetic part of L_k when $i = k$.

An equivalent electrical diagram corresponding to the problem is shown in Figure 1.

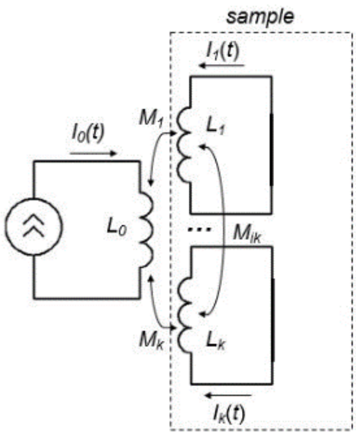

Figure 1. An equivalent electrical diagram corresponding to the problem.

Full magnetic fluxes through the excitation coil Φ_0 and model turns Φ_k can be written as:

$$\Phi_0(t) = L_0 I_0(t) + \sum_{i=1}^{N} M_i I_i(t),$$

$$\dots$$

$$\Phi_k(t) = M_k I_0(t) + \sum_{i=1}^{N} M_{ik} I_i(t).$$

When the sample is fully in superconducting state (linear mode) the current distribution in the sample is determined by the system of equations:

$$\frac{dI_k(t)}{dt} = -\frac{1}{L_k^{mag} + L_k^{kin}} \left\{ M_k \frac{dI_0(t)}{dt} + \sum_{i=1, i\neq k}^{N} M_{ik} \frac{dI_i(t)}{dt} \right\}$$

For the voltage across the coil terminals we write:

$$E(t) = -\frac{d\Phi_0(t)}{dt} = -L_0 \frac{dI_0(t)}{dt} - \sum_{i=1}^{N} M_i \frac{dI_i(t)}{dt},$$

where the second term represents the response of the film $\Delta E(t)$.

Consider,

$$\left[\frac{dI_k(t)}{dt} \right] = -\mathbf{W} \cdot \left[M_k \frac{dI_0(t)}{dt} \right],$$

$$\boldsymbol{W} = \boldsymbol{M}^{-1},$$

where the expression in square brackets meets the k-th elements of the corresponding vector-columns, $\mathbf{M} = [M_{ik} + \delta_{ik} L_i^{kin}]$ - the matrix of mutual inductances of the turns and δ_{ik} - the Kronecker symbol . The main diagonal of this matrix contains full inductance L_i. In this case, we get the formula:

$$I_k(t) = -I_0(t) \sum_{i=1}^{N} W_{ik} M_i ,$$

and the voltage response of the sample is expressed as:

$$\Delta E(t) = \frac{dI_0(t)}{dt} \sum_{k=1}^{N} M_k \sum_{i=1}^{N} W_{ik} M_i$$

The value

$$\Delta L = -\sum_{k=1}^{N} M_k \sum_{i=1}^{N} W_{ik} M_i$$

represents the change in self-inductance of the coil under the film influence and can be determined from the experiment. On the other hand, in the framework of the presented model it can be calculated as a function of the penetration depth λ. This gives the possibility of experimental estimation of λ value. Let us consider results of numerical calculation of $\Delta L(\lambda)$ dependence.

3. Results of the numerical calculation

Dependence of the coil inductance change ΔL on the magnetic field penetration depth λ calculated for different films thicknesses d is presented in Figure 2. As seen, the thinner the film the stronger the change of $\Delta L(\lambda)$ for small λ values. As low-temperature values of the London penetration depth λ lie in submicron lengths for both type-I and type-II superconductors, so it is reasonable to use thin films in measurements.

An important condition for the model is smallness of the circuits thickness in comparison with the London penetration depth, $dz << \lambda$. But reduction of the turns thickness is limited by the capabilities of the computational techniques used. Excessive fragmentation of the thickness may lead to significant increase of the computation time.

Figure 3 shows results of the calculations made for several film thicknesses and different numbers of circuits on the plate thickness N_z. It is clearly seen for the thick plates (d=1000 nm) that calculated $\Delta L(\lambda)$ values are less for N_z = 40 than for N_z = 1 and the difference is more pronounced for small λ values.

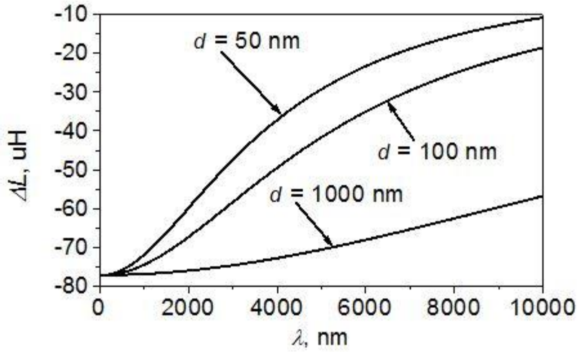

Figure 2. Dependences $\Delta L(\lambda)$ calculated for different film thicknesses. The calculations were made for N_r = 200, and N_z = 1. ΔL is in μH units.

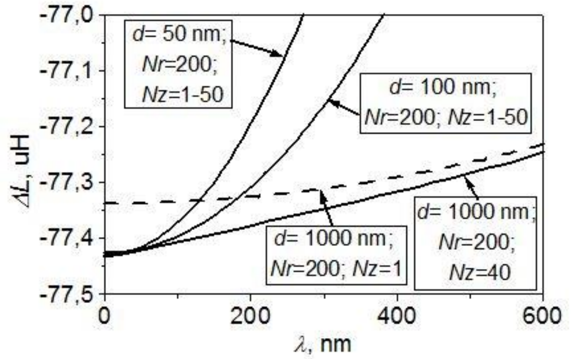

Figure 3. Dependences $\Delta L(\lambda)$ calculated for different film thicknesses. The calculations were made for N_r = 200 and different N_z. ΔL is in μH units.

Let us estimate the calculation error for different thickness of the circuits. Assuming an exponential decrease of the magnetic field through the sample thickness the dependence of the kinetic inductance of the turn with rectangular cross section has the form:

$$L^{kin}{}_{k\text{in}\,\text{hom}} = \frac{\pi\mu_0\lambda r_k \,\text{cth}(dz/2\lambda)}{dr}.$$

For a homogeneous distribution of magnetic field in the turn cross section, we can use more simple formula:

$$L^{kin}{}_{k\,\text{hom}} = \frac{\mu_0\lambda^2 2\pi r_k}{dzdr}.$$

In the described model this case corresponds to $dz \ll \lambda$. Ratio of the kinetic inductances calculated from these formula for different thicknesses of turns is shown in Table 1.

Table 1. Ratio of the kinetic inductances calculated for homogeneous and inhomogeneous distribution of magnetic field in circuits with different thicknesses. The thickness is in λ units.

dz/λ	1- $L^{kin}{}_{hom}$/ $L^{kin}{}_{inhom}$
0,01	0,00%
0,1	0,08%
0,2	0,33%
0,3	0,75%
0,4	1,33%
0,5	2,07%

It should be noted that accuracy of the model depends on the thickness dz of the circuits and dz should be small for precise estimation of the penetration depth λ.

4. Conclusions

The impact of superconducting film on low-frequency self-inductance of the measuring coil was analysed and the model for estimation of the London penetration depth λ from the experimental data was developed. We found that precision of λ estimation increases with decrease of both film thickness and thickness of the model turns dz. We obtained that for $dz = 0.3\lambda$ a contribution of the kinetic inductance value uncertainty to total error of λ estimation is less than 1%. There are no severe restrictions for the model turns width dr.

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References

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