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Non-stationary transformation of neutron energy by a moving grating

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Abstract. The report gives a short review of the studies dedicated to the investigation and applications of non-stationary quantum effects in neutron diffraction from moving periodic structures. The studies were performed in 1994-2015.

1. Introduction

It is known that the neutron is a very appropriate object for demonstration and investigation of wave properties of massive particles. In most performed or proposed experiments a neutron beam or wave are permanent and experimental results may be described with the stationary Schrödinger equation. However, neutron optics acquires new properties, when parameters describing interaction of the neutron wave with an object, depend on time. Non-stationary effect on the wave allows changing considerably such wave properties, as energy spectrum, spin, intensity, phase, direction of propagation, etc.

Apparently, Moshinsky [1] was the first to discuss the problem of the non-stationary quantum effect in the optics of the massive particle. He studied the evolution of the wave function after instantaneous extraction of a perfect absorber from a beam of monochromatic particles. Gerasimov and Kazarnovsky [2] considered the possibility of observation of a number of non-stationary quantum phenomena arising from the interaction of ultracold neutrons (UCN) with a potential barrier oscillating in time. Later on, a number of non-stationary effects were investigated theoretically and experimentally in [3-16].

Under certain conditions neutron diffraction may also be considered as a non-stationary phenomenon. In [17] UCN diffraction by surface Rayleigh waves was regarded as a cause of inelastic neutron scattering resulting in a decrease in the neutron storage time in traps.. Later, the diffraction of cold neutrons by surface acoustic waves generated on the surface of a quartz crystal was observed in [18].

Almost two decades after publishing paper [17], the effect of neutron energy change at diffraction by a moving grating was predicted again in [19]. The starting point of this paper was the idea of a moving grating as of a quantum chopper [5-7] or neutron wave modulator, as moving a periodic structure across the neutron beam modulates the transmitted flux with the frequency *V/d.* In fact, paper [19], where not only the quantum phenomenon itself, but also the possibility of its observation

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was discussed, initiated a long period of investigation of neutron diffraction by a moving grating (NDMG). In this report we present a short review of these papers continued up to now.

2. Prediction and first observation of the energy quantization at NDMG

In Ref. [19], theoretical analysis of the NDMG problem was based on several assumptions. Firstly, it was supposed that in the laboratory system of reference the neutron wave is normally incident on the grating surface $\Psi_{in}(z,t)=exp\left[i\left(k_0z-\omega_0t\right)\right]$, and the grating moves in the positive direction of the X-axis with the velocity V (see Figure 1). Secondly, the kinematic approach was used, according to which the grating was supposed to be ideally thin and characterized only by a periodic function of transmission $T(x)$.

As in Ref. [17], the diffraction problem was solved in the moving system of reference, where the grating is at rest. The diffraction results in appearance of a set of waves with identical wave numbers $k'_n=\left[k_0^2+k'^2_{xn}\right]^{1/2}$ and amplitudes

$$a_n=d^{-1}\int_0^d T(x)exp(-iq_nx)dx\,, \tag{1}$$

where $k'_{xn}=q_n-k_V$, $k_V=mV/\hbar$, $q_n=nq_0$, and $q_0=2\pi/d$ is the reciprocal lattice vector. Z -component k'_{zn} corresponds to each component k'_{xn}. It is obvious that z-components both in the moving and laboratory systems of reference are identical $k'_{zn}=k_{zn}$. The wave function in the laboratory system is

$$\Psi(z,x,t)=\sum_n a_n\,exp[i(q_nx+k_{zn}z-\omega_nt)]\,, \tag{2}$$

where

$$k_{zn}=\left(k_0^2+2k_Vq_n-q_n^2\right)^{1/2},\quad \omega_n=\omega_0+n\Omega,\quad \Omega=2\pi\left(V/d\right)\text{ and }\Omega<<\omega_0. \tag{3}$$

It is seen from equations (2) and (3) that the solution is characterized by a discrete energy spectrum and in a limit of small diffraction angles, when $q_0^2<<k_0^2$, the waves of different diffraction orders differ only by z-components of the wave numbers and correspondent frequencies. In this limit $k_{zn}\cong k_0\left(1+n\Omega/\omega_0\right)^{1/2}$ and the wave function is almost identical to the state formed by the periodical quantum chopper [6, 9].

The cases of the amplitude and phase modulation were both considered in [19]. A special case of the so-called π-grating was analyzed separately. In this case, function $T(x)$ takes the value of 1 or $exp(i\pi)$ altering each half of a period. It is known that at the normal incidence on such a grating the amplitudes of even orders (including zero order) are $a_n=0$, whereas the odd-ordered ones are $a_n=2i/\pi n$ $(n=2s-1)$.

Discussing the possibility of experimental observation of the new quantum phenomenon, it was proposed to use ultracold neutrons passing through two interference filters [20-23], one of which serves as a monochromator and the other as an analyzer. A spectrometer based on this idea was later constructed [24] and the effect of spectrum splitting at UCN diffraction by a moving grating was demonstrated [25, 26] (see Figure 2). Instead of a linearly moving grating, a rotating grating manufactured on the surface of a silicon disk was used. In its peripheral region the radial grooves were made using the lithographic technique. Its angular period was 3.33×10^{-4} rad^{-1} corresponding to a space period of 20 mkm at a diameter of 120 mm. The grooves covered a half of the grating period. The grating could be spinned around the vertical axis. UCN reach the grating through an annular corridor. Owing to the refraction in silicon, neutron waves, which have passed through different elements of the grating, possess different phase shifts $\Delta\varphi=k_0(1-n)h$, where n is the refractive index.

The depth of the grooves h was selected to be approximately 0.14 mkm, which corresponds exactly to a phase shift $\Delta\varphi = \pi$.

In the following experiment [27] UCN spectra appearing due to diffraction at a moving grating were measured at several rotating rates. From the common analysis of these results the intensities of ± 1 diffraction orders were found as $|a_1|^2_{exp} = 0.383(8)$, being in reasonable agreement with the theoretical value $|a_1|^2_{th} = 0.405$ found in kinematic approximation. At the same time it was realized that neglecting the grating form is a very rough approximation, as in the moving system of reference neutrons are incident on the grating at a glancing angle (see figures 3, 4).

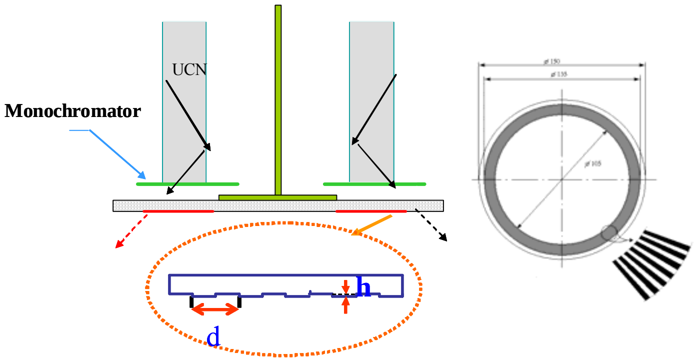

Figure 1. Principle of the experiment. Phase grating is prepared on the surface of the silicon disc, which can be rotated. On the right - geometry of grating and orientation of grooves.

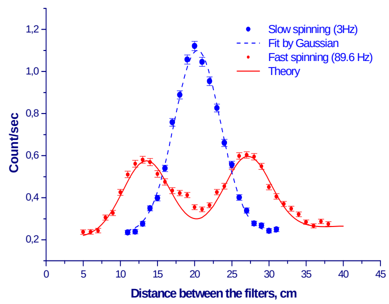

Figure 2. First demonstration of the energy splitting at diffraction of UCN by a moving (rotating) grating [25, 26].

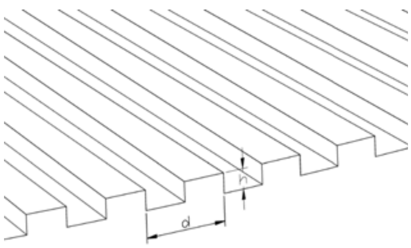

Figure 3. 3D structure of grating.

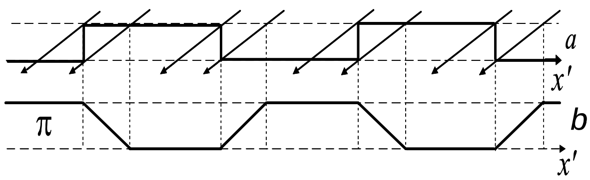

Figure 4. a - grating profile, b - phase profile.

Therefore, in Ref. [28] it was proposed to modify the transmission function of the grating for the phase profile to be of a trapezoidal form, as it is illustrated in Figure 4. In this case, the amplitudes of even and odd orders become equal to $a_n = CB_n$, and $a_n = -B_n/n$, respectively, where $B_n = [1+\exp(-i\pi nC)]/[i\pi(1-n^2C^2)]$, $C = 2hV/dv_{oz}$ and v_{oz} is the neutron velocity.

The intensities of the diffracted waves predicted by this model differ remarkably from the model of a plane grating (see below Figure 8). While the grating velocity V and dimensionless parameter C are both increasing, the first-order intensities of waves should decrease, and in contrast to this, the intensities of the zero and second orders are increasing. However, under the conditions of experiments [25, 26] parameter C was of the order of 0.1 and the effect was insignificant.

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3. Applications of neutron diffraction by a moving grating

Concluding paper [26], the authors wrote “We believe that this effect, which has been observed for the first time, may be used for controlled changing of the neutron energy in other experiments. The idea of neutron time-focusing, as proposed earlier, now looks more realistic”. Herein they referred to papers [29, 30], where the possibility of application of non-stationary acting at a neutron wave in order to create a neutron time lens was discussed. This idea was discussed in connection with the proposal of pumping UCN produced by a pulse source into a trap [31].

Soon afterwards, the experiment demonstrating the possibility of creation of the neutron time lens was implemented. For the time focusing of neutrons an aperiodic rotating grating was used. Neutrons passed through a small fragment of the grating in such a way that its effective space frequency ξ, i.e. the number of grooves per a length unit, and, accordingly, the transferred energy $\varDelta E = 2\pi\hbar V\xi$ varied during the rotation period of the grating. Neutrons of +1 diffraction order were accelerated and those of -1 were decelerated (see Figure 5). The first results of the experiment on neutron time focusing were reported at ECNS-2003 conference [32] and described in greater detail in Ref. [33]. The acting of the neutron time lens is illustrated in Figure 6, which displays the result never published before

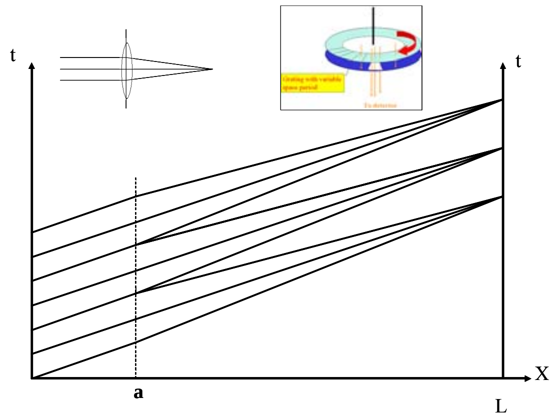

Figure 5. Coordinate versus time scheme of the experiment with the time lens.

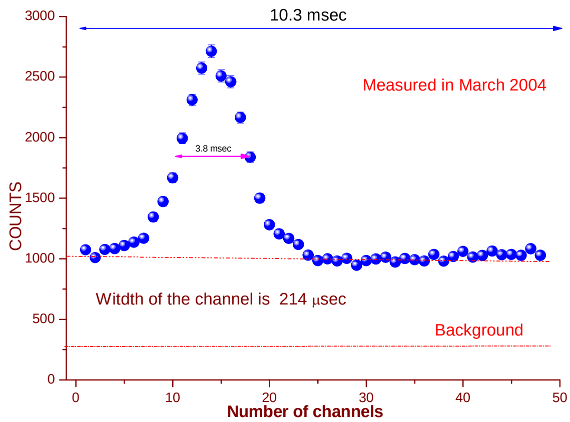

Figure 6. Time focusing peak representing distribution of the neutron arrival times plotted on a time scale equal to the grating rotation period.

Later on, it was realized that the energy quantization effect at NDMG can be used for the investigation of neutron gravity properties [34]. The idea was to compare the gravity energy *mgH* that a neutron gets, when falling from the height *H* with the energy $\varDelta E = \hbar\varOmega$ transferred to the neutron due to interaction with a non-stationary device. The proper experiment was performed in 2006 [35, 36]. The narrow energy spectrum of neutrons *f*(*E*) was formed by a monochromator located at the height *H* = 0. In order to arrive at the detector and to be detected, neutrons had to pass through an analyzer, which was located below the monochromator by height *H* and had the same spectrometric properties. Neutrons were accelerated in the Earth’s gravitational field in the path between the monochromator and analyzer. Since the spectral function *f*(*E*) was narrow, the system already became opaque for a relatively small distance *H*. However, if the neutron energy additionally decreased by $\varDelta E = \hbar\varOmega$, it was possible to find the position, in which the analyzer transmitted neutrons. In this case, the dependence of the system transmittance on the analyzer position was described by the symmetric function

$$\varPhi(H) = \int f(E) f(E + m_i g_n H - \hbar\varOmega) dE \,, \tag{4}$$

where m_i is neutron inertial mass and $m_i g_n$ is the force acting at the neutron in the gravity field of Earth. The maximum of this function or scanning curve was determined by the condition $m_i g_n H = \hbar\Omega$. Neutron interference filters were used as a monochromator and analyzer in this experiment. The rotating grating with the angular period $\alpha = 8.333\times10^{-5}$ rad corresponding to a space period of 5 mkm at a diameter of 120 mm was used in this experiment. It was obtained for the factor of correspondence $\gamma = 1-\left(m_i g_n / m_n g_{loc}\right)$, $\gamma = (1.8\pm2.1)\cdot10^{-3}$ where m_n and g_{loc} are neutron table mass and local value of the free fall acceleration, respectively. The scanning curves (4) measured at different rotating rates are shown in Figure 7.

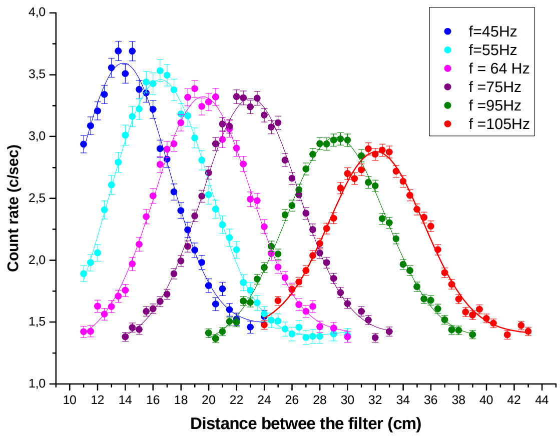

Figure 7. Scanning curves for the line of -1 diffraction order measured for various grating rotation speeds and the fitting Gaussians.

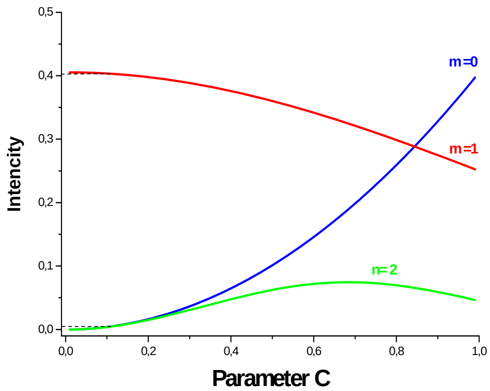

Figure 8. Intensities $J_m = |a_m|^2$ of the waves of the diffraction order m as a function of parameter *C*

As it is seen in Figure 7, the intensity of the first-order diffraction line decreases remarkably with an increase in the rotation rate. Qualitatively, it corresponds to the prediction in [28] (see Figure 8), whereas quantitative comparison was not performed.

The intension to increase precision of the experiment in order to test the equivalence principle stimulated the improvement of the experimental method. The main idea of comparing the gravity energy *mgH* with the transferred quantum of energy $\Delta E = \hbar\Omega$ was the same as before, but the approach to the UCN spectrometry changed significantly. The implementation of this project is still in progress; nevertheless, it became evident that the role of even diffraction orders was seriously underestimated in the process of experiment planning. Factor *C* was four times larger than in [25, 26], and the intensities of even orders rising with the growth of factor *C* increased remarkably. Due to the fact that the value of the energy splitting $\Delta E = \hbar\Omega$ was much larger than in the previous experiment, it was not possible to measure the spectrum using the existing spectrometer [37]. Moreover, the theory did not give any reliable answer to the question of the relation of different diffraction orders. It became obvious that the phenomenon of NDMG had to be investigated in further detail.

4. Modern stage of NDMG investigation

4.1. Dynamic theory of multiwave diffraction

A significant breakthrough in understanding theoretical aspects of the problem was achieved with appearance of paper [38]. In contrast with Ref. [19, 28] the dynamic approach was developed, taking into account both the three dimensional structure of the grating (see Figures 3 and 4) and the cross-interaction of the diffracted waves at their propagation in the matter of the grating. Further on, we summarize the main results of this work.

We shall consider the problem in the moving system of reference, where the grating is at rest. In this system the wave is obliquely incident on the grating:

$$\Psi'_{in}(x',z,t) = A_{in}(x')\exp\left(ik_0 z - i\omega' t\right), \tag{5}$$

where $A_{in}(x')=A_0\exp(-ik_V x')$, $k_V = mV/\hbar$, $\omega' = \omega_0 + \omega_V$, $\omega_V = \hbar k_V^2/2m$.

Neutron propagation in a medium is described by the Schrödinger equation

$$\Delta\psi\left(\mathbf{r},t\right) + \left[k^2 - k_b^2(\mathbf{r})\right]\psi(\mathbf{r},t) = 0, \tag{6}$$

where k is the wave number of neutrons in vacuum, $k_b^2(\mathbf{r}) = 4\pi N(\mathbf{r})b(\mathbf{r})$, $N(\mathbf{r})$ is the density of nuclei, $b(\mathbf{r})$ is the coherent scattering length of the medium. Let us assume that the function $k_b^2(x)$, periodic in the region $0 \le z \le h$, has the form

$$k_b^2(x) = \sum_{n=-\infty}^{\infty} \chi_n \exp(iq_n x), \tag{7}$$

where $q_n = 2\pi n/d$ are reciprocal lattice vectors,

$$\chi_n = \tfrac{1}{d}\int_0^d k_b^2(x)\exp(-iq_n x)dx\ . \tag{8}$$

A zero Fourier amplitude χ_0 determines the value of the average grating refractive index $n_e = (1 - \chi_0/k^2)^{1/2}$ in a layer of thickness h.

We can write the wave function of neutrons in the region $0 \le z \le h$ in the moving coordinate system using (5) as the sum of Bloch functions with amplitudes $\Psi_m(z)$ depending on the vertical coordinate z:

$$\Psi'(x',z,t) = \sum_{m=-\infty}^{\infty} \Psi_m(z)\exp(i\mathbf{q}_m\mathbf{r}' - i\omega' t), \tag{9}$$

where the projections of the wave vectors are $q_{mx} = q_m - k_V$, $q_{mz} = k_{0z} = (k_0^2 - \chi_0)^{1/2}$. Herein, it was assumed that the wave number in vacuum is $k' = (k_0^2 + k_V^2)^{1/2}$, and in the layer $0 \le z \le h$ it is $k_{0z} = (k'^2 n_e^2 - k_V^2)^{1/2}$.

By inserting (7) and (9) into (6), we equate the terms with equal exponents and neglect the second derivative of Ψ_m with respect to z. As a result, we obtain a system of differential equations:

$$\frac{d\Psi_m}{dz} = i\alpha_m \Psi_m - i\sum_{n\neq 0}\beta_n \Psi_{m-n}\ , \tag{10}$$

where $\alpha_m = q_m(2k_V - q_m)/2k_{0z}$, $\beta_n = \chi_n/2k_{0z}$. The system of equations (11) should be supplemented by boundary conditions: $\Psi_0(z = 0)= A_0$, $\Psi_{m\neq 0}(z = 0) = 0$. It may be solved digitally.

4.2. Fourier time-of-flight spectrometry

New theoretical results [38] have brought us to the understanding of the need for more detailed experimental investigation of UCN spectra in diffraction by a moving grating. As it was mentioned above, gravity spectrometry with interference filters used in Refs. [23, 24, 37] appeared to be inadequate to the problem. The most appropriate method for the solution of the problem was time-of-flight (TOF) gravity spectrometry [39]. For this purpose the spectrometer [37] was upgraded significantly and converted into a TOF Fourier UCN spectrometer. This spectrometer was used for the new measurements of UCN spectra at diffraction by a moving phase grating. The description of the spectrometer and particular results of diffraction spectrum measurements are reported in [40].

One of the measured spectra compared with theoretical calculations based on the solution of system (10) is shown in Figure 9. It can be observed that the theory corresponds to the results of the experiment quite satisfactorily. Using time-of-flight Fourier spectroscopy, it is possible to measure the

UCN spectra appearing at neutron diffraction by a moving grating in a wide energy range comparable with the initial neutron energy.

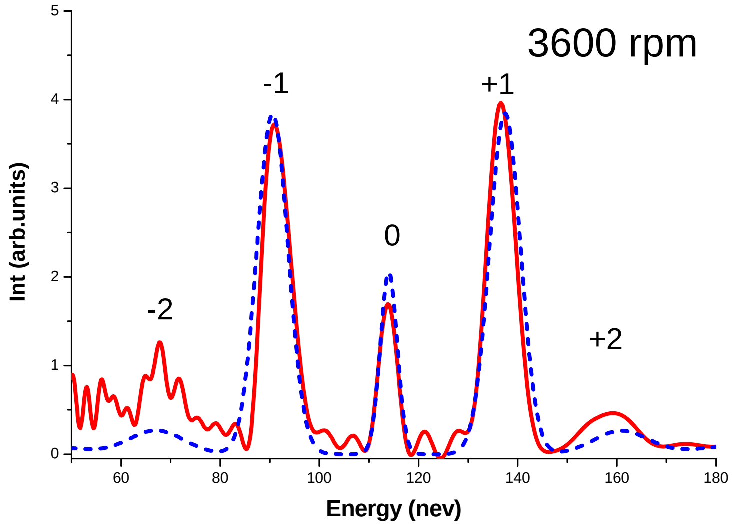

Figure 9. Energy spectra of neutrons diffracted by a moving grating measured by TOF Fourier spectrometer. Experimental (red solid) and theoretical (blue dotted) spectra are shown.

5. Prospects

Presently, it is rather difficult to predict any future application of this relatively new phenomenon. The theory stated in [38] provides us with a wide range of possibilities of formation of diffraction spectra with defined parameters. In particular, it is possible to increase considerably the intensities of higher diffraction orders, for example, the second one, simply by varying the depth of grooves. Using the glancing moving grating [41], it is possible to concentrate the entire flux only in one diffraction order, and consequently, to accelerate and decelerate neutrons without any noticeable widening of the spectrum. The interferometer with a moving grating [42] can have a unique sensitivity to small forces acting at the neutron.

Summarizing the above-said, we have shortly discussed gravity experiments methodically based on the condition $\delta E = \hbar\Omega - mgH = 0$. Let us briefly consider another possibility of using non-stationary neutron diffraction based on the circumstance that the amount of energy $\Delta E = \hbar\Omega$ transferred to the neutron can be accurately defined.

Assumingly, the aim of the experiment is to measure parameters of the motion equation $F(x,E,m,a,t)=0$, where coordinate x, mass m, energy E or velocity $\upsilon = (2E/m)^{1/2}$, acceleration a, or force $f = ma$ and time t. It is evident that in order to measure one of the parameters, we have to define the others independently. For example, to measure the free-fall gravity acceleration g of the neutron in the so-called 'free falling experiment', it is necessary to accurately determine the height of falling H and the time of falling t. However, having any periodical non-stationary device such as a moving grating, it is possible to split the initial spectrum and get a state with a set of energies $E_n = E_0 + n\hbar\Omega$, $n = 0, \pm 1, \pm 2$.. with precisely known values $n\hbar\Omega = n\Delta E$. By measuring the time of flight, for instance, for five diffraction orders, we will have not one, but five equations $F_n(x,E_0+n\Delta E,m,a,t_n)=0$ with four unknown values x,E_0,m,a. The accuracy of this common measurement can be improved by overdetermining the equation system applying the method of multiplication of the number of frequencies Ω_j.

The possibility of implementation of this idea to test the equivalence principle for the neutron is considered in the report of Zakharov et al. at the present conference [43].

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