

Modeling of the surface-to-surface radiation exchange using a Monte Carlo method

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Abstract. The thermal conditions inside of switch cabinets are strongly influenced by radiation heat transfer. To achieve accurate simulation results for the temperature distribution inside of switch cabinets a surface-to-surface radiation model is developed, which calculates the net heat flows for given temperatures of the plane, grey, diffuse surfaces. The accuracy of this calculation strongly depends on exact values for the radiation view factors. A Monte Carlo method is applied to compute the view factors. Several variations for grid spacing, the number of emitted photons and a quasi-Monte Carlo method are discussed. The radiation model is validated using numerical comparative data for a switch cabinet of given size with and without electrical components inside.

Keywords: radiation, surface-to-surface model, Monte Carlo method

1. Introduction

Packaging density and volumetric dissipation power of electronic components inside of switch cabinets for manufacturing increased over the years. This leads to high temperatures inside of switch cabinets and with that to a reduction of component life time, which endangers the functionality of the whole switch cabinet. That is why most switch cabinets must be equipped with active cooling devices [1]. In order to improve the design process of the cooling devices, reliable simulation models for the temperature distribution are required. Because of the level of the component surface temperatures inside of switch cabinets radiation heat transfer has a strong influence on air temperature distribution. Due to unavailable suitable models a surface-to-surface radiation model is developed to compute the radiation net heat flows among the component surfaces. The radiation view factors, which depend on the arrangement of the surfaces inside the switch cabinet, are computed by a Monte Carlo method.

2. Surface-to-Surface radiation model

For modeling the surface-to-surface radiation it is assumed, that the radiation heat transfer is not influenced by the air inside of the switch cabinet. Moreover the radiating surfaces are considered to be grey Lambert radiators [2]. That means, that the surfaces are both diffuse grey emitters and reflectors. In addition Kirchoff's law is applied, because of small temperature differences between the radiators.



Thus the total hemispherical emissivity ε of the radiating surfaces is equal to the absorptivity α and is assumed to be constant.

To compute the net heat flow for each surface within the switch cabinet, it is handy, to define the radiosity H , which is the sum of the emissive power M and the reflected part of surface irradiance E .

$$H = M + (1 - \varepsilon)E \quad (1)$$

Thus, for the prescribed assumptions, the following system of linear equations (2) can be formulated to calculate the radiosities H_j for each surface for given temperatures T_i of all surfaces in dependence of the view factors φ_{ij} [2]. In equation (2) δ_{ij} denotes the Kronecker symbol and σ the Stefan-Boltzmann constant.

$$\sum_{j=1}^n [\delta_{ij} - (1 - \varepsilon_i)\varphi_{ij}]H_j = \varepsilon_i\sigma T_i^4 \quad (2)$$

Once the system of linear equations (2) has been solved for H_j , the radiation net heat flows \dot{Q}_i for each surface can be computed for given areas A_i using equation (3).

$$\dot{Q}_i = A_i[H_i - E_i] = \frac{A_i\varepsilon_i}{1 - \varepsilon_i} (\sigma T_i^4 - H_i) \quad (3)$$

The main difficulty in applying the proposed surface-to-surface model is the computation of the n^2 view factors, where n denotes the number of surfaces. Advantageous in the application of the model is, that for given view factors and temperatures the net heat flows are calculated by solving a system of linear equations, which can be computed easily applying robust and fast numerical algorithms.

3. Calculation of view factors using the Monte Carlo method

The basis for the Monte Carlo method are random experiments, which are repeated very often, to obtain an estimation of the real solution. To calculate radiation view factors using the Monte Carlo method discrete particles of energy, so called photons, are randomly emitted from each surface within the switch cabinet. By a ray tracing algorithm the relative frequencies, how often the photons emitted from surface i hit the surfaces j , are computed. These relative frequencies represent an estimation of the radiation view factors.

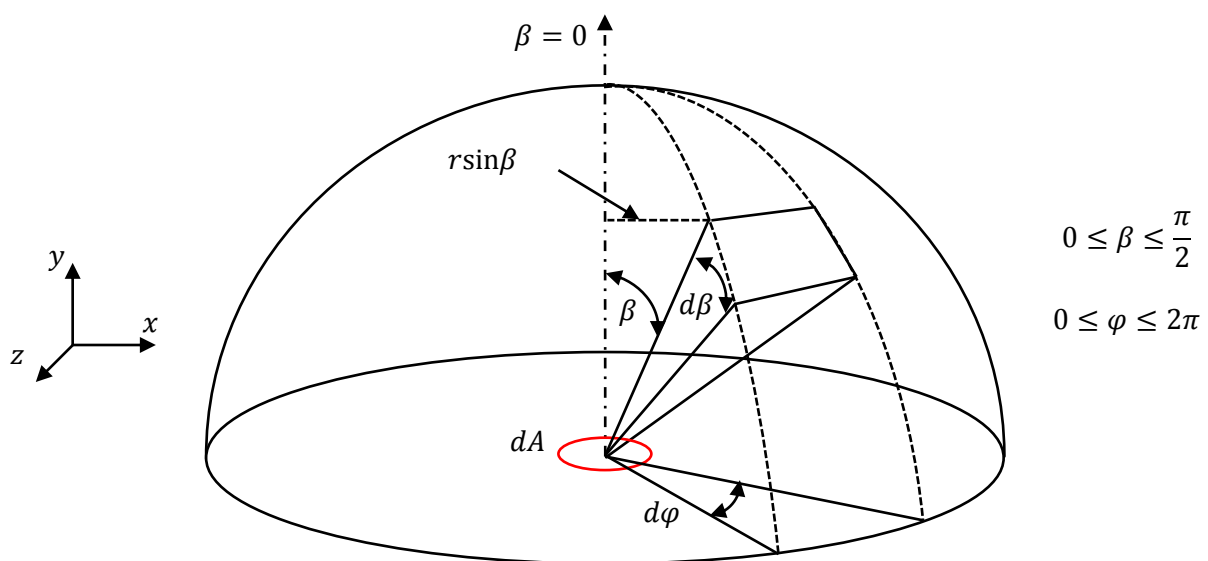


Figure 1. Polar angle β and azimuth angle φ for a radiating surface

In order to obtain results, which converge to the real solution for an infinite number of emitted photons, the theoretically known distribution of radiant emission must be taken into account. Figure 1 shows the geometric conditions of a radiating surface. The infinitesimal energy per unit time $d\phi$ emitted by the surface element dA in a definite direction in the half space can be calculated applying the following equation in spherical coordinates.

$$d\phi = I \cos \beta \sin \beta d\beta d\varphi dA \quad (4)$$

For a grey diffuse surface the total intensity I is independent from the polar angle β and the azimuth angle φ . For the Monte Carlo method equation (4) is interpreted in a statistical way and thus distribution functions both for the azimuth angle φ and for the polar angle β can be determined. The following describes the procedure for the polar angle β . The probability density function for a photon in a definite polar angle direction β can be calculated by normalization of the terms containing β in equation (4) [3].

$$f(\beta) = \frac{\cos \beta \sin \beta}{\int_0^{\pi/2} \cos \beta \sin \beta d\beta} = 2 \cos \beta \sin \beta \quad (5)$$

Most computer programs have random number generators, which output uniformly distributed random numbers R in the interval $[0,1]$ [4]. Thus for the polar angle β the following correlation between the random number R_β and the probability density function (5) is valid.

$$R_\beta = \int_0^\beta 2 \cos \beta \sin \beta d\beta = \sin^2 \beta \quad (6)$$

If equation (6) is solved for β , the quantile function of $f(\beta)$ results.

$$\beta = \arcsin \sqrt{R_\beta} \quad (7)$$

In an analogous way a relation between the azimuth angle φ and the uniformly distributed random number R_φ can be deduced.

$$\varphi = 2\pi R_\varphi \quad (8)$$

For a full description of the photon path besides the polar and azimuth angles the point of emission must be determined. In principle it is possible to formulate a relation between two more random numbers R_x and R_y and the related coordinates on the surface x and y as described in [3]. In the present work the procedure according to Hoff & Janni [5] is applied. The surfaces are divided into squares depending on a user defined grid spacing. The point of photon emission is determined using the midpoint of each squared subarea. The investigations of Vujicic et al. [6] showed that neither randomly chosen points of photon emission nor the midpoints of subareas for photon emission indicate any advantage. Besides the grid spacing n_{gs} the user has to define the number of emitted photons per squared subarea N_p .

3.1. Ray tracing algorithm

For the present work only plane geometries are considered. Thus it is useful to introduce a global Cartesian coordinate system (x, y, z) as shown in Figure 1. For the ray tracing algorithm an equation of a straight line vector \vec{g} is defined for each emitted photon in the global coordinate system.

$$\vec{g} = \vec{M} + p \cdot \vec{d} \quad (9)$$

In equation (9) p represents a parameter and the vector \vec{M} is the point of photon emission in the global Cartesian coordinate system. The direction vector \vec{d} depends on the polar angle β , the azimuth

angle φ and the orientation of the emitting surface, which is defined by a normal vector of unit length \vec{n} . The direction vector \vec{d} is calculated in dependence of the rotation matrix \bar{R} using equation (10).

$$\vec{d} = \bar{R} \cdot \vec{n} \quad (10)$$

with

$$\bar{R} = \begin{bmatrix} \cos \beta & \sin \beta \cos \varphi & \sin \beta \sin \varphi \\ \sin \beta \cos \varphi & \cos \beta & \sin \beta \sin \varphi \\ \sin \beta \sin \varphi & \sin \beta \sin \varphi & \cos \beta \end{bmatrix} \quad (11)$$

The next step is to check, which surface j^* is hit by the photon emitted from surface i . For this purpose the intersection between the photon patch defined by the equation of a straight line (9) and any component surface is computed. Usually more than one intersection exists. The surface j^* , which has the intersection with the closest distance to the point of emission is the one, which is hit by the photon. The ray tracing algorithm is executed for the total number of emitted photons of the surface $N_{photons}$ defined in equation (12), whereas N_{sub} denotes the number of squared subareas of the surface.

$$N_{photons} = N_{sub} N_p \quad (12)$$

Finally the view factors from the emitting surface i to any other surface j can be computed using equation (13).

$$\varphi_{i,j} = \frac{n_{ij}}{N_{photons}} \quad (13)$$

n_{ij} represents the number of photons emitted from surface i , that hit surface j . The described procedure is executed for every surface within the switch cabinet.

3.2. Quasi-Monte Carlo method

Standard Monte Carlo methods use independent, randomly distributed numbers R_φ and R_β for the ray tracing algorithm. One disadvantage of this method is the quite slow convergence rate of $N_{photons}^{-1/2}$, which is associated with the standard deviation of random methods [6]. One way to improve the convergence rate, is to use so called low discrepancy sequences for R_φ and R_β , which are more uniformly distributed than a random sequence.

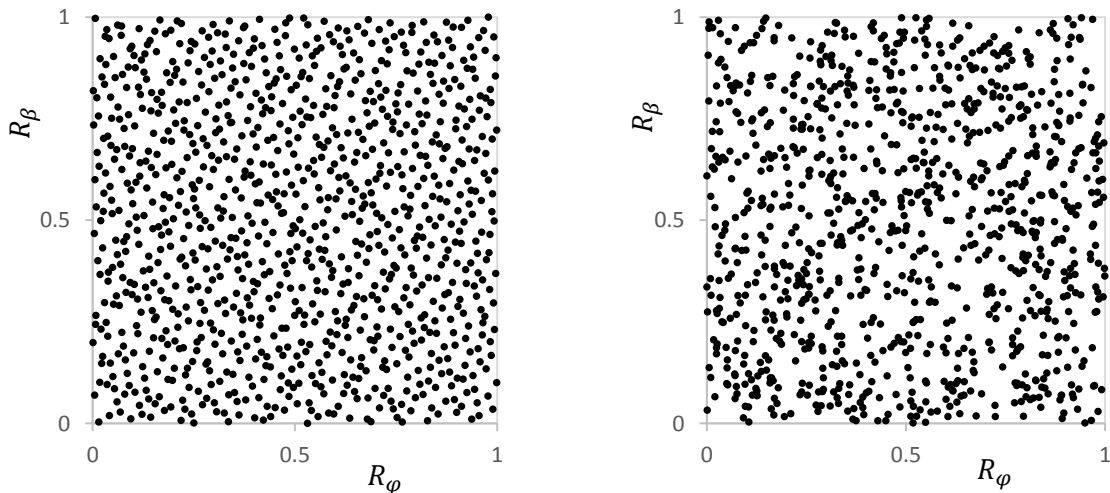


Figure 2. R_β on R_φ generated by a Halton sequence (left) and pseudo-random sequence (right)

Thus they are called quasi-random sequences and consequently the Monte Carlo method becomes a quasi-Monte Carlo method. One example for a low discrepancy sequence is the Halton sequence [7]. In Figure 2 R_β is plotted over R_ϕ . The points on the left-hand side are calculated using a Halton sequence, the points on the right-hand side are pseudo-random numbers generated using Matlab. The points of the Halton sequence are more uniformly and more smoothly distributed compared to the pseudo-random points. Therefore a faster convergence rate is expected for the quasi-Monte Carlo method than for the Monte Carlo method.

4. Test cases and results

Two test cases are examined to verify the radiation model. Test case 1 considers an empty switch cabinet, for which the view factors can be computed analytically. Figure 3 shows the configuration of test case 1 with associated dimensions and numbering of the thermal irradiating surfaces. Correlations for the analytical calculation of the view factors can be found in [8].

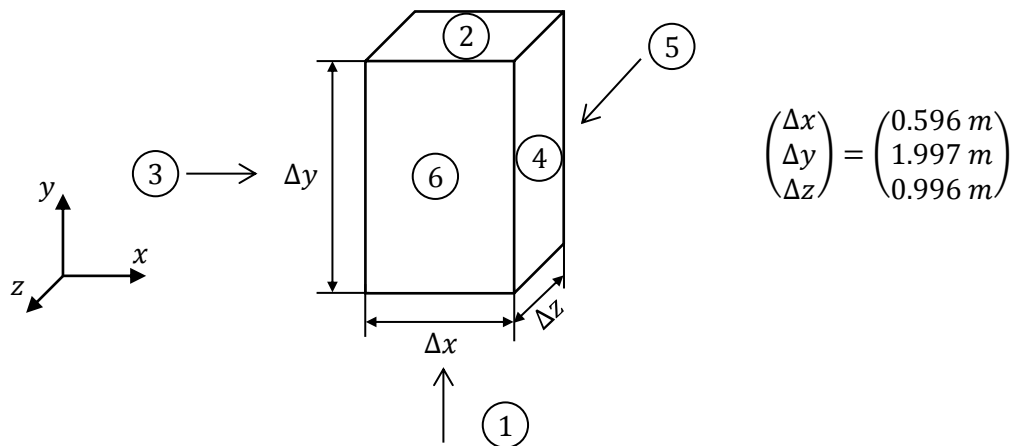


Figure 3. Test case 1: empty switch cabinet with numbering of wall surfaces

The precise view factor matrix $\bar{\bar{\phi}}$ for the empty switch cabinet (see Figure 3) can be described as follows.

$$\bar{\bar{\phi}}_{analytic} = \begin{bmatrix} 0 & 0.043 & 0.295 & 0.295 & 0.184 & 0.184 \\ 0.043 & 0 & 0.295 & 0.295 & 0.184 & 0.184 \\ 0.088 & 0.088 & 0 & 0.451 & 0.186 & 0.186 \\ 0.088 & 0.088 & 0.451 & 0 & 0.186 & 0.186 \\ 0.091 & 0.091 & 0.312 & 0.312 & 0 & 0.193 \\ 0.091 & 0.091 & 0.312 & 0.312 & 0.193 & 0 \end{bmatrix} \quad (14)$$

The results for view factor calculation applying the Monte Carlo method and the quasi-Monte Carlo method are compared with the analytical solution (see equation (14)). The grid spacing n_{gs} and the number of emitted photons per squared subarea N_p were varied extensively when applying the different Monte Carlo methods. Using the Monte Carlo method for each variation of n_{gs} and N_p five simulations were executed and arithmetically averaged, to reduce the statistical scattering of the results. In Figure 4 the maximum relative error σ_{max} between the analytical view factors and the corresponding view factors computed by Monte Carlo methods are shown for different values of n_{gs}

and N_p . For most cases the maximum relative error for the quasi-Monte Carlo method is smaller than for the Monte Carlo method.

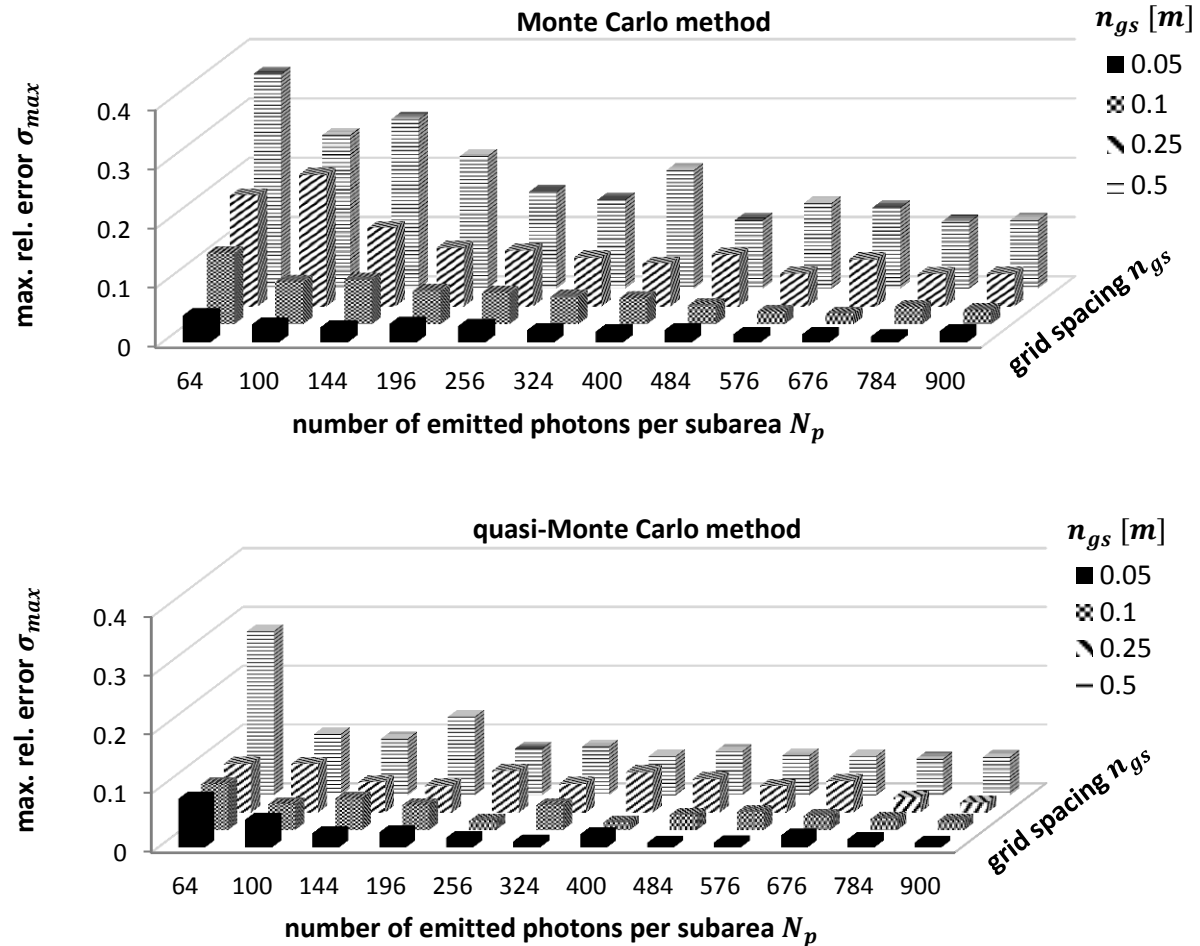


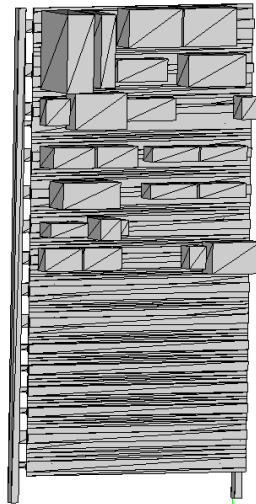
Figure 4. Maximum relative error σ_{max} between analytical view factors and view factors calculated by Monte Carlo methods for different variations of grid spacing and emitted photons

For test case 2 a switch cabinet is considered equipped with electrical components inside. The geometry of the cabinet corresponds to the sketch shown in Figure 3. Figure 5 shows the frame with fitted electrical devices, in which the outer walls are not depicted for reasons of clearness. For test case 2 partly or completely obstructed surfaces exist within the switch cabinet. These obstructed parts of the surfaces do not participate in radiation heat transfer, thus obstructed areas must be subtracted from the real surfaces.

Due to unavailable analytical results for the view factors of this geometry, the results of the Monte Carlo simulations are compared with numerical comparative data generated using the software FloTHERM [10]. This software is widely used for CFD simulations within the scope of cooling electrical components. The radiation heat transfer in FloTHERM is computed using a surface-to-surface radiation model. The view factor calculation is also based on a Monte Carlo method.

The comparative data consist of calculated net heat flows for both the walls and the components within the switch cabinet. For the calculations of the net heat flows the temperatures of the walls and the components are set to fixed values as shown in Figure 5. The emissivity of all walls is set to $\varepsilon = 0.6$, the emissivities for the components are ranging from 0.1 to 0.95 depending on material and

surface condition. For test case 2 a grid spacing $n_{gs} = 0.1 \text{ m}$ and a number of emitted photons per subarea $N_p = 100$ was used. The results of the net heat flows for the walls of the switch cabinet and the components including frame and electrical devices are shown in Table 1.



| parameters | walls | frame and components |
|-------------------|-------|----------------------|
| T [K] | 300 | 350 |
| ε [-] | 0.6 | 0.1 – 0.95 |

Figure 5. Frame and electrical components inside of the switch cabinet considered for test case 2

Table 1. Simulation results for test case 2

| Simulation method | FloTHERM | Monte Carlo | quasi-Monte Carlo |
|---------------------------------------|----------|-------------|-------------------|
| $\dot{Q}_{walls} [W]$ | 721.1 | 725.8 | 726.9 |
| $\dot{Q}_{components} [W]$ | -722.1 | -777.4 | -774.9 |
| rel. error in energy conservation [%] | 0.1 | 7.1 | 6.6 |

Table 1 shows, that in contrast to the FloTHERM results the energy conservation is not fulfilled very well for the Monte Carlo simulations. This is due to the fact, that for complicated geometries such as test case 2 a very large number of emitted photons and infinitely fine grid spacing is necessary, to get results for the view factors, which fulfill the reciprocity rule and with that the second law of thermodynamics [9]. To improve that, smoothing algorithms are applied, which lead to matrices, that are less sensitive to errors in the view factors [10].

To determine the accuracy of the radiation model test case 1 is also computed using FloTHERM. The maximum relative error σ_{max} between the analytical view factors and view factors calculated by FloTHERM is about 3.0 %.

5. Conclusion

A surface-to-surface radiation model for grey Lambert radiators is presented. The view factors, which are necessary for the application of the radiation model, are determined by means of a Monte Carlo method and a quasi-Monte Carlo method. The applicability of the Monte Carlo methods for the view factor calculation is verified based on two test cases. The comparison of analytical and calculated view factors for a not equipped, empty switch cabinet (test case 1) shows an excellent agreement. The smallest maximum error for both the Monte Carlo based view factor values and the quasi-Monte Carlo based view factor values is below 1 %. For most simulations the results obtained with the quasi-Monte

Carlo method are more accurate than the values obtained with the Monte Carlo method for a corresponding grid spacing and number of emitted photons. The application of the presented radiation model to determine the net heat flows in a switch cabinet equipped with electrical components (test case 2) shows relative errors in the energy conservation of about 7 % for both Monte Carlo methods. To improve the accuracy of the proposed radiation model, a smoothing algorithm has to be implemented, which also minimizes the deviation between the FloTHERM results and the results obtained by the presented Monte Carlo methods.

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