Error estimation of single phase effectiveness and LMTD methodologies when applied to heat exchangers with phase change

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Abstract. Single phase formulas based on logarithmic mean temperature difference (LMTD) or Effectiveness (ε-NTU) are widely and wrongly used for the thermal analysis of evaporators and condensers. Those formulas do not take into account that temperature variation during phase change is due to pressure variation and/or concentration changes when using nonazeotropic refrigerant mixtures. This paper first presents the correct evaluation of the mean temperature difference and effectiveness, for parallel and counter flow arrangements, under the hypothesis of linear temperature variation, for both evaporator and condenser cases. Then, the analytical solution is employed to evaluate the error of applying the single phase formulas of LMTD and Effectiveness to the phase change part of evaporators and condensers.

1. Introduction

In practice, there is frequently the need to evaluate an average Overall Heat Transfer Coefficient (OHTC) $\overline{U}$ in order to characterize heat exchangers (Hx). Analytical solutions exists for a lot of processes, designs and flow arrangements [1]. If the temperature evolution of the refrigerant along the Hx can be measured or numerically evaluated, then a conveniently discretized solution can be developed and $\overline{U}$ estimated. However, in most of the cases detailed local data is not available and only the temperature of the fluids at the outlet and inlet of the Hx is known. It is then quite common to employ LMTD or ε-NTU single phase definitions to evaluate OHTC: $\overline{U}$.

In the particular case of pure refrigerants or azeotropic mixtures with negligible pressure variation the refrigerant temperature can be assumed as constant along the two phase region and therefore the single phase formulas for LMTD and Effectiveness ε become the exact analytical solution. This is a good approach in general for condensers but for evaporators, typical designs lead to some 1 to 2 K temperature decrease due to pressure drop when the fluid is evaporating, while the fluid should have to increase its temperature if single phase is assumed. In the case of nonazeotropic mixtures the temperature glide can be quite large, around 5 K for instance for R407C.

Figure 1 shows the evolution of the refrigerant and secondary fluid temperatures, for a R407C condenser and for a R290 evaporator, both working in counter current flow arrangement. The evolution have been calculated with an accurate model [2] which employs a fine discretization of the HX following a special numerical technique to provide high accuracy in the integration as well as short computation time and high numerical robustness [3].The evaluation of the void fraction, thermodynamic properties as well as friction and heat transfer coefficients is local.

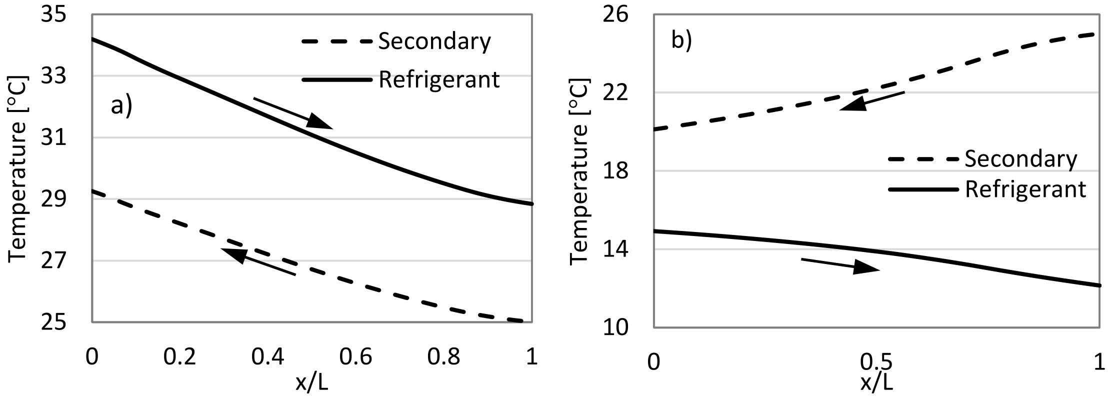

Figure 1. Actual temperature profile for the two-phase flow region in a counter-current heat exchanger: (a) condenser working with a nonazeotropic refrigerant mixture: R407C, (b) evaporator working with a pure refrigerant: R290.

Given that nonazeotropic refrigerant mixtures lead to a variation of temperature (glide) along the condensation or evaporation processes Granryd [4] developed a heat transfer thermal analysis with a varying specific heat for the refrigerant with respect to enthalpy, and presented the solution for the Effectiveness for the cases of assuming linear and quadratic dependence of the specific heat with respect to quality.

Later on, Poz [5] analyzed the general problem of heat exchangers using pure refrigerants and nonazeotropic refrigerant mixtures and presented the analytical solution for the secondary fluid (air in this case) temperature assuming linear or quadratic variation of the refrigerant temperature along the heat exchanger. With this solution the authors then analysed the heat transfer performance of two tube and fin air evaporators (one with plain fins and the other with offset-strip fins) for a nonazeotropic 50% mixture of R22 and R114.

Variation of the fluid properties as well as of the heat transfer coefficient lead of course to an additional uncertainty in the estimation of the average OHTC. Claesson [6] performed a very interesting analysis on the LMTD correction adequate for a brazed plate heat exchanger. He integrated the heat transfer equations assuming no pressure drop but variation of the heat transfer coefficient in the refrigerant side depending on the local heat flux. For this, he employs the Cooper correlation [7]. From that analysis, he was able to deduce an expression for the average OHT as well as for the integral mean of the temperature difference, which then he compared with the LMTD standard definition.

As it can be seen in figure 1, the refrigerant temperature evolution is quite linear although it has some quadratic component. In any case, a linear evolution is much more close to reality than the single phase assumption, and in general leads to a reasonable good estimation of the average OHTC. This paper targets first the development of a general solution of the mean temperature difference and effectiveness, for parallel and counter flow arrangements, under the hypothesis of linear temperature variation, for both evaporator and condenser cases, and then, the use of the developed solution to evaluate the error of applying the single phase formulas of LMTD and Effectiveness to the phase change part of evaporators and condensers.

2. Governing equations

For illustration of the employed nomenclature, Fig. 2 depicts a generic example of the temperature distribution along a parallel flow evaporator assuming linear variation of the refrigerant temperature. The secondary fluid cools down while the refrigerant evaporates. ΔT_{in} is the absolute temperature difference between the secondary fluid inlet and refrigerant inlet, which will be negative for the case of a condenser.

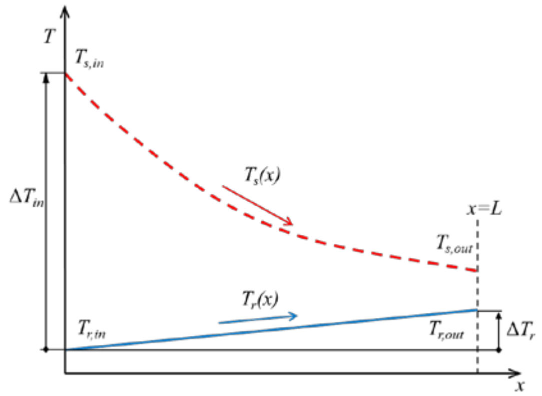

Figure 2. Temperature evolution in a generic parallel flow evaporator

As discussed in the introduction, the two-phase flow undergoes a saturation temperature variation due to pressure drop or gliding effect when using nonazeotropic mixtures. Fig. 2 shows ΔT_r as the total temperature variation for the refrigerant, which has been defined as ($T_{r,out}$ - $T_{r,in}$), so a positive value means a temperature rise. Gliding effect results in a positive ΔT_r value for evaporators or negative value for condensers. For scenarios where pressure drop is negligible or gliding effect is dominant, ΔT_r ranges between 2÷5 K in evaporators working for nonazeotropic mixtures like R407C while ΔT_r can reach up to -5K for condensers working with the same refrigerant. ΔT_r with pure refrigerant or azeotropic mixtures will be only due to pressure drop, so it will be almost negligible for condensers while or the order of 1 to 3 K for evaporators, and it will be negative with the adopted nomenclature.

In a heat exchanger with a parallel flow or counter flow arrangement, the differential equation that governs the process, for the secondary fluid, can be written as:

$$U(x)\,P\left[T_s(x)-T_r(x)\right]dx=-C_s\,dT_s \tag{1}$$

Where U is the local OHTC, P is the perimeter and C_s is the heat capacity of the secondary fluid flow. Eq. (1) can be rewritten as:

$$\frac{dT_s}{dx}+\frac{U(x)P}{C_s}T_s(x)=\frac{U(x)\,P}{C_s}T_r(x) \tag{2}$$

This is a first order non-homogenous pseudo-linear differential equation, whose general solution is:

$$T_s(x)=e^{-\int_0^x \frac{U(x)\,P}{C_s}dx}\left[\int_0^x \frac{U(x)\,P}{C_s}T_r(x)\,e^{\int_0^x \frac{U(x)\,P}{C_s}dx}dx\right]+K\,e^{-\int_0^x \frac{U(x)\,P}{C_s}dx} \tag{3}$$

Now, if we assume that the overall heat transfer coefficient $U(x)$ is constant along the heat exchanger and equal to the average value along the heat exchanger. The solution (3) becomes:

$$T_s(x)=\frac{\overline{U}\,P}{C_s}e^{\frac{-\overline{U}\,P\,x}{C_s}}\left[\int_0^x T_r(x)\,e^{\frac{\overline{U}\,P\,x}{C_s}}dx\right]+K\,e^{\frac{-\overline{U}\,P\,x}{C_s}} \tag{4}$$

Where K is the constant of integration. Notice that Eq. (4) is valid for either a condenser or an evaporator as well as for parallel and counter-current flow. In order to complete the solution, it is necessary to have the temperature distribution of the refrigerant $T_r(x)$ to complete the integration. If the function for this temperature profile is known, the integral could be solved for that specific case. However, in common experiments, only inlet and outlet temperatures are known.

As commented above, assuming a linear refrigerant temperature variation is a good approximation for both evaporators and condensers working with nonazeotropic mixtures and negligible pressure drop (or less important than the temperature glide). For the other cases the assumption is still good as far as the temperature drop due to pressure drop is moderate (e.g. 2 K). Therefore, in the following a linear refrigerant temperature variation will be used to solve Eq. (4) for parallel and counter flow arrangements. With the nomenclature of figure 2, the refrigerant temperature can be written as:

$$T_r(x) = T_{r,in} + \frac{\Delta T_r}{L} x \tag{5}$$

2.1. Parallel Flow Arrangement

Substituting Eq. (5) in Eq. (4), we can obtain the solution for the secondary fluid temperature:

$$T_s(x) = T_{r,in} + \frac{\Delta T_r}{\frac{\overline{U}\,P\,L}{C_s}}\left[\frac{\overline{U}\,P}{C_s}(x-1)\right] + \left(T_{s,in} - T_{r,in} + \frac{\Delta T_r}{\frac{\overline{U}\,P\,L}{C_s}}\right) e^{\frac{-\overline{U}\,P}{C_s}x} \tag{6}$$

And the total heat transfer can be evaluated with the following expression:

$$q = C_s\,\Delta T_{in}\left[1 - \frac{\Delta T_r}{\Delta T_{in}}\,\frac{1-\frac{1-e^{-NTU}}{NTU}}{1-e^{-NTU}}\right]\left(1-e^{-NTU}\right) \tag{7}$$

Where the Number of Heat Transfer Units (*NTU*) has been defined with the average OHTC ($\overline{U}$):

$$NTU = \frac{\overline{U}PL}{C_s} = \frac{\overline{U}A}{C_s} \tag{8}$$

If we now introduce the non-dimensional temperature variation of the refrigerant along the Hx,

$$\theta = \frac{\Delta T_r}{\Delta T_{in}}$$

Eq. (7) can be expressed in a more compact manner,

$$q = C_s\,\Delta T_{in}\,\varepsilon_0\left[1-\theta\,\frac{1-\frac{\varepsilon_0}{NTU}}{\varepsilon_0}\right] \tag{9}$$

Where $\varepsilon_0 = (1-e^{-NTU})$, is the Hx Effectiveness if the refrigerant temperature were constant. Employing the following definition of $q_{\max} = C_s\,\Delta T_{in}$, the actual effectiveness of the heat exchanger can be deducted from Eq. (9) as follows,

$$\varepsilon = \frac{q}{q_{\max}} = \frac{q}{C_s \Delta T_{in}} = \varepsilon_0\left[1-\theta\,\frac{1-\frac{\varepsilon_0}{NTU}}{\varepsilon_0}\right] \tag{10}$$

Notice that for $\Delta T_r = 0$, which means $\theta = 0$, the effectiveness becomes equal to ε_0 which is the corresponding analytical solution. Eq. (10) shows that the actual effectiveness only depends on *NTU* and θ.

The integrated mean value of the temperature difference ($\overline{\Delta T}$) between both fluids can be evaluated from,

$$\dot{q} = \overline{U}A\,\overline{\Delta T} \longrightarrow \overline{\Delta T} = \frac{\dot{q}}{\overline{U}A} = \Delta T_{in}\,\frac{\varepsilon}{NTU} \tag{11}$$

2.2. Counter flow Arrangement

For a counter flow arrangement, the linear assumption for the refrigerant temperature can be expressed in the following way:

$$T_r(x) = T_{r,in} - \frac{\Delta T_r}{L}(x-L) \tag{12}$$

Correspondingly, the heat transferred for this case results in,

$$\dot{q} = C_s\;\Delta T_{in}\;\varepsilon_0\left[1-\theta\left(1+\frac{\dfrac{\varepsilon_0}{NTU}-1}{\varepsilon_0}\right)\right] \tag{13}$$

And the effectiveness solution is:

$$\varepsilon = \varepsilon_0\left[1-\theta\left(1+\frac{\dfrac{\varepsilon_0}{NTU}-1}{\varepsilon_0}\right)\right] \tag{14}$$

3. Results and discussion

Expressions (9) and (13) allow the estimation of the heat exchanger *NTU* from measured values of the refrigerant and secondary fluid temperatures at inlet and outlet of the heat exchanger. Then the value of the OHTC, $\overline{U}$ can simply be calculated from (8).

Expressions (9) to (14) allow the evaluation of the actual heat exchanger effectiveness, as well as the heat transferred and the integral value of the temperature difference. It is interesting now to use them to evaluate the error between the developed expressions and the values obtained if the single phase solutions are employed for that purpose, i.e.,

$$\varepsilon_{sp} = \varepsilon_0 = \left(1-e^{-NTU}\right) \qquad\qquad LMTD\big)_{sp} = \frac{\Delta T_{x=0}-\Delta T_{x=L}}{\ln\left(\dfrac{\Delta T_{x=0}}{\Delta T_{x=L}}\right)}$$

In the following, the error committed when the above expressions are employed is presented and discussed for the different flow arrangements, i.e. parallel and counter flow for both evaporator and condenser scenarios. The errors are defined in the following way,

$$Er_{\varepsilon} = \frac{\varepsilon_0-\varepsilon}{\varepsilon} \qquad\qquad Er_{LMTD} = \frac{LMTD\big)_{sp} - \overline{\Delta T}}{\overline{\Delta T}}$$

3.1. Parallel flow

Fig. 3 shows the error on effectiveness, which is analogous to the error in heat transfer estimation. In general, the error in absolute terms is larger for high *NTU* and for positive values of θ, due to the absolute value of the effectiveness. The error can be up to almost 100% for high θ, but even for a small glide of θ=0.2 we would have a 20% error.

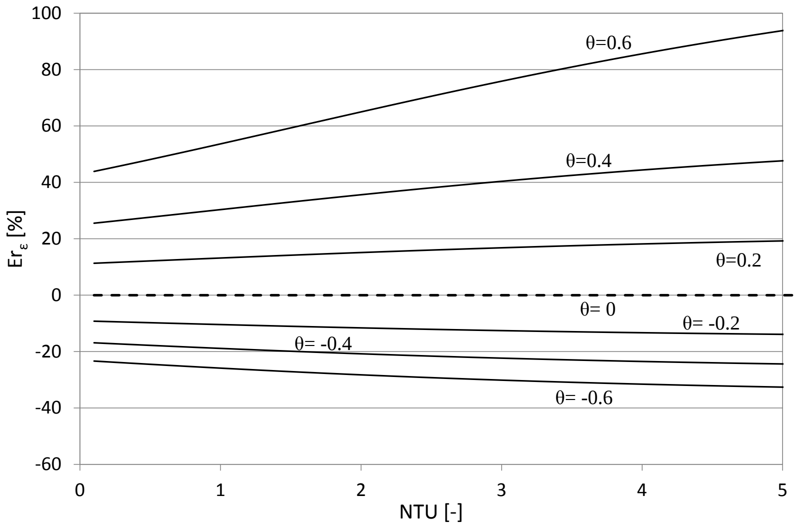

Figure 3. Error in the evaluation of Hx effectiveness as a function of *NTU* for different values of θ in a parallel flow configuration.

Figure 4 shows the Error in the evaluation of the integral mean of the temperature difference when the single phase definition of $LMTD_{sp}$ is employed (Er_{LMTD}). As it can be observed, the error increases as the *NTU* increases, and is much larger for positive values of θ, increasing drastically even with low *NTU*. When θ is positive, there is a value of *NTU* when the line does not exist. The reason is that in such a situation the outlet refrigerant temperature would cross the secondary fluid temperature because the refrigerant temperature does not depend on the heat transfer. Worth to mention the cases where the error is 0%, which correspond with the points where $\theta = -\varepsilon$. For such a condition $\Delta T_r = \Delta T_s$ and the formulas become indeterminate. Solving the indetermination $LMTD = \Delta T_{in}$ and the result for Er_{LMTD} is zero.

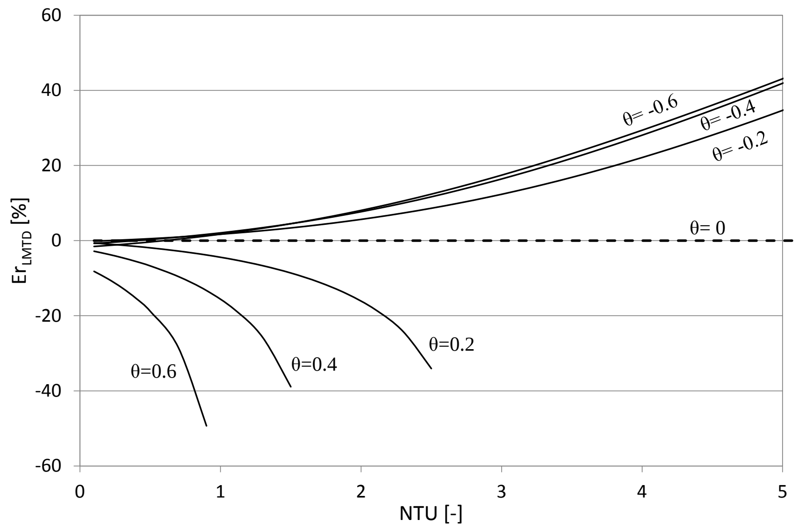

Figure 4. Error in the evaluation of the mean temperature difference $\overline{\Delta T}$ as a function of *NTU* for different values of θ, in a parallel flow configuration.

Summarizing for parallel flow, the error of using the single phase formulas is very important for nonazeotropic mixtures. This effect is the same for both evaporator and condenser. The effect in evaporators where pressure drop is the dominant effect is much less important than the glide effect.

3.2. Counter flow

Fig. 5 shows the error on the effectiveness for counter flow situation. It can be noticed that errors are much lower than for parallel flow, tending all of them to zero when *NTU* increases.

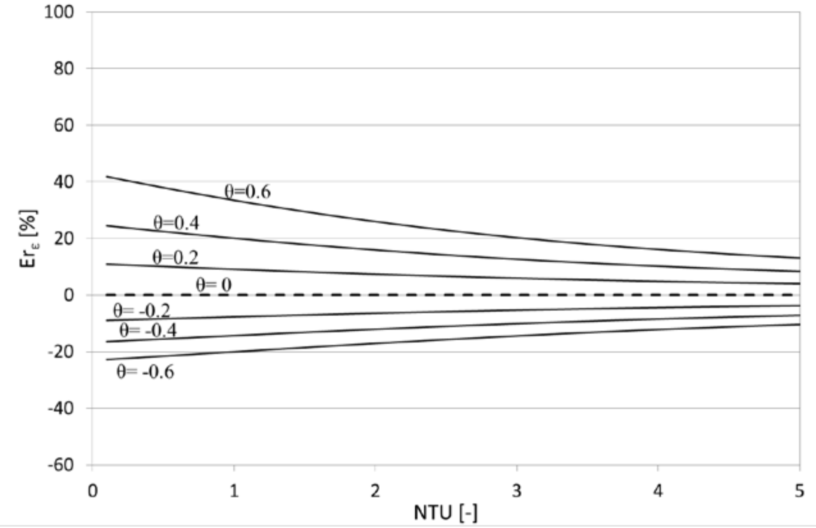

Figure 5. Error in the evaluation of Hx effectiveness as a function of *NTU* for different values of θ, in a counter flow configuration.

Fig. 6 shows the error when using the LMTD to calculate $\overline{\Delta T}$. The plot is very similar to the case of parallel flow (Fig. 4) but now the curves with $\theta < 0$ have larger error than those with $\theta > 0$. In contrast with parallel flow, the influence of the glide on the prediction of effectiveness in counter flow is much lower. The difference can be even negligible for heat exchangers with high effectiveness. In contrast, in counter flow the temperature drop due to pressure drop leads to much higher errors.

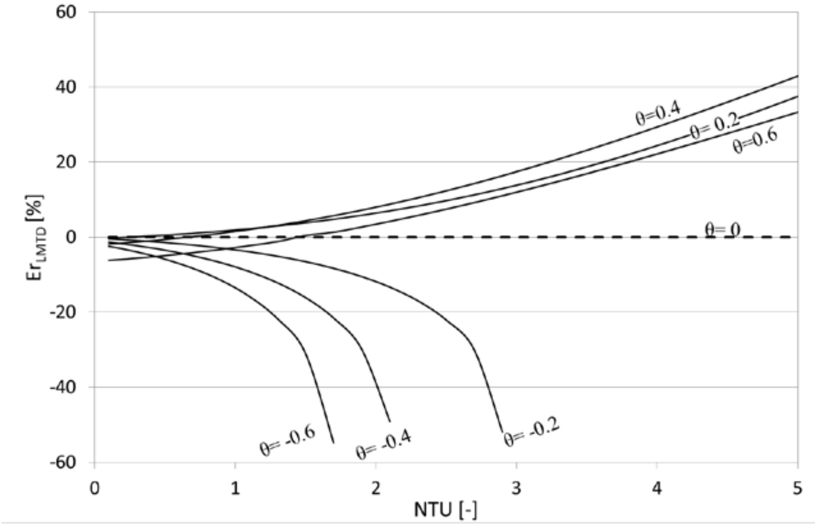

Figure 6. Error in the evaluation of the mean temperature difference $\overline{\Delta T}$ as a function of *NTU* for different values of θ, in a counter flow configuration.

4. Conclusions

Assuming constant refrigerant temperature is a widely used assumption for the estimation of heat exchanger effectiveness, or of the OHTC, in evaporators and condensers. The paper discusses the validity of this assumption for heat exchangers operating with a nonazeotropic refrigerant mixture and for non-negligible pressure drop. It presents the expressions to obtain the effectiveness and the mean temperature difference, for both parallel and counter flow configurations, under the assumption of a linear variation of the refrigerant temperature.

In regard to the error in the estimation of the Hx effectiveness, or the mean temperature difference, when the single phased formulas are employed, the following conclusions can be drawn:

- In parallel flow, the estimation of the effectiveness with the single phase formula is large (20% to 100%) for condensers and evaporators working with negligible pressure drop ($\theta > 0$) and nonazeotropic refrigerant mixtures. The larger θ, the larger the error on effectiveness prediction. For counter flow configuration, the error on the effectiveness prediction is lower than 30% when $\theta > 0$, and almost negligible for high effectiveness Hx.
- Actual effectiveness larger than 1 can be obtained with negative values of θ. A significant negative value of θ is only possible in evaporators for pure refrigerants or azeotropic refrigerant mixtures with considerable pressure drop. Effectiveness is much larger in parallel flow when $\theta < 0$ than in counter flow, reaching values of up to 1.5.
- In parallel flow, the estimation of $\overline{\Delta T}$ by using the definition of single phase LMTD is significant for $\theta < 0$ at high NTU, and can become very large at $\theta > 0$. The opposite is true for counter flow.

5. References

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