

## Heat source estimation in low diffusive materials

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**Abstract.** In this work, a method to estimate unsteady 2D heat sources is developed. The sources are estimated from Infra-Red (IR) temperature mapping on the front face of thermally thin material. A comparison of the estimation accuracy as well as the evaluation of time cost is also carried out between a direct method (coupled with filtering techniques) and an iterative method (of conjugate gradient type). The present work was realised on experimental data.

### 1. Introduction

Composite materials, such as matrix resin reinforced with glass fibers are designed because of their noticeable mechanical properties and can be found in various applications and especially in aeronautics. Their mechanical behaviour is strongly coupled to their thermal behaviour when in use. Such a behavior is known as the thermo-mechanical behaviour law of the material. It is important to know this law particularly to understand the damage mechanisms. Heating ignition of such materials is present when subjected to an inelastic deformation (e.g. in a tensile test experiment), either in quasi-static regime or in fatigue regime. Thermal ‘observation’ of such material during mechanical stress can thus be useful to understanding the mechanisms of its damage (fiber/matrix rubbing, creating cracks in the matrix, rupture of fibers, etc.).

This observation is, however, not sufficient if it is not associated with a method of estimating heat sources allowing the precise determination of the energies involved during these tests. These estimates consist in determining the cause (the source) from the consequence (the temperature field measured by IR thermography) and correspond to the category of inverse problems in heat conduction. Some methods have already been developed mainly in 1-D framework for homogeneous materials. They allow the estimation of the heat sources involved during tensile tests on homogeneous materials [1]. Very recently, direct 2-D methods have been developed on numerical [2] and experimental data. These methods are classified according to three main families [3]:

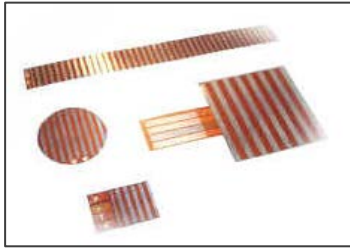
- methods based on calculation of derivatives (regularization by data filtering) [4]
- methods based on the research of “quasi-solutions” (regularization of the inverse operators) [5] [6]
- methods based on successive optimization (iterative algorithms on direct problem) [7]

In this work, two methods have been developed for comparison of their performances (precision and speed). The first method (corresponding to the first family) is a 2-D direct inversion by identifying the source term of the diffusion equation coupled to filtered data [8]. The second method (corresponding to the third family) uses the conjugated gradients in 2-D [7].

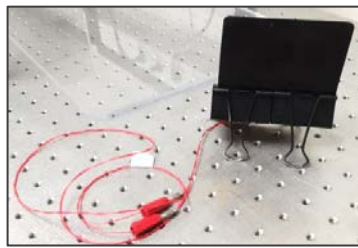


## 2. Experimental setup

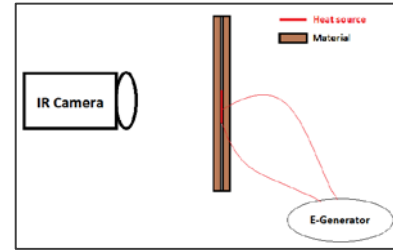
A linear heating resistor (and flat) (Figure 1b) is positioned between two thin sheets of vinyl. The 'sandwich' is painted in black to overcome the problems of reflection and emissivity to form a test sample (Figure 1b). The complete thermal experiment description is presented in Figure 1c with the following dimensions:  $L_x=6.33\text{cm}$  x  $L_y=5.63\text{cm}$



**Figure 1a.** Plane electrical resistors

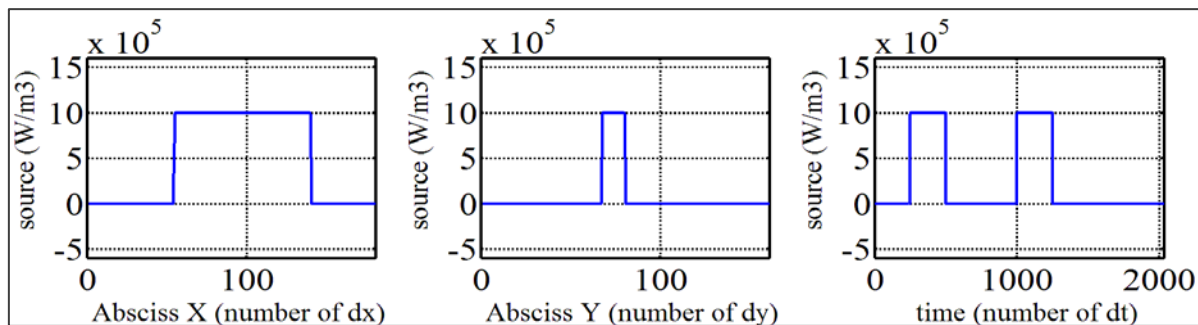


**Figure 1b.** Test sample



**Figure 1c.** Scheme of the thermal experiment

The spatial and temporal distribution of this heat source is presented in Figure 2. The boundary conditions in  $x=0$ ,  $x=L_x$ ,  $y=0$  and  $y=L_y$  ( $dx=dy=3.5175\times10^{-4}\text{m}$ ) are considered as adiabatic because of the small involved areas and the embedded heating source is far enough from the edges and the material has a low thermal diffusivity. The acquisition is made by an IR camera to a frequency of 50Hz ( $dt=0.02\text{s}$ ,  $t_{\text{final}}=40\text{s}$ ).



**Figure 2.** Space and time experimental profiles of heat sources

$$e = 1.5 \text{ mm} ; dV = e \times dx \times dy = 1.85592 \times 10^{-10} \text{ m}^3 ; h_c = 10 \text{ W} / \text{m}^2 \cdot \text{K}$$

$$P_{\text{conv\_max}} = 2 \times (T_{\text{max}} - T_{\text{amb}}) \times h_c \times dx \times dy \text{ (W)} = 12.73 \times 10^{-6} \text{ W}$$

$$S_{\text{prod}} = 1.03 \times 10^6 \text{ (W} / \text{m}^3) \times dV = 191.16 \times 10^{-6} \text{ W}$$

Convective losses represent less than 6% of the heat source we want to estimate. Therefore, during this study, we will neglect these convective losses.

We work with the following physical configuration:

$$\rho = 1380 \text{ kg} / \text{m}^3 ; c_p = 1160 \text{ J} / \text{kg} \cdot \text{K} ; k_{\text{vinyl}} = 0.355 \text{ W} / \text{m} \cdot \text{K}$$

$$\text{Imposed source: } S = \frac{P}{V} = \frac{U \times I}{l \times L \times e} = \frac{2\text{V} \times 0.07\text{A}}{0.003\text{m} \times 0.03\text{m} \times 0.0015\text{m}} = \frac{0.14 \text{ W}}{135 \times 10^{-9} \text{ m}^3} = 1.03 \times 10^6 \text{ W} / \text{m}^3$$

### 3. Two dimensional unsteady heat sources estimation

#### 3.1. Direct Method

##### 3.1.1. Description

The equation governing heat diffusion in the material is given by:

$$\rho C_p \frac{\partial T(x, y, t)}{\partial t} - \vec{\nabla} \cdot (\lambda \vec{\nabla} T(x, y, t)) = S(x, y, t) \quad (1)$$

The resolution of the inverse problem is based on the discretization by an implicit finite difference scheme (order 1 in time and order 2 in space). The estimation of sources terms field is done by identifying the first and second derivatives terms of the temperature. The implicit discretization allows the stability of the numerical scheme and to obtain a matrix ‘operator’ linking directly the causes (Boundary Conditions and Sources) to the consequences (temperature field). The discretization is done so that the searched sources are at the pixel scale. The material is isotropic and thermally thin, thermally thin. Its thermal properties do not depend on the temperature. No prior information on the spatial-temporal form is given. The initial temperature is known and uniform ( $T_{init} = 20^\circ\text{C}$ ). Using the thermal diffusivity  $\alpha = \frac{\lambda}{\rho \cdot C_p}$ , the discretization of the heat diffusion equation gives [3], [6]:

$$[A]\{T\}^{N+1} = \{T\}^N + \{S\}^{N+1} \quad (2)$$

[X] – Matrix Representation ; {X} – Vectorial Representation ; N = time index ; N+1 = future time index

In this equation (2), the 2D matrix T(x,y) is reorganized in a vector T(k) where k contains the two XY coordinates.

To find the source terms, it only needs to apply the tridiagonal operator [A] on the temperature field and subtract it from the temperature field in the previous step.

$$\{S\}^{N+1} = [A] \cdot \{T\}^{N+1} - \{T\}^N \quad (3)$$

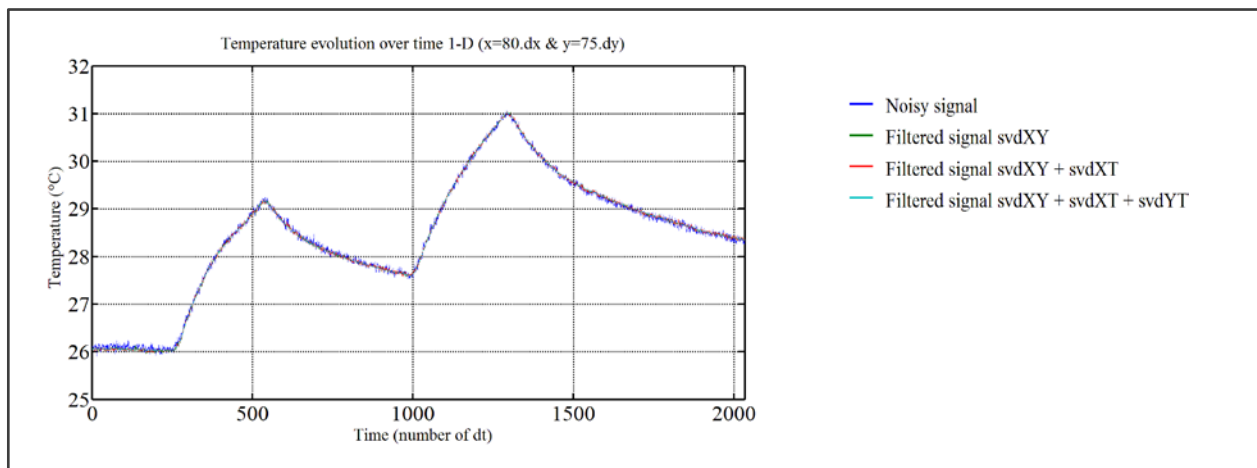
Therefore, the ‘direct’ calculation of  $\{T\}^{N+1}$  would requires a matrix inversion (of A) from eq. (2) whereas the inverse calculation (calculation of the 2D source term) only requires a vector matrix product and a subtraction [9], [10]. Anyhow, the measurement noise is amplified by the matrix [A], which justifies a preliminary filtering of input data that will be realized thanks to the singular value decomposition (SVD).

##### 3.1.2. Data filtering by singular values decomposition (SVD)

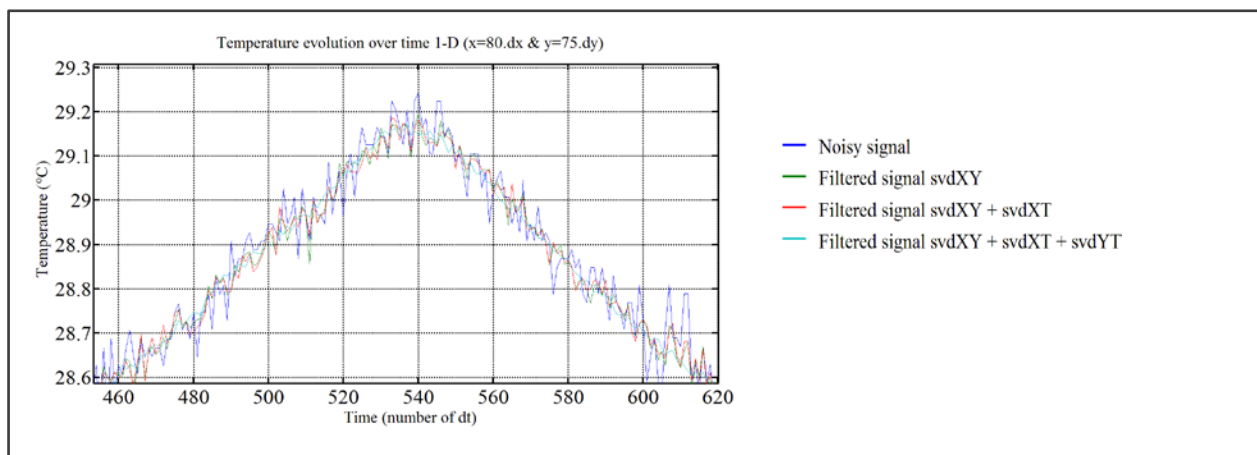
Singular value decomposition of a rectangular matrix X (m, n) (i.e. the 2-D field temperature at a given time) is given by the expression:

$$X = U S V^T \quad (4)$$

This mathematical decomposition [11] is a matrix factorization that directly gives a diagonal rectangular matrix [S] in a direct way containing the singular values of the matrix X (in decreasing order). It is thus possible to implement a low-pass filter by truncating these modes from a certain rank. The matrices U and V contain a set of orthonormal vectors: U is the matrix of eigenvectors of  $X^T X$ , and V is the matrix of eigenvectors of  $XX^T$ . Truncating a part of the singular values is a way to compress the input data (matrix X). The filtering is performed in three steps on 2-D matrices extracted from our data T (x, y, t). The first step is a purely spatial SVD filtering process applied on each temperature map T(x,y) obtained at each time step. This gives the filtered field T\_f1 (x,y,t). This field is then subjected to a second SVD filtering process, at each y value, on the extracted T\_f1 (x,t) 2D matrices; that gives the filtered field T\_f2 (x,y,t). The third step is also a SVD space and time filtering applied, at each x value, on the extracted T\_f2 (y,t) 2D matrices.



**Figure 3.** 1-D section (over the time, at  $x=80.dx$  &  $y=75.dy$ ) of errors with perfect signal for noisy and filtered signals at different filtering levels



**Figure 4.** 1-D section (over the time) of errors with perfect signal for noisy and filtered signals at different filtering levels (**ZOOMED**)

The Figure 3a shows also a maximum heating of 5°C above the initial temperature. Therefore, it is possible to neglect the convective losses of the sample.

The new standard deviations of the residual noise measurement (post filtering) as well as the noise's reduction ratio according to different axes of the data matrix are presented below:

$$\frac{Std(X_{old})}{Std(X_{new})} = \frac{0.65}{0.023} = 28.26 \quad \frac{Std(Y_{old})}{Std(Y_{new})} = \frac{0.65}{0.019} = 34.21 \quad \frac{Std(t_{old})}{Std(t_{new})} = \frac{1.3}{0.014} = 92.86$$

### 3.1.3. Estimation results

Despite the filtering effort, the identified heat source is finally too noisy, and does not enable to retrieve the imposed heat source profiles (see green curves on Figure 5). Therefore, in order to stronger regularize our estimation, we proceed to an oversampling of our input data (temperatures) followed by a moving temporal average of our output data (source terms fields obtained by the first inverse method). The estimation thus appears correct for a temporal average of at least 20 time steps ( $newdt = 20.dt$ ) [Figure 6]. This method is very interesting because of its short computation time and its simplicity; but the precision of the estimation requires material that would allow a high frequency acquisition, and of low diffusivity in order to suppose the quasi-steady state of the system during the 20 time steps required for the moving average.

### 3.2. Conjugate Gradient Method (CGM)

#### 3.2.1. Description

The conjugate gradients method consists in solving the direct problem corresponding to our experiment, with a given prescribed heat source, and to minimize the squared difference between the results and temperature measurements, by modifying the prescribed heat source in the model. The subsequent resolution of adjoint and sensitivity problems leads to the determination of the direction of descent and the step size respectively. The direct problem is defined by the heat diffusion equation presented in paragraph 3.1.1 (inputs: sources / outputs: temperatures).  $T_{init} = 20^\circ\text{C}$

#### 3.2.2. Inverse problem

The inverse problem consists in estimating  $S$  in order to minimize the squared difference defined by the following criterion:

$$J(S) = \frac{1}{2} \sum_{i=1}^{N_x N_y} \int_0^t (T_{cal}(x, y, t, S) - T_{meas}(x, y, t, S))^2 dt \quad (5)$$

$T_{meas}$  - Measured temperature by IR thermography

$T_{cal}$  - Calculated temperature by direct problem resolution

The conjugate gradient method is an iterative method which consists in approaching the new iterated  $S^{it+1}$  from the previous iterated  $S^{it}$  as follows:

$$S^{it+1}(x, y, t) = S^{it}(x, y, t) - P_{descente}^{it} \cdot D_{descente}^{it}(x, y, t) \quad (6)$$

$S^{it}$  - Estimated heat source at iteration it

$P_{descente}^{it}$  - Step size calculated at iteration it

$D_{descente}^{it}$  - Direction of descent calculated at iteration it

The specificity of this method is to build the direction of descent  $D_{descente}^{it}$  in such a way that the successive directions of descent are combined together. To do this, the direction of descent at each iteration is calculated as follows:

$$D_{descente}^{it}(x, y, t) = \Psi^{it}(x, y, t) + \beta^{it} \cdot D_{descente}^{it-1}(x, y, t) \quad (7)$$

With  $\beta^{it}$ , the conjugation PRP [7] coefficient is defined by:

$$\beta^{it} = \frac{\iint [\Psi^{it}(x, y, t) \cdot (\Psi^{it}(x, y, t) - \Psi^{it-1}(x, y, t))] d\Omega dt}{\|\Psi^{it-1}(x, y, t)\|^2} \quad (8)$$

#### 3.2.3. Adjoint problem & Gradient equations

The adjoint problem enables to obtain the expression of the criterion gradient  $\nabla J$  in order to determine the direction of descent. We show mathematically that in our case of heat sources estimation, the criterion gradient is equal to the Lagrange multipliers:

$$\nabla J = \Psi \quad (10)$$

The resolution of the adjoint problem enables to calculate the Lagrange multipliers:

$$\rho C_p \frac{\partial \Psi(x, y, t)}{\partial t} + \vec{\nabla} \cdot (\lambda \vec{\nabla} \Psi(x, y, t)) = T_{cal}(x, y, t) - T_{meas}(x, y, t) \quad (11)$$

This problem is solved in a backward way in time; we then need to impose a final condition in order to start this backward process:  $\Psi(x, y, t_{final}) = 0$

### 3.2.4. Sensitivity problem & Step size

The resolution of the sensitivity problem aims to give the equations that enable to obtain the sensitivity function according to the unknowns to finally calculate their step size in the direction of descent:

$$\frac{\partial \rho C_p \delta T(x, y, t)}{\partial t} - \vec{\nabla} \cdot (\lambda \vec{\nabla} \delta T(x, y, t)) = D_{descente}(x, y, t) \quad (12)$$

Calculation of the step size:

$$P_{descente}^{it} = \frac{\sum_1^{N_x \cdot N_y} \int_0^t (T_{cal}^{it}(x, y, t) - T_{meas}(x, y, t)) \cdot \delta T^{it}(x, y, t) dt}{\sum_1^{N_x \cdot N_y} \int_0^t (\delta T^{it}(x, y, t))^2 dt} \quad (13)$$

### 3.2.5. Stopping criterion

In accordance with the "discrepancy principle", the iterations continue while the criterion is strictly greater than the threshold value defined by:

$$Threshold = \frac{1}{2} N_x \cdot N_y \cdot N_t \cdot \sigma^2 \quad (14)$$

$N_x, N_y, N_t$  - Number of point X, Y, t

$\sigma$  - Standard deviation of noise

The problems encountered by this type of method generally fall within the slow convergence of the iterative calculation, or the tendency of the algorithm to adapt the model to fit the noise fluctuations into the signal. To avoid 'explaining noise' from the model, and for acceleration of convergence, an interesting trick consists in using the preliminary filtered data (in order to reduce the measurement noise) and to rely on stopping criterion the local algorithm at pixel. By choosing a local criterion, the expression of the threshold changes because it is specific to each pixel:

$$\text{Here, with } \sigma = 0.03 K \quad Threshold = \frac{1}{2} N_t \cdot \sigma^2 \quad (15)$$

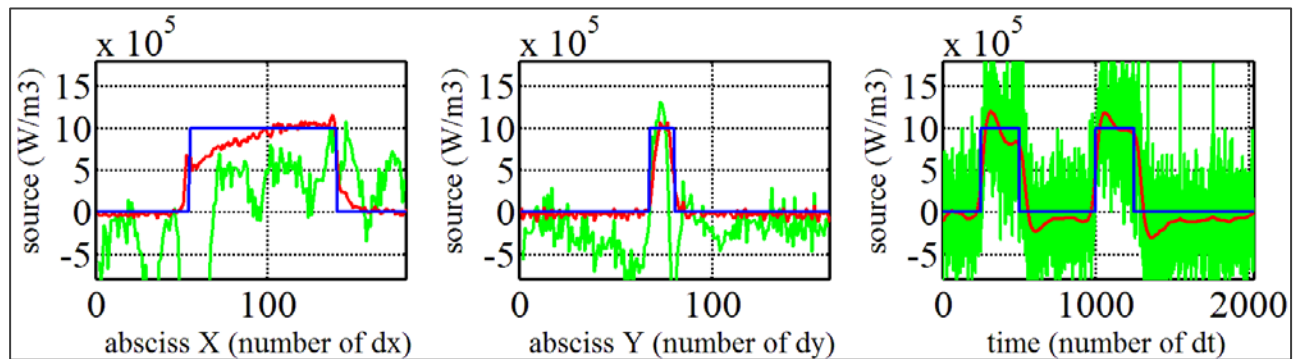
### 3.2.6. Optimizing computational time

The set of calculations are performed on the Matlab software. The simplicity of the problem geometry allows optimizing the computation time. The three problems i.e. direct, adjoint and sensitive are solved by an explicit finite difference scheme. This scheme is preferred for its resolution simplicity; it is moreover possible to replace the matrix products by simple products terms by terms by tri-diagonal aspect of transfer matrices which further optimizes the speed of the calculations. It is simply needed to ensure the stability of scheme by choosing a mesh Fourier number Fo such as:

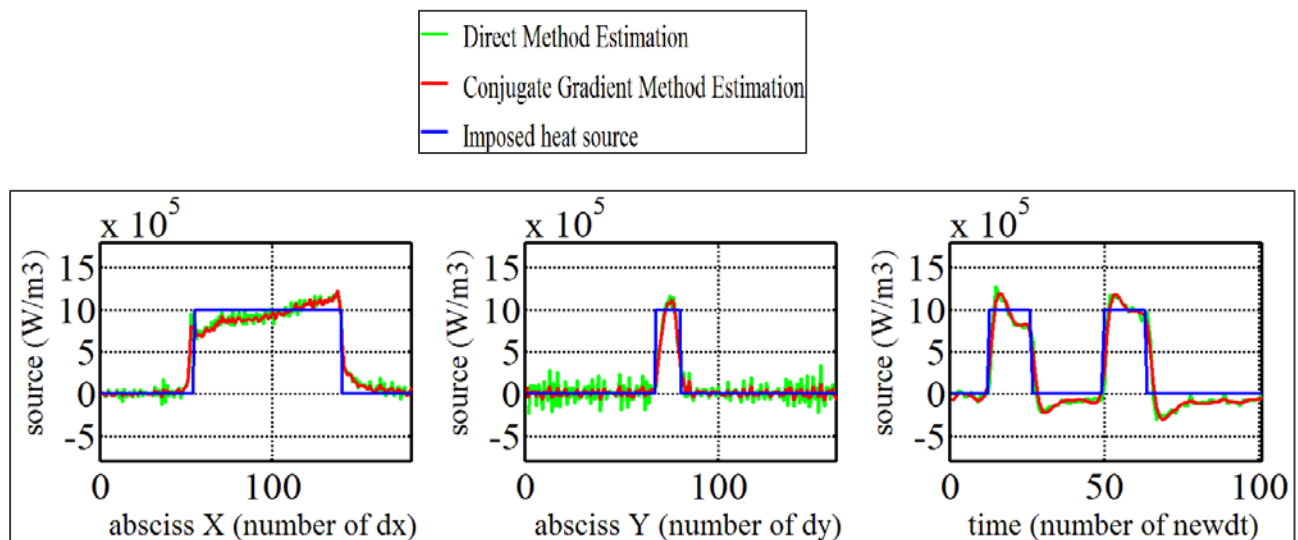
$$Fo = \frac{\alpha \Delta t}{(\Delta x)^2} < \frac{1}{4}, \quad \alpha = \text{thermal diffusivity}$$

## 3.3. Comparison between CG Method & Direct Method

To estimate heat productions imposed inside the material, both methods give correct results. However, some over and under source estimations appear. If we want to estimate an intense threshold (rectangular function) it is important to not over-regularize the inversion, which explains why some oscillations emerge. This error set is largely due to the two-dimensional approximation. Indeed, although the "thermally thin" supposition is valid, the sample includes several layers (vinyl-paste-vinyl paint), and it becomes very difficult to predict the behavior of the 2D thermal diffusion. Mistakes made on the material's properties thermo-physical characterization are also very important as we have seen in the previous part. We can also wonder if the ignition resistance is really instantaneous and uniform across the resistance's surface.



**Figure 5.** 1D heat sources estimations by direct and iterative methods, with SVD filtering, without moving average



**Figure 6.** 1D heat sources cut estimations by direct and iterative methods, with SVD filtering, with 20 frames moving average

The direct method is simple to implement. Indeed, the implicit finite difference scheme (unconditionally stable) discretizes only the time with order 1 but shows only one operator, therefore the inversion only requires a matrix-vector product; moreover, this matrix operator possesses a lower conditioning ( $\sim 100$ ) which limits the amplification of random perturbations of the input signal. This method is very interesting to use if the studied material is low diffusive and subject to have an acquisition material allowing an oversampling of the input data, because the estimation result (with implicit scheme, SVD filtering and post moving average) is quantitatively satisfactory and also very fast in these exploitation conditions. The method is also very effective for estimating the spatial and temporal steep fronts.

The conjugate gradient method is generally much more stable. It is important to correctly initialize the domain temperature during each resolution of the direct problem. This stability also implies a tendency (due to the diffusive nature of the equation solved) to mitigate steep fronts sources. However, convergence and estimation are optimized due to the introduction of a local convergence criteria which reduces the number of iterations (with a low number of iterations, estimation is stable but very imprecise because regularization is very important) and at a given filtering data which reduces the standard deviation of the measurement noise and increases the number of iterations (to better approach the exact solution); this combination is a very good compromise to represent all the source grades all by regularising the estimation.

The programming optimization of the numerical scheme in both cases makes these methods relatively quick methods in terms of calculation time (5 minutes for the direct method and ~ 2 hours for the iterative method)  $N_x=181$  ;  $N_y=161$  ;  $N_t=2000$ .

#### 4. Conclusion

To conclude, we can say that the difficulty of an inverse problem is principally to correctly regularize the resolution, but without creating bias in the estimation. Every configuration corresponds to a method or an optimal combination of methods. In case of heat sources estimation, it seems better to use conjugate gradient method than direct method because of its stability and for its estimation way; this method has a more general behaviour than the other. The execution times are acceptable for both methods (in the physical configuration presented in this article), it is possible to consider (in a thermomechanical test) to use both methods to check the accuracy of the estimations and deduce a relevant physical analysis. The solution for a better estimation could be working on a thermomechanical experiment, where heat sources would take place in homogeneous way in the material, without multilayer problems.

#### 5. References

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