

# The accuracy of a 2D and 3D dendritic tip scaling parameter in predicting the columnar to equiaxed transition (CET)

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**Abstract.** The dendrite tip kinetics model accuracy relies on the reliability of the stability constant used, which is usually experimentally determined for 3D situations and applied to 2D models. The paper reports authors' attempts to cure the situation by deriving 2D dendritic tip scaling parameter for aluminium-based alloy: Al-4wt%Cu. The obtained parameter is then incorporated into the KGT dendritic growth model in order to compare it with the original 3D KGT counterpart and to derive two-dimensional and three-dimensional versions of the modified Hunt's analytical model for the columnar-to-equiaxed transition (CET). The conclusions drawn from the above analysis are further confirmed through numerical calculations of the two cases of Al-4wt%Cu metallic alloy solidification using the front tracking technique. Results, including the porous zone-under-cooled liquid front position, the calculated solutal under-cooling and a new predictor of the relative tendency to form an equiaxed zone, are shown, compared and discussed two numerical cases. The necessity to calculate sufficiently precise values of the tip scaling parameter in 2D and 3D is stressed.

## 1. Introduction

Dendritic crystal structures which form during solidification of metallic alloys have their enormous influence on the mechanical properties of solid alloys, thus, the dendritic growth problem has been a topic of long-term interest within the academia and metal industry. An exact solution for the non-dimensional temperature distribution ahead of an isolated dendritic crystal in the form of an isothermal and semi-infinite paraboloid of revolution, suggested by Papapetrou in 1935, with the fixed radius of tip curvature  $\rho$ , growing at a constant velocity,  $V$ , was first developed by Ivantsov [1] in 1947. The ability to correctly predict  $\rho$  is a problem of fundamental importance to the theory of dendritic growth with an additional dendritic tip selection constant (stability parameter)  $\sigma^*$  needed to determine the operating conditions (the combination of radius  $\rho$  and growth rate  $V$ ) at the dendrite tip. Values of the  $\sigma^*$  have been calculated from considerations of two main dendritic growth theories: marginal stability [2] and microscopic solvability [3]. The marginal stability estimates the tip selection parameter as constant for all materials under all conditions to be  $1/4\pi^2 \approx 0.025$  which is very close to the numerical value of  $\sigma^* = 0.025$  evaluated by Langer and Müller-Krumbhaar [2] for the symmetric problem in 3D, but approximately twice smaller than a value calculated using 3D phase-field simulations by Oguchi

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and Suzuki [4] for Al-4.5wt%Cu as a one-sided problem (the solute diffusion in the solid phase is negligible). Based on the marginal stability theory the Kurz-Giovanola-Trivedi (KGT) constrained (columnar) dendritic growth model was developed for steady-state conditions with the stability parameter which is equal to  $1/4\pi^2$  [5]. It must be stressed that the KGT model or similar models, like the LKT (Lipton, Kurz and Trivedi) one, are an intrinsic part of a range of alloy solidification models like cellular automaton (CA) [6], front tracking method (FTM) [7] or one-domain multiphase models based on volume averaging [8]. Gäumann et al. [9] have numerically modified Hunt's columnar to equiaxed transition (CET) analytical model [10] by combining the KGT model. Rebow and Browne [11] estimated dendritic tip stability parameters  $\sigma^*$  of two aluminium alloys, namely Al-4wt%Cu and Al-2wt%Si, based on measured values of their crystal-melt surface energy anisotropy strength  $\varepsilon$  and a simple linear scaling law of microscopic solvability theory. Subsequently they showed that the stability parameter has the significant influence on columnar dendritic growth models and consequently on a columnar to equiaxed transition with help of modified Hunt's analytical maps and meso-scale front tracking simulations. Recently, Mullis [12] presented results from a phase field model and found that the tip stability parameter  $\sigma^*$  is non constant, but varies as a function of tip undercooling for different alloy concentration, Lewis number, and equilibrium partition coefficient.

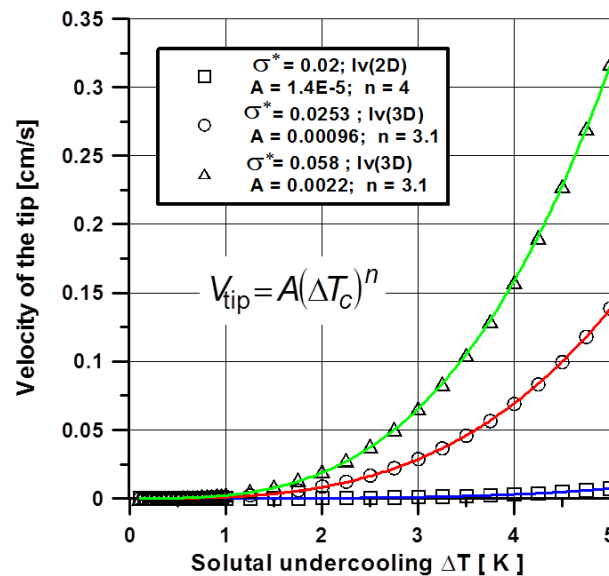
Usually the tip stability parameter  $\sigma^*$ , which is experimentally determined for 3D situations, is applied to 2D models. In this study, authors attempt to cure the situation by deriving 2D dendritic tip scaling parameter for aluminium-based alloy Al-4wt%Cu. The obtained parameter is then incorporated into the KGT dendritic growth model in order to compare it with the original 3D KGT counterpart and to derive two-dimensional and three-dimensional versions of the modified Hunt's analytical model for the columnar-to-equiaxed transition (CET). The conclusions drawn from the above analysis are further confirmed through numerical calculations of the two cases of a Al-4wt%Cu metallic alloy solidification using the front tracking technique on a fixed control-volume grid. Results, including the mush-liquid front position, the calculated solutal under-cooling and a new predictor of the relative tendency to form an equiaxed zone, are shown, compared and discussed.

## 2. Dendritic Growth Model and Columnar to Equiaxed Transition (CET) Analytical Map

Following the same approach developed by Rebow and Browne [11] dendritic tip stability parameters  $\sigma^*$  of Al-4wt%Cu for 2D equal to 0.02 and 3D equal 0.0253 (the typical value used in dendritic growth models) and 0.058 based on a simple linear scaling law of microscopic solvability theory, were derived. These values of  $\sigma^*$ AlCu were introduced into the KGT model with the following properties used for Al-4wt%Cu: diffusivity of solute in liquid ( $D_l$ )  $2.4 \times 10^{-9}$  m<sup>2</sup>/s; the Gibbs-Thomson coefficient ( $\Gamma$ )  $2.36 \times 10^{-7}$  mK; the partition coefficient ( $k$ ) 0.17; the liquidus slope ( $m$ )  $-2.6$  K/ wt%. The dendrite tip velocity  $V$  vs. the solutal (constitutional) tip undercooling  $\Delta T_c$  for Al-4wt%Cu is presented in figure 1 and fitted to the following relationship:  $V = A (\Delta T_c)^n$  where  $A$  ms<sup>-1</sup>K<sup>-n</sup> and  $n$  are fitting parameters, which values are shown in that figure. The dendrite tip velocity  $V$  for the 2D dendritic tip stability parameter is much lower than for 3D ones.

A Columnar to Equiaxed Transition (CET) analytical map can be significantly changed due to a selection of dendritic growth models as proved by Rebow and Browne [11]. The sensitivity of CET maps for 2D and 3D different stability parameters incorporated into the KGT model is compared. Based on a modification of Hunt's model [10], a relationship between the temperature gradient,  $G$  and the dendrite tip solutal undercooling,  $\Delta T_c$  for fully equiaxed growth for 2D and 3D can be written as,

$$\begin{aligned} G_{2D} &= M_{2D} N_0^{1/2} \Delta T_c \left( 1 - \frac{\Delta T_N^{n+1}}{\Delta T_c^{n+1}} \right) \\ G_{3D} &= M_{3D} N_0^{1/3} \Delta T_c \left( 1 - \frac{\Delta T_N^{n+1}}{\Delta T_c^{n+1}} \right) \end{aligned} \quad (1)$$



**Figure 1.** Comparison of the KGT dendritic growth model for 2D and 3D stability parameters  $\sigma^*$  of Al-4wt%Cu alloy.

where  $N_0$  is the total number of heterogeneous nucleation sites per unit volume,  $\Delta T_N$  is nucleation undercooling,  $M$  is constant for the particular growth model:

$$M_{2D} = \frac{1}{n+1} \sqrt{\frac{-\pi}{\ln(1-\Phi)}} \quad (2)$$

$$M_{3D} = \frac{1}{n+1} \sqrt[3]{\frac{-4\pi}{3\ln(1-\Phi)}}$$

where  $n$  is a parameter, see equation (1) and the volume fraction of equiaxed grains  $\phi = 0.49$  for a 3D map [10] and  $\phi = 0.544$  for a 2D map.

Substituting  $V = A (\Delta T_c)^n$  into equation (1) along with values of the parameter  $n$ ,  $\Delta T_N = 0.75$  K,  $N_{02D} = 1.24 \cdot 10^6 \text{ m}^{-2}$  and  $N_{03D} = 10^9 \text{ m}^{-3}$ , the CET map presented in figure 2 for Al-4wt%Cu alloy and 2D and 3D stability parameters are established.

In general, results show that for the same  $G$  value, the CET shifts to a higher  $V$  value for the 3D  $\sigma^*$  stability parameter proving that there is a significant difference in 2D and 3D representing of the CET effect.

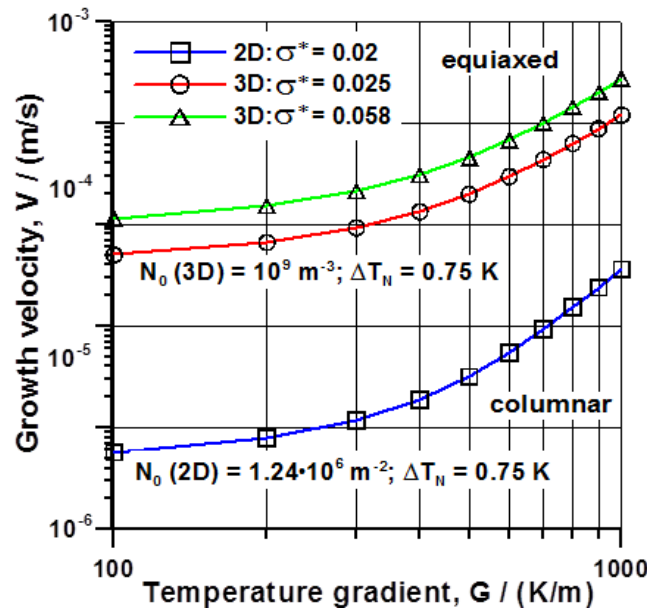


Figure 2. CET map for an Al-4wt%Cu alloy.

### 3. Mathematical and numerical models

The volume averaged heat balance equation, describing the transient, diffusive heat transfer in the solidifying binary alloy is discretised on the control volume mesh. The integral on the single control volume (CV) reads

$$\int_V \frac{\partial(\rho c T)}{\partial t} dV = - \int_{\partial V} k \frac{\partial T}{\partial x_j} n_j dS + \int_V \rho L \frac{\partial g_s}{\partial t} dV \quad (3)$$

where  $\rho$ ,  $c$ ,  $k$  and  $L$  are density, specific heat, thermal conductivity and latent heat, respectively. All material properties are assumed constant and equal in both phases. The last term of the right side of the equation (3) is due to heat release accompanying phase change. The solute micro-diffusion model obeys the Scheil's model, so the volumetric solid fraction  $g_s$  can be expressed as a function of temperature  $T$ :

$$g_s = \begin{cases} 0 & \text{for } T_{liq} \leq T \\ 1 - \left( \frac{T_M - T}{T_M - T_{liq}} \right)^{\frac{1}{k_p - 1}} & \text{for } T_E \leq T < T_{liq} \\ 1 & \text{for } T < T_E \end{cases} \quad (4)$$

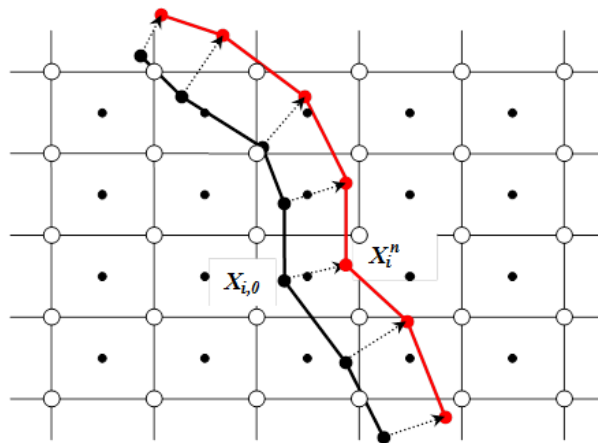
where  $T_M$ ,  $T_E$ ,  $T_{liq}$ ,  $k_p$  are the melting temperature of the solvent, the eutectic temperature, the liquidus temperature and the partition coefficient.

Between the liquidus and solidus isotherms, a region of coexistence of solid and liquid phases develops. Typically, two dendrite morphologies are observed, namely motionless columnar dendrites fixed to cooled walls of the mould and equiaxed grains in the central part of the domain. To distinguish these regions the virtual surface, representing an envelope of columnar dendrites tips is defined which is moving across the domain with respect to predefined columnar dendrite tips growth kinetics. Such defined interface is represented with mass-less markers connected with line segments

(figure 3). Their motion mimics the growth of columnar dendrites tips and is dependent on the local under-cooling  $\Delta T$ , which is a difference between the liquidus temperature and the interpolated temperature at the markers. The two-linear interpolation scheme is used to determine temperatures in front markers on the basis of nodal temperatures got from adjacent cells centres (black dots in figure 3). Positions of the markers are known from the previous time step, they are denoted as black dots connected with black segments in figure 3. The new positions of markers (red dots in the figure 3) shifted along the normal vectors to front, are calculated with the formula

$$\mathbf{X}_i^n = \mathbf{X}_{i,0} + \mathbf{n}_i \cdot w_i \cdot \Delta t \quad (5)$$

where  $\mathbf{X}_{i,0}$  is the position of the  $i$ -th marker in the previous time step,  $\mathbf{X}_i^n$  is the position of the moved  $i$ -th marker in the  $n$ -th iteration,  $\mathbf{n}_i$  is the vector normal to the front determined in the  $i$ -th marker,  $w_i$  dendrite growth rate calculated in the  $i$ -th marker according to the KGT model, and  $\Delta t$  is the time step. Details of the procedure of the front tracking are given in [7] and [13].

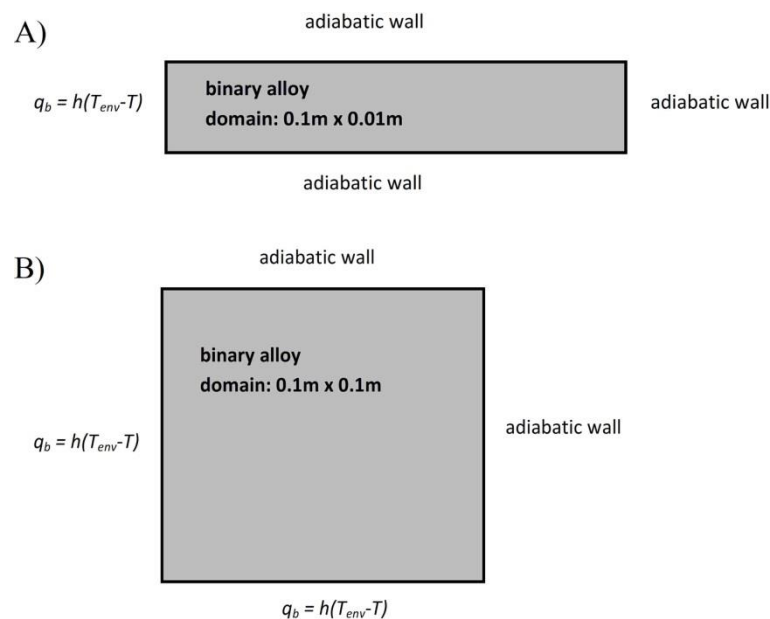


**Figure 3.** A fragment of the front crossing the control volume mesh in previous (black line) and next time step (red line).

The originally proposed by Browne [14] predictor for equiaxed solidification is determined to investigate the impact of the dendrite tip kinetics on the tendency to formation of CET. In the author's opinion the blockage of growth of columnar dendrites by thickening equiaxed grains growing in the under-cooled liquid could take place till the equiaxed index parameter achieves its maximum value. The definition of equiaxed index taken from [14] needs the information on a volume of part of each c.v. which is behind the front. To determine this function the switching function, presented in papers [7] and [13] can be utilized. However, in this analysis the immersed front tracking approach, originally introduced by Peskin [15] is utilized. The indicator function, which is distributed to nearest points of the front using the Peskin's cosine function, is determined. The details of this method are available elsewhere, e.g. in [16].

#### 4. Numerical results for considered cases

To examine the influence of the dendritic tip kinetics model on the rate of development of the undercooled region and columnar dendrites zone, the solidification of the binary alloy Al-4wt%Cu problem was solved in two geometries (figure 4). Configuration of boundary conditions ensures the 1D heat transfer and flat front (figure 4A) or 2D heat transfer and curved front (figure 4B). The initial temperature,  $T_0$  is equal to 687°C, heat transfer coefficient,  $h$  is 1000 W/(m<sup>2</sup>K) and environment temperature,  $T_{env}$  is 50°C.



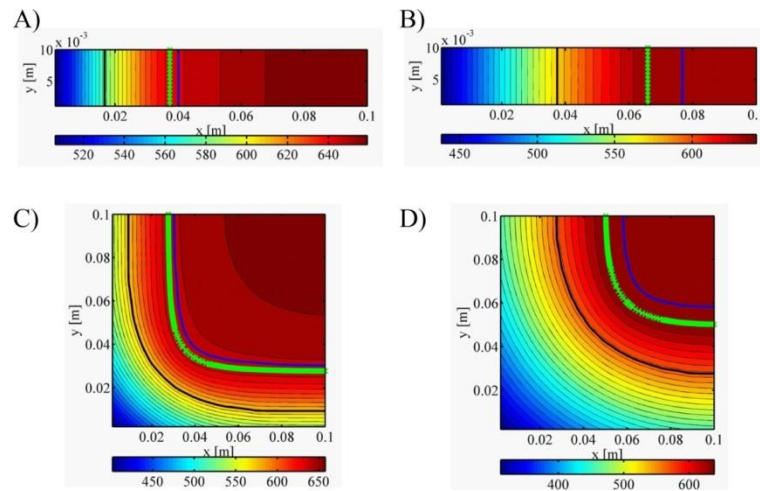
**Figure 4.** Configurations of geometry and boundary conditions of considered cases.

Thermophysical properties of considered alloy are listed in table 1. Calculations were performed on a uniform mapped control volume (c.v.) grid. Numbers of divisions was 100 x 10 for the 1D heat transfer case and 50 x 50 for the 2D heat transfer case. The fully implicit Euler time integration scheme was utilized with time step equal to 0.005s. At the end of each time step the Poisson equation for the indicator function was solved and equiaxed index was determined.

**Table 1.** Thermophysical properties of Al-4wt%Cu.

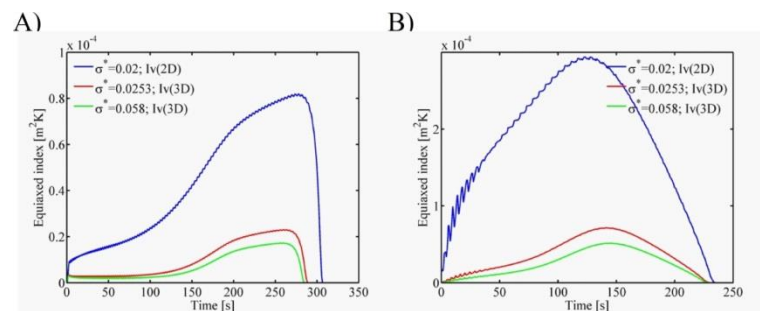
Symbol	Unit	Value
$\rho$	[kg/m <sup>3</sup> ]	2600
$c$	[J/(kgK)]	1000
$k$	[W/(mK)]	90
$L$	[J/kg]	390000
$T_M$	[°C]	661.48
$T_{liq}$	[°C]	648
$T_E$	[°C]	580
$k_p$	[-]	0.17

Temperature distribution in the considered domains are presented in figure 5. Together with a position of the front of columnar dendrites, both solidus and liquidus isotherms are shown. They bound the regions where one of two grain morphologies is predominant. In the under-cooled liquid region, between liquidus isotherm and the columnar dendrites front equiaxed grains develop. In considered model of equilibrium solidification, driven by the Scheil law, solid phase appear at the liquidus temperature, so nucleation under-cooling is set equal to zero. The under-cooling liquid regions predicted in two considered cases are rather narrow, which corresponds to simulations of Banaszek and Browne [17] performed for the same alloy and similar boundary conditions. A width of the under-cooled region increases in time while the temperature profile flattens so conditions for equiaxed growth are more pronounced.



**Figure 5.** Temperature distributions in two considered domains. Temperature maps A) and B) relates to 1D heat transfer 60s and 120s after process start, C) and D) relates to 2D heat transfer after 90s and 180s. Position of the front is marked with green line, liquidus and solidus isotherms are marked with thick blue and black lines, respectively.

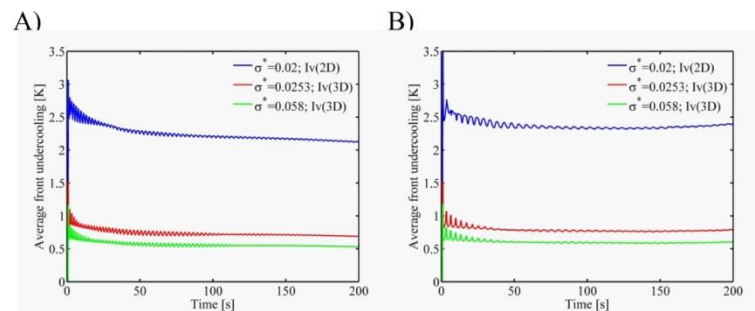
The indicator function, predicted with the methodology given in [14] and [17], is presented in figure 6. For the 1D heat flow case the maximum in the indicator function is observed at the same time, independent of the imposed kinetics. For the 2D heat transfer case the maximum is shifted towards the earlier times. In general, results for both numerical cases show that there is a large discrepancy between equiaxed predictors for FT numerical simulations using 2D and 3D dendritic growth models.



**Figure 6.** Equiaxed index as a function of time for: A) 1D heat transfer case and B) 2D heat transfer case for three considered kinetics.

Under-cooling averaged along the front, after short transient time period stabilizes and tends to a virtually constant value, dependent on the used kinetics. This value is slightly more elevated for higher dimension of the domain.





**Figure 7.** Average under-cooling as a function of time for: A) 1D heat transfer case and B) 2D heat transfer case for three considered kinetics.

## 5. Conclusions

In the proposed paper three dendrite tip kinetics, derived for various tips stability parameters, valid for 2D and 3D geometry of dendrites are examined for solidification cases involving planar and curved front. Extents of the under-cooled zone, positioned between line of the front and the liquidus isotherm are determined. They show the relatively narrow under-cooled liquid region developing in time. The computed equiaxed index (figure 6) shows tendency to exhibit a maximum, which is more pronounced for the "slowest" kinetics (figure 1) for which a scope of parameters resulting the equiaxed growth is widest (figure 2). Under-cooling averaged along the front shown tendency to stabilize at constant value. In general, analytical and numerical results show that there is a significant difference in a 2D and 3D representation of the CET effect and special consideration should be paid to the selection of stability parameters  $\sigma^*$  for dendritic growth models.

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