

# Study on Vibration Reduction Design of Suspended Equipment of High Speed Railway Vehicles

Yu Sun, Dao Gong and Jinsong Zhou

Institute of Rail Transit of Tongji University, Shanghai 201804, China

gongdao@tongji.edu.cn

**Abstract.** The design methods of the under-chassis equipment of a high speed railway vehicle based on dynamic vibration absorber (DVA) theory and vibration isolation theory are proposed, respectively. A detailed rigid-flexible coupled dynamic model of a high speed railway vehicle which includes car body flexibility and the excitation of the suspended equipment is established. The vibrations of the car body and the suspension equipment with the proposed design methods are studied. Results show that the elastic vibration of the car body can be decreased effectively by mounting the under-chassis equipment with elastic suspension. Comparing with vibration isolation theory, the method based on DVA theory is more effective for suppressing the car body flexible vibration, but it will increase the vibration of the equipment to a certain extent. The method based on vibration isolation theory can reduce the vibration of both the car body and the equipment at the same time. Therefore, the design method should be selected appropriately according to the specific requirement.

## 1. Introduction

For the distributed power electric multiple unit (EMU) high speed trains, lots of the equipment, such as the traction convertor, is suspended under the chassis of the car body, and some of which has excitation sources and weights up to several tons. To reduce the adverse effects of the car body vibration caused by the equipment and avoid the resonance of the car body and the equipment, elastic suspensions, such as rubber springs, are usually used for the vibration reduction. According to the different design concepts, the vibration reduction design methods can be divided into three types, which are the active vibration reduction, the vibration absorption and the vibration isolation, respectively. The active vibration reduction is not available because of the high cost, difficult to maintain and the limited space of the equipment cabin. Therefore, the vibration absorption and the vibration isolation are the main methods of the vibration reduction design.

Gong et al.<sup>[1]</sup> took a under-chassis equipment as a DVA and used an Euler-Bernoulli beam model of railway vehicles to analyze the inhibition of the car body elastic vibration. Results showed that the under-chassis equipment can effectively reduce the vibration of the car body, and after adopting the DVA design, the requirement of the first vertical bending frequency of the car body is greatly reduced. To solve the serious damage problem of the under-chassis equipment of high speed trains, Zhang et al.<sup>[2]</sup> established a vehicle-track coupling dynamic model and calculated the eigenfrequencies of the car body. Based on the vibration reduction theory and the characteristics of the rubber, they chose the vibration frequency range of the equipment and confirming the stiffness and damping of the rubber spring. Gong et al.<sup>[3,4]</sup> created a rigid-flexible coupled vertical dynamic model of a railway vehicle that



contained a suspended equipment and designed the rubber element parameters of the suspended equipment based on the vibration isolation theory, and the influences of the rubber element parameters, the equipment mass and the position of the equipment on the ride quality of the vehicle and the vibration of the equipment were analyzed. Results showed that the static deflection of the rubber element played a major role in vibration isolation; With the change of vehicle speed, the optimal value of the static deflection would shift, and the equipment which had large mass should be mounted close to the center of the car body.

To investigate a reasonable vibration reduction design method of the under-chassis equipment, a detailed rigid-flexible coupled dynamic model of a railway vehicle which includes the car body flexibility and the excitation of the under-chassis equipment is established in this study. The vibrations of both the car body and the equipment with the two kinds of vibration reduction design methods of the under-chassis equipment, i.e. the DVA method and the vibration isolation theory method, are calculated and compared on the condition that the equipment is excited when the vehicle is running.

## 2. Modelling

With the further research of the vehicle vibration, considered only the car body rigidity is unable to reflect its dynamic characteristics accurately, especially for a vehicle runs at a higher speed. Therefore, the car body should be considered as a flexible system with local deformation in the dynamic studies<sup>[5,6]</sup>. Compared with the rigid car body system, the flexible system mainly considered the coupling vibration of the flexible car body deformation and the system, which could make the virtual prototype simulation test more close to the real situation. In this paper, a detailed rigid-flexible coupled dynamic model of railway vehicle which includes the car body flexibility and the excitation of the under-chassis equipment is established by utilizing the SIMPACK multi-body dynamic simulation software and the simulation calculation is carried out.

To obtain the rigid-flexible coupled dynamic model, the flexible car body is modelled and calculated by means of the commercial finite element software MSC. Nastran and imported into SIMPACK through the FEMBS interface. The finite element model of the car body contains 630191 nodes and 775146 elements, which is shown in Fig.1. The frequency results of the first five modes of the car body are shown in Tab.1.

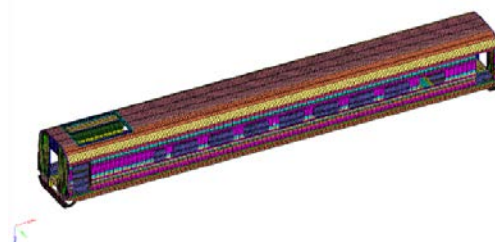


Fig.1 The car body finite element model

Tab.1 The car body eigenfrequency results

Order	Mode shape	Frequency/Hz
1	Diagonal distortion	9.701
2	First vertical bending	11.625
3	Torsion	12.421
4	First lateral bending	14.163
5	Breathing	14.353

A detailed rigid-flexible coupled dynamic model of the vehicle is established (Fig. 2), including one elastic car body, two bogies, eight axle boxes and four wheelsets. Compared with the car body, the bogies, the axle boxes and the wheelsets are still considered as rigidity because of the small elastic

deformation. According to the contribution of the eigenmodes of the car body to the vibration energy<sup>[8]</sup>, the car body flexibility only considers its first five eigenmodes in Tab.1. And the nonlinear characteristics of the dampers and the wheel/rail contact are also considered in the model. The calculation results of the rigid modes of the car body are shown in Tab.2.

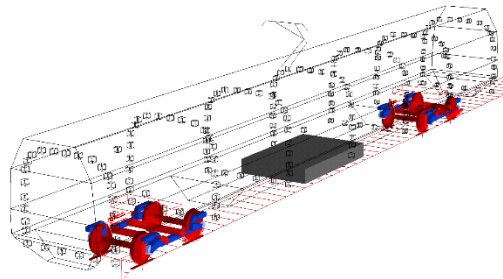


Fig.2 Rigid-flexible coupled dynamic system model

Tab.2 The car body rigid modes

Order	Car body rigid modes	Frequency/Hz	damping ratio/%
1	Upper sway mode	0.468	29.3
2	Bounce mode	0.713	9.92
3	Lower sway mode	0.735	33.8
4	Pitch mode	0.940	12.1
5	Yaw mode	1.029	86.3

In the study, the under-chassis equipment contains a cooling fan which will generate an excitation when the vehicle operates. The excitation consists of harmonic waves in three directions, and its working frequency is 25Hz and with the amplitude of 3200N. The excitation curve is shown in Fig.3.

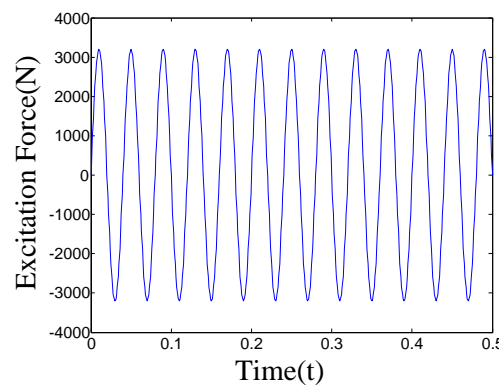


Fig.3 The suspended equipment excitation

### 3. The vibration reduction design based on dynamic vibration absorber

The DVA consists of the auxiliary mass, springs and dampers. Its working principle is the use of the anti-resonant characteristic of the multiple-degree-of-freedom system. Due to the dynamic action of the absorber, the force that acts on the main system by the absorber is in the opposite direction of the external exciting force but with a similar amplitude. Therefore, the forces act on the main system is counteracted, which the vibration of the main system can be improved<sup>[9]</sup>. Fig.4 shows a typical dynamic model of an absorber, in which  $m_1$  and  $k_1$  are the mass and stiffness of the main system, respectively;  $m_2$ ,  $k_2$  and  $c_2$  are the mass, stiffness and damping of the absorber, respectively. In the

detailed rigid-flexible coupled dynamic model shown in Fig.2,  $m_1$  represents the car body while  $m_2$  represent the equipment.

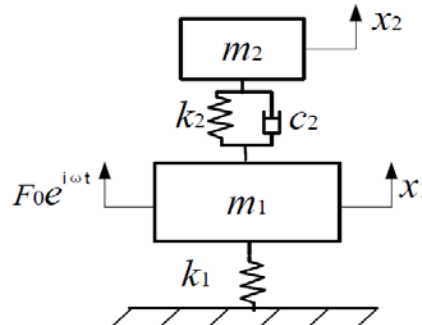


Fig.4 A dynamic vibration absorber model

It is assumed that the main system is excited by a harmonic excitation  $F_0 e^{i\omega t}$ , then the motion equations of the main system and the absorber are

$$m_1 \ddot{x}_1 + k_1 x_1 + c_2 (\dot{x}_1 - \dot{x}_2) + k_2 (x_1 - x_2) = F_0 e^{i\omega t} \quad (1)$$

$$m_2 \ddot{x}_2 = k_2 (x_1 - x_2) + c_2 (\dot{x}_1 - \dot{x}_2) \quad (2)$$

When the main system is excited, the response of the main system and the absorber can be written as  $x_1 = X_1 e^{i\omega t}$ ,  $x_2 = X_2 e^{i\omega t}$ , respectively, then the acceleration of the main system is

$$\ddot{x}_1 = -\omega^2 X_1 e^{i\omega t} = \bar{A}_1 e^{i\omega t} \quad (3)$$

where,  $\bar{A}_1$  is the complex amplitude of the acceleration. Taking the amplitude ratio  $\bar{A}_1/F_0$  into account, one obtains,

$$\frac{\bar{A}_1}{F_0}(\omega) = \frac{-\omega^2(m_2\omega^2 + k_2 + i\omega c_2)}{(-m_1\omega^2 + k_1)(-m_2\omega^2 + k_2) - m_2k_2\omega^2 + i\omega c_2(-m_1\omega^2 + k_1 - m_2\omega^2)} \quad (4)$$

Dividing both of the numerator and denominator of equation (4) by  $(m_1 m_2)^2$ , and introducing the following physical quantities: the static displacement of the main system which caused by the static force that equals to the external excitation amplitude  $\delta_{st} = F_0/k_1$ , the natural frequency ratio of the DVA to the main system  $\gamma = \sqrt{k_2/m_2}/\sqrt{k_1/m_1}$ , the mass ratio of the DVA to the main system  $\mu = m_2/m_1$ , the damping ratio of the DVA  $\xi = c_2/2\sqrt{m_2 k_2}$ , the ratio of the external excitation frequency to the natural frequency of the main system  $\lambda = \omega/\sqrt{k_1/m_1}$ , then

$$\frac{A_1}{\delta_{st}}(\omega) = \sqrt{\frac{(\lambda^2)^2[(\gamma^2 - \lambda^2)^2 + (2\xi\lambda)^2]}{[(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu\gamma^2\lambda^2]^2 + [1 - (1 + \mu)\lambda^2]^2(2\xi\lambda)^2}} \quad (5)$$

According to the DVA theory and the vibration characteristics of the vehicle system, the car body and the equipment can be simplified as a system of two degrees of freedom, in which the car body is the main system, and the equipment is the absorber. In order to get the optimum design of the absorber parameters, the following parameters are defined:

$$\omega_d = \sqrt{\frac{k_d}{m_d}}, \mu = \frac{m_d}{m_b}, \gamma = \frac{\omega_d}{\omega_a} \quad (6)$$

The first vertical bending frequency is chosen as the car body natural frequency  $\omega_a$ , which makes the greatest contribution to the vertical elastic vibration of the car body. Combined with the fixed-point theory<sup>[9]</sup>, the optimum frequency ratio between the tuned frequency of the DVA and the natural frequency of the main system is:

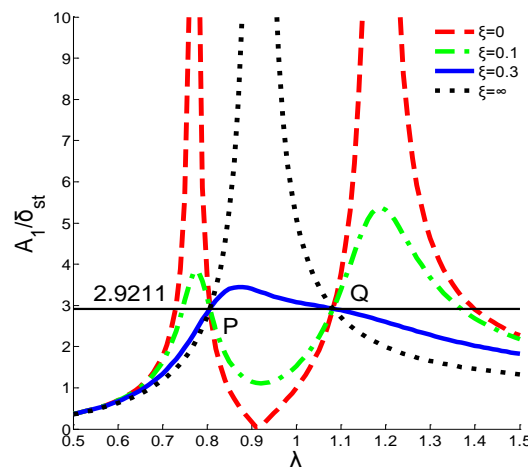
$$\gamma_{opt} = \frac{1}{\sqrt{1 + \mu}} \quad (7)$$

The notation and the value of the parameters in equations (6) and (7) are shown in Tab.3. It can be seen that, the tuned frequency of the equipment is close to the first vertical bending frequency of the car body.

Tab.3 Parameters of the dynamic vibration absorber

Parameter	Value	Unit
The tuned frequency of the equipment	10.63	$\omega_d/$ (Hz)
The stiffness of the rubber spring	28550	$k_d/$ (N/mm)
The equipment mass	6400	$m_d/$ (kg)
The car body mass	32650	$m_b/$ (kg)
The mass ratio	0.196	$\mu$
The car body first vertical bending frequency	11.625	$\omega_a/$ (Hz)
The optimum frequency ratio	0.9144	$\gamma_{opt}$

According to equation(5) and the values of Tab.3, the relation between  $A_1/\delta_{st}$  and  $\lambda$  on the optimum condition is shown in Fig.5. In which, it can be seen that no matter what the value of the damping ratio  $\xi$  is, the four curves in Fig.5 all pass through the fixed points  $P$  and  $Q$ .

Fig.5 The relation between  $A_1/\delta_{st}$  and  $\lambda$ 

#### 4. The vibration reduction design based on vibration isolation theory

Vibration isolation is another method to reduce the system vibration by using the isolating element, such as rubber springs. According to the different transmission direction of the vibration, vibration isolation can be divided into two categories: one is negative vibration isolation which aims reduce the vibration transmitted from the supporting base to the equipment and the other is positive vibration isolation which aims to reduce the vibration transmitted from the equipment to the surrounding objects<sup>[10,11]</sup>. Although the concept of positive and negative vibration isolation is different, the basic principle is the same.

According to the vibration characteristics of the suspended equipment of the railway vehicle, the vibration transmission of the car body to the suspended equipment is designed as negative vibration isolation, and the reversed transmission is designed as positive vibration isolation. Based on the vibration isolation theory, no matter what the value of  $c/c_0$  is, only when the ratio of the car body eigenfrequency  $\omega$  to the natural frequency of the equipment  $\omega_n$ , i.e.  $\omega/\omega_n$ , is greater than  $\sqrt{2}$ , the vibration transmissibility  $T_A$  can be less than 1. Therefore, to achieve the purpose of vibration isolation, the choice of the natural frequency of the suspended equipment should satisfy the condition  $\omega/\omega_n > \sqrt{2}$  (condition 1). If not, then it should satisfy the condition  $\omega/\omega_n < 0.4$  (condition 2)<sup>[11]</sup>. Fig.6 shows the vibration transmissibility from the car body to the under-chassis equipment.

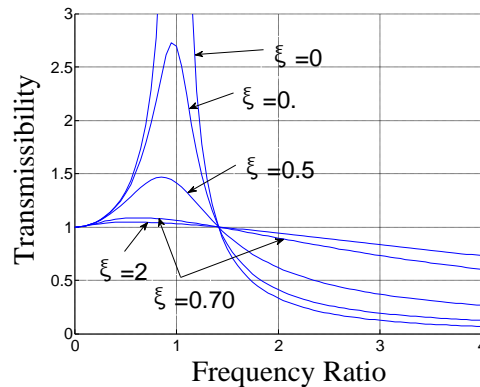


Fig.6 The vibration transmissibility curves

According to condition 1  $\frac{\omega_1}{\omega_n} > \sqrt{2}$ , considering the car body eigenfrequencies,  $\omega_1$  equals 11.625 Hz, one obtains

$$\omega_n < \frac{11.625}{\sqrt{2}} = 8.22 \quad (3)$$

According to condition 2  $2 \frac{\omega_2}{\omega_n} < 0.4$ , considering the frequency of the car body rigid modes,  $\omega_2$  equals 1.029 Hz, one obtains

$$\omega_n > \frac{1.029}{0.4} = 2.57 \quad (4)$$

Therefore, for the negative vibration isolation, the optimum natural frequency of the equipment is in the range of 2.57~8.22 Hz. However, a low frequency of the equipment will lead to a large static deflection, then the frequency range of 5~9Hz is selected in the research. As for positive vibration isolation, the excitation frequency of the suspended equipment is 25Hz, which is much larger than 5~9Hz, so it can also meet the requirement of positive vibration isolation.

Although the frequency range of the suspended equipment can be obtained based on the vibration isolation theory, the final selection of which, should be considered the dynamic analysis and take the optimal vibration of both car body and equipment as the objective. According to the frequency range of the suspended equipment, the vertical stiffness of the rubber spring can be obtained. Making both the longitudinal-vertical stiffness ratio and vertical-horizontal stiffness ratio equal to 2, the stiffness in the three directions of the rubber spring can be obtained,

$$k_z = (2\pi\omega_n)^2 \times m, \quad k_x = k_z \times 2, \quad k_y = k_z/2 \quad (5)$$

The damping ratio of each kind of rubber element is different. If the damping ratio is too high, the rubber is easy to be heated and the process of aging and creep will be accelerated. In the study, the natural rubber is chosen and its damping ratio is 0.06<sup>[12]</sup>.

In the simulation calculation, the low excitation high-speed spectrum is adopted as the track irregularity<sup>[13]</sup>, when the frequency range of the suspended equipment is 5~9Hz, the vertical ride quality, the vertical acceleration RMS(root-mean-square) value of the car body and the vertical acceleration RMS value of the equipment are calculated under different speeds. The results are shown in Fig.7~Fig.9. It can be seen from Fig.7 that the vertical ride quality decreases with the increasing of the frequency of the suspended equipment, and it reaches to a minimum value when the frequency of the suspended equipment is up to 9Hz.

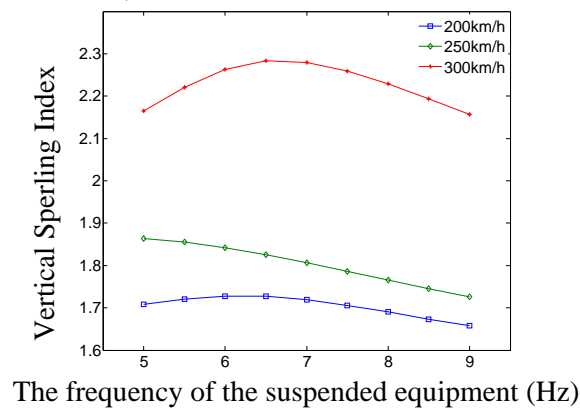


Fig.7 The vertical ride quality (Sperling index)

It can be seen from Fig.8 that, the vertical acceleration RMS value of the car body decreases with the increasing of the frequency of the suspended equipment, which is similar to the tendency of the vertical ride quality.

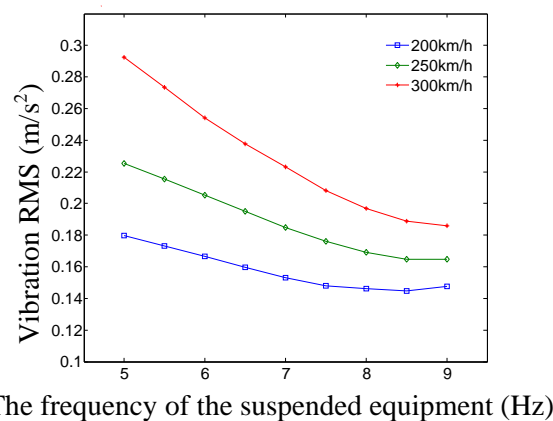


Fig.8 Vertical acceleration RMS value of the car body

According to Fig.9, the vertical acceleration RMS value of the equipment increases rapidly with the increasing of the frequency of the suspended equipment. Based on the results of Fig7~Fig.9, the effects of the frequency of the suspended equipment on the vibration of car body and equipment are opposite. The optimum frequency of the suspended equipment is 8Hz with the consideration of both of the vibrations of the car body and the equipment.

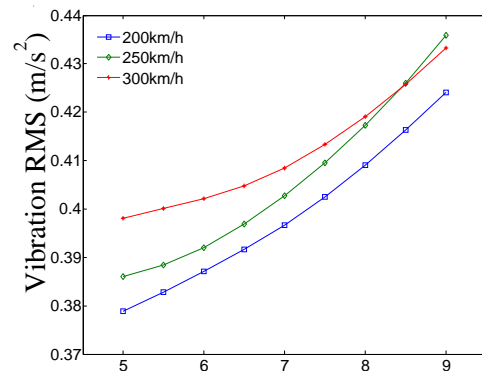


Fig.9 Vertical acceleration RMS value of the equipment

### 5. Comparative analysis of the vibration reduction design

According to section 2 and 3, the optimum frequency of the suspended equipment based on dynamic vibration absorber method and vibration isolation theory are 10.63Hz and 8Hz, respectively. The two design methods above will be compared in this section, and the equipment which mounted rigidly under the car body chassis is also added in the section as a reference.

Fig.10~Fig.12 show the results of the vertical ride quality of the car body, the vertical acceleration RMS value of the car body and the equipment, respectively, when changing the running speed. One can find that, when the equipment is mounted rigidly under the car body chassis, the vibration and ride quality of the car body is the worst due to the decrease of the eigenfrequency of the car body with fixed equipment<sup>[13]</sup>. Besides, the excitation of the equipment transmits directly to the car body without any reduction. Compared with the design method based on vibration isolation theory, the method based on DVA has a much better effect on reducing the elastic vibration of the car body, but the vibration of the suspended equipment will increase to a certain extent. Moreover, the design method based on vibration isolation theory can reduce the vibration of the car body and equipment at the same time.

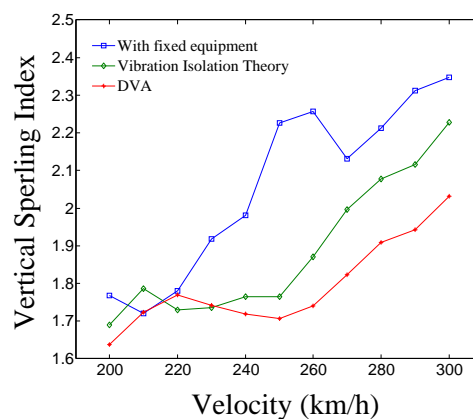


Fig.10 The vertical ride quality of the vehicle



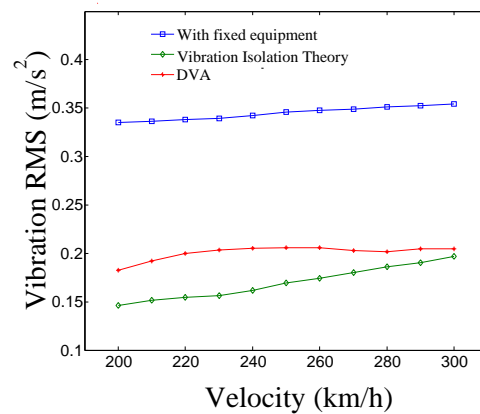


Fig.11 The vertical acceleration RMS value of the car body

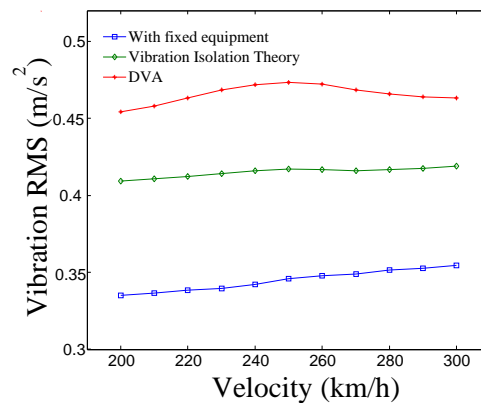


Fig.12 The vertical acceleration RMS value of the equipment

The vertical acceleration PSDs (power spectrum density) of the car body and the equipment are shown in Fig.13~Fig.14, respectively. In which, the peak value of 25Hz is caused by the suspended equipment excitation. It can be seen from Fig.13 that, the vibration of the car body based on the design of the vibration isolation theory is the lowest, then is that based on the design of DVA, and that of fixed equipment is the worst. It can be seen from Fig.14 that, compared with DVA method, the vibration isolation theory is more effective on reducing the impact of the equipment excitation.

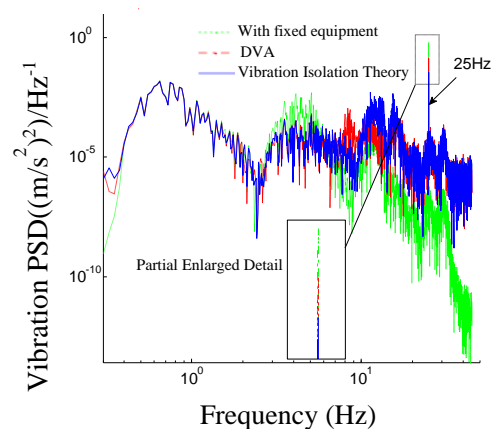


Fig.13 The vertical acceleration PSD of the car body

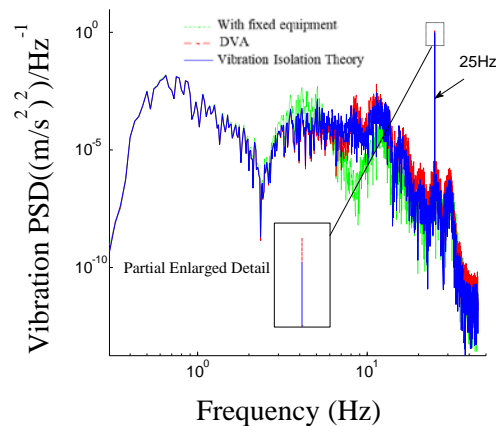


Fig.14 The vertical acceleration PSD of the equipment

## 6. Conclusions

- (1) The vibration of the car body can be reduced obviously by mounting the equipment elastically.
- (2) Compared with the design method based on vibration isolation theory, the method based on DVA has a much better effect on reducing the elastic vibration of the car body, but the vibration of the suspended equipment will increase to a certain extent. It is suggested to give priority to using this method when reducing the car body vibration and the suspended equipment has no excitation.
- (3) The design method based on vibration isolation theory can reduce the vibration of the car body and the equipment at the same time. It is better to use this method to reduce the vibrations of the car body and the equipment when the equipment has excitation.

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