

Random seismic response and sensitivity analysis of uncertain multi-span continuous beams subjected to spatially varying ground motions

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Abstract. An analytical method is formulated for the seismic analysis of multi-span continuous beams with random structural parameters subjected to spatially varying ground motions. An earthquake-induced ground motion is modelled as a stationary random process defined by power spectral density function, and the spatial variation is considered. The physical parameters of the multi-span beams are random and modelled as continuous random Gaussian variables. The stationary random responses are determined as approximate explicit functions of the structural parameters. Direct differentiation of these functions with respect to the structural parameters provides analytical expressions of the sensitivities of the stationary responses. On the basis of Taylor expansion, the statistic moments of the random responses are obtained. Taking the four-span beam as an illustrative example, the mean value and standard deviation of the random responses are computed and compared with those from Monte Carlo simulation to demonstrate the accuracy of the proposed method. Results are illustrated for the influence of different structural parameters on the statistic moments of the random responses. It is found that randomness in Young's modulus and the mass per unit length has approximate equivalent and significant influence on the random responses, while that of damping is negligible.

1. Introduction

Due to the influences of the *wave passage effect*, *incoherence effect* and *site-response effect*, variations can be found during the seismic wave propagation along the length of long-span structures, which result differences in amplitude and phase of ground motions at supports of the structures [1]. This phenomenon is termed *spatially varying ground motions*, which have been a fundamental problem of interest for decades [1-8]. Harichandran and Vanmarcke [2] investigated the accelerograms recorded by the SMART 1 seismograph array in Lotung, Taiwan, using the second-order theory of random fields and proposed a frequency-dependent spatial correlation model of earthquake ground motions. Der Kiureghian and Neuenhofer [3] proposed a new response spectrum method for seismic analysis of linear multi-degree-of-freedom, multiply supported structures subjected to spatially varying ground motions, which was based on the fundamental principles of random vibration theory. Lee and Penzien [4] developed a stochastic method for seismic analysis of structures and piping systems subjected to multiple support excitations in both the time and frequency domains. Zerva [5, 6] analyzed two- and three-span beams subjected to different types of spatially varying ground motions. The results indicated that the spatially varying effects of the ground motions have complex and remarkable



influences on the dynamic responses of long-span structures. On the basis of the pseudo-excitation method, Zhang et al. [7] presented a random vibration methodology for the seismic analysis of multi-supported structures, which had advantage in computation time. In most of the above literatures, based on the modal dynamic concept, the quasi-static decomposition method was widely used. Nevertheless, as mentioned by Chen and Tsaur [8], the quasi-static solution might still be very difficult to find for a continuous system, and in a discrete system, the quasi-static part was obtained at the cost of a large matrix inversion.

On the other hand, the structural parameters may be also random for the reason of inhomogeneity of material, or randomness resulting from the assembly process, measurement and manufacturing tolerances. It was reported that the uncertainty in parameters of a structural system might have equal or even greater influence on the response than the randomness in excitations [9]. Therefore, dynamic analysis for structural systems with random parameters under random excitations has been the subject of research for many years [9-15]. Jensen and Iwan [10] developed a method in time domain for the dynamic analysis of linear systems with uncertain parameters to non-stationary stochastic excitation. Gao and Kessissoglou [11] investigated the dynamic characteristics and responses of stochastic truss structures under non-stationary random excitations, and a new method called the *random factor method* was put forward. The influences of the randomness of the structural parameters on the structural seismic responses were studied. Muscolino et al. [12] presented a semi-analytical approach based on the so-called rational series expansion, which could be used for the sensitivity analysis of the response of linear discretized structures subjected to stationary multi-correlated Gaussian random excitations. Wall and Bucher [13] studied the dynamic effects of uncertainty in structural properties when the excitation was random by using the *perturbation stochastic finite element method*. Bhattacharyya and Chakraborty [14] studied the stochastic sensitivity of structures with random structural parameters subjected to random earthquake loadings by Neumann Expansion technique. The random structural parameters were modeled as homogeneous Gaussian stochastic field and discretized by the local averaging method. Li and Liao [15] investigated the use of orthogonal expansion method with the pseudo-excitation method for analyzing the dynamic response of structures with uncertain parameters under external random excitations.

From the above literatures, it can be found that the random seismic responses of the long-span structures are greatly affected by (i) the spatial variation of ground motions, (ii) the randomness in excitations, and the (iii) uncertainties in structural parameters. However, the research which includes the influences of all the above three factors is very little. This provides the initial motivation for the present work, in which the seismic responses of multi-span continuous beams with random structural parameters subjected to spatially varying ground motions are investigated. A new analytical method is presented for the dynamic analysis of multi-span continuous beams in frequency domain. Since the present method is not a mode-based method, the solutions for normal modes and the quasi-static displacements are avoided, which may be difficult for a continuous system. The influences of the random structural parameters on the statistic moments of the random responses are converted to a sensitivity problem through the second order Taylor expansion.

2. Problem formulation

The schematic of the multi-span beam subjected to the spatially varying ground motions is shown in figure 1. The beam is modeled based on Bernoulli-Euler beam theory and the transverse deformation is considered. The governing equation and boundary conditions can be written as

$$\begin{cases} m \frac{\partial^2 y(x,t)}{\partial t^2} + c \frac{\partial y(x,t)}{\partial t} + EI \frac{\partial^4 y(x,t)}{\partial x^4} = 0 \\ y(x_i, t) = u_i(t) \end{cases} \quad (1)$$

where $y(x, t)$ is the displacement of beam, m is the mass per unit length, c is the viscous damping per unit length, E is Young's modulus and I is the moment of inertia of the cross-section. x_i and $u_i(t)$ is the location and displacement of the i -th support, respectively. N is the total number of supports.

The arbitrary parameter of beam, b , is assumed to be an random variable and expressed as

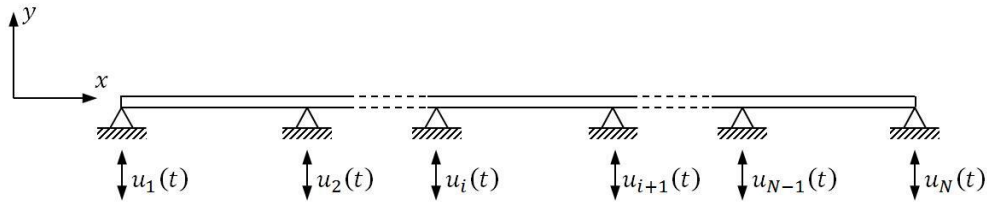


Figure 1. Schematic of a multi-span beam.

$$b = b_0(1 + \epsilon) \quad (2)$$

where b_0 is the mean value, ϵ is the zero mean Gaussian and dimensionless fluctuation. As the physical parameters, such as Young's modulus and mass, must be strictly positive, ϵ is required to satisfy the condition $P[(1 + \epsilon) \leq 0] = 0$. This requirement, strictly speaking, rules out the use of Gaussian models for the random variables. However, for small ϵ , it is expected that Gaussian models can still be used if the primary interest in the analysis is on finding the first few response moments and not on the response behavior near the tails of the probability distributions. For brevity, all the fluctuations of the random physical parameters of the multi-span beams can be expressed as the following vector

$$\epsilon = [\epsilon_1 \quad \epsilon_2 \quad \cdots \quad \epsilon_r]^T \quad (3)$$

where r is the total number of random parameters.

The seismic ground motion is assumed to be a normal stationary random process, and the spatial effects are considered. Therefore, the random displacements of supports are characterized by the power spectral density function (PSD) matrix [1]

$$\mathbf{S}_{uu} = \begin{bmatrix} S_{u_1u_1} & S_{u_1u_2} & \cdots & S_{u_1u_N} \\ S_{u_2u_1} & S_{u_2u_2} & \cdots & S_{u_2u_N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{u_Nu_1} & S_{u_Nu_2} & \cdots & S_{u_Nu_N} \end{bmatrix} \quad (4)$$

where $S_{u_iu_j} = \gamma_{ij} \sqrt{S_{u_iu_i} S_{u_ju_j}}$ and γ_{ij} are the cross-PSD and coherency function of the displacements at the i -th and j -th supports, respectively; and $S_{u_iu_i}$ is the auto-PSD of the displacement at the i -th support.

3. Random responses of deterministic structures subjected to random excitations

3.1. Stationary random responses

According to the superposition principle of linear system, an arbitrary response of the structure, $z(x, t)$, can be expressed as [16]

$$z(x, t) = \sum_{i=1}^N \int_{-\infty}^{+\infty} G_i(x, t, \theta_i) u_i(t - \theta_i) d\theta_i \quad (5)$$

where $G_i(x, t, \theta_i)$ is the Green's function corresponding to the i -th support. As $u_i(t)$ is a stationary random process, $z(x, t)$ is also stationary. Accordingly, the autocorrelation of $z(x, t)$ can be given by

$$\begin{aligned}
R_{zz}(x, \tau) &= E[z(x, t)z(x, t + \tau)] \\
&= E \left[\sum_{i=1}^N \sum_{j=1}^N \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} G_i(x, t, \theta_i) G_j(x, t, \theta_j) u_i(t - \theta_i) u_j(t - \theta_j + \tau) d\theta_i d\theta_j \right] \\
&= \sum_{i=1}^N \sum_{j=1}^N \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} G_i(x, t, \theta_i) G_j(x, t, \theta_j) R_{ij}(\tau + \theta_i - \theta_j) d\theta_i d\theta_j
\end{aligned} \quad (6)$$

where $E[\]$ represents the expectation operator. By applying the Wiener-Khinchin theorem, the PSD at the angular frequency ω of $z(x, t)$ can be obtained as

$$\begin{aligned}
S_{zz}(x, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{zz}(x, \tau) e^{-i\omega\tau} d\tau \\
&= \frac{1}{2\pi} \sum_{i=1}^N \sum_{j=1}^N \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} G_i(x, t, \theta_i) G_j(x, t, \theta_j) \int_{-\infty}^{+\infty} R_{ij}(\tau + \theta_i - \theta_j) e^{-i\omega\tau} d\tau d\theta_i d\theta_j \\
&= \sum_{i=1}^N \sum_{j=1}^N S_{u_i u_j}(\omega) \int_{-\infty}^{+\infty} G_i(x, t, \theta_i) e^{i\omega\theta_i} d\theta_i \int_{-\infty}^{+\infty} G_j(x, t, \theta_j) e^{-i\omega\theta_j} d\theta_j
\end{aligned} \quad (7)$$

where $R_{ij}(\tau)$ is the cross-correlation function of the displacements of the i -th and j -th supports. The frequency response function, $H_i(x, \omega)$, corresponding to the i th support, can be written as

$$H_i = \int_{-\infty}^{+\infty} G_i(x, t, \theta_i) e^{-i\omega\theta_i} d\theta_i = e^{-i\omega t} \int_{-\infty}^{+\infty} G_i(x, t, \theta_i) e^{-i\omega(\theta_i - t)} d\theta_i \quad (8)$$

Equations (7) and (8) may be combined to yield the PSD of $z(x, t)$ in the form

$$S_{zz}(x, \omega) = [H_1^* \ H_2^* \ \cdots \ H_N^*] \begin{bmatrix} S_{u_1 u_1} & S_{u_1 u_2} & \cdots & S_{u_1 u_N} \\ S_{u_2 u_1} & S_{u_2 u_2} & \cdots & S_{u_2 u_N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{u_N u_1} & S_{u_N u_2} & \cdots & S_{u_N u_N} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_N \end{bmatrix} \quad (9)$$

in which $*$ denotes complex conjugate. According to equation (5), equation (9) may be rewritten as

$$S_{zz}(x, \omega) = [\mathbf{H}(x, \omega)]^* \mathbf{S}_{uu}(\omega) [\mathbf{H}(x, \omega)]^T \quad (10)$$

where $\mathbf{H}(x, \omega) = [H_1 \ H_2 \ \cdots \ H_N]$ is a $1 \times N$ frequency response function matrix. After integrating $S_{zz}(x, \omega)$ within the frequency domain, the mean square value $\sigma_z^2(x)$ can be obtained as

$$\sigma_z^2(x) = 2 \int_0^{+\infty} S_{zz}(x, \omega) d\omega \quad (11)$$

To summarize the present subsection, it has been shown that the stationary random response analysis of multi-span beams may reduce to the solution of the deterministic frequency response function matrix $\mathbf{H}(x, \omega)$.

3.2. Frequency response function of multi-span beams

The middle supports of the multi-span beam can be replaced by the unknown reaction forces $p_i(t)$, as shown in figure 2. Then, the original problem is converted to the forced vibration analysis of a single span beam subjected to the external forces. The equation of motion may be written as

$$m \frac{\partial^2 y(x, t)}{\partial t^2} + c \frac{\partial y(x, t)}{\partial t} + EI \frac{\partial^4 y(x, t)}{\partial x^4} = \sum_{i=2}^{N-2} p_i(t) \delta(x - x_i) \quad (12)$$

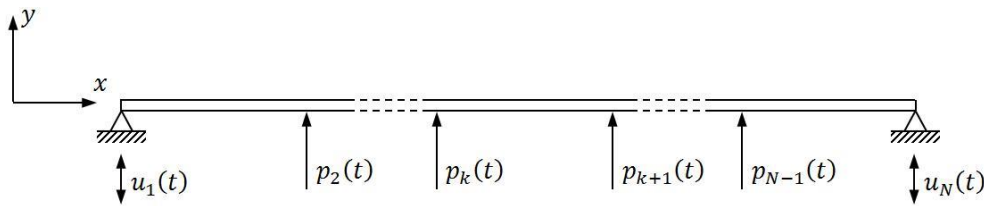


Figure 2. Schematic of a single span beam subjected to the external forces.

in which $\delta(\cdot)$ is the Kronecker delta function. Assuming that the external forces and motions of the extreme supports are harmonic, i.e., $p_i(t) = P_i e^{i\omega t}$, $u_1(t) = U_1 e^{i\omega t}$, $u_N(t) = U_N e^{i\omega t}$, the solution to the equation of motion can be expressed in the form

$$y(x, t) = Y(x) e^{i\omega t} \quad (13)$$

where $Y(x)$ is the displacements amplitude along the beam.

Substituting equation (13) into equation (12) and eliminating the terms related to time t , results in an ordinary differential equation as

$$EI \frac{d^4 Y(x)}{dx^4} - (m\omega^2 - ic\omega) Y(x) = \sum_{i=2}^{N-2} P_i \delta(x - x_i) \quad (14)$$

By performing a Laplace transformation, equation (14) may be written as

$$\begin{aligned} (s^4 - k^4) \mathcal{L}[Y(x)] &= s^3 Y(0) + s^2 Y'(0) + s Y''(0) + Y'''(0) \\ &+ \sum_{i=2}^{N-2} \frac{P_i}{EI} [\cosh(sx_i) - \sinh(sx_i)] \end{aligned} \quad (15)$$

where $k^4 = (m\omega^2 - ic\omega)/EI$, $\mathcal{L}[\cdot]$ denotes the Laplace transformation. After inverting equation (15), the general solutions to equation (14) can be written as

$$\begin{aligned} Y(x) &= A \sinh(kx) + B \sin(kx) + C \cosh(kx) + D \cos(kx) \\ &+ \sum_{i=2}^{N-2} \frac{P_i}{2k^3 EI} \{ \sinh[k(x - x_i)] - \sin[k(x - x_i)] \} h(x - x_i) \end{aligned} \quad (16)$$

in which $h(x - x_i)$ is the *Heaviside function*, A , B , C , D and P_i are undetermined coefficients. Equation (16) can be written in the vector form

$$Y(x) = \mathbf{Q}(x) \mathbf{V} \quad (17)$$

where $\mathbf{Q}(x)$ is a $1 \times (2 + N)$ row vector related to x ; $\mathbf{V} = \{A \ B \ C \ D \ P_2 \ \dots \ P_{N-1}\}^T$ is a $(2 + N) \times 1$ column vector of the undetermined coefficients, which can be determined by satisfying the boundary conditions at the supports. For the sake of clarity, brief derivations are provided only for the case of simply supported at both extreme supports. The corresponding boundary conditions are

$$\frac{d^2 Y(0)}{dx^2} = 0, \quad \frac{d^2 Y(x_N)}{dx^2} = 0, \quad Y(x_i) = U_i \quad (18)$$

Combining equation (17) with the conditions (18) yields

$$\mathbf{T} \mathbf{V} = \mathbf{W} \quad (19)$$

in which \mathbf{T} is a $(2 + N) \times (2 + N)$ constant matrix related to 2 bending moment and N displacement boundary conditions, $\mathbf{W} = \{0 \ 0 \ U_1 \ \dots \ U_N\}^T$ is a $(2 + N) \times 1$ vector. By using equation (19), equation (17) can also be written in the form

$$Y(x) = Q(x)T^{-1}W \quad (20)$$

Setting the displacement of the i th support equal to 1, and those of the other supports equal to 0, let W_i be the displacement vector of the supports in this case. Substituting it into equation (18), the frequency response function corresponding to the i -th support can be written as

$$H_i(x, \omega) = Q(x)T^{-1}W_i \quad (21)$$

The frequency response functions corresponding to the other supports can be obtained by repeating the above procedure for the remaining supports. Thereafter, rearranging those functions as the form of N -dimensional row vector, the frequency response functions matrix may be expressed as

$$H(x, \omega) = Q(x)T^{-1}Y \quad (22)$$

where $Y = [W_1 \ W_2 \ \cdots \ W_N] = \begin{bmatrix} \mathbf{0}_{2 \times N} \\ \mathbf{I}_{N \times N} \end{bmatrix}$ is a $(N+2) \times N$ matrix indicating the displacements of the supports, in which $\mathbf{0}_{2 \times N}$ is a $2 \times N$ zero matrix, $\mathbf{I}_{N \times N}$ is a $N \times N$ identity matrix.

The relationship between the arbitrary response and displacement PSD of the supports is derived in this section, following which the stationary random response analysis of the multi-span beam subjected to the spatially varying ground motions reduces to the solution of the deterministic frequency response function matrix. Finally, an analytical expression is introduced to obtain the frequency response function. It should be cautioned that most of the expressions during the above derivation are explicit, which provides great convenience for the subsequent solution of the sensitivity.

4. Influences of random parameters on random responses

The derivation that follows will be specific to the influences of the randomness of structural parameters, which are not included in the previous section. The sensitivities of the random responses with respect to random structural parameters can be evaluated by using the above explicit expressions. Subsequently, the mean values and variances of random responses are obtained from the second order Taylor expansion of center $\bar{\epsilon}$.

4.1. Sensitivity of random responses

The first and second order sensitivities of the displacement PSD $S_{yy}(x, \omega)$ corresponding to the i -th structural parameter ϵ_i are expressed, by the definition given in [17], as the first and second partial derivatives of $S_{YY}(x, \omega)$ with respect to ϵ_i , i.e.

$$\frac{\partial S_{YY}(x, \omega)}{\partial \epsilon_i} = \left(\frac{\partial H^*}{\partial \epsilon_i} S_{uu} H^T + H^* S_{uu} \frac{\partial H^T}{\partial \epsilon_i} \right) |_{\epsilon=\bar{\epsilon}} \quad (23a)$$

$$\frac{\partial^2 S_{YY}(x, \omega)}{\partial \epsilon_i^2} = \left(\frac{\partial^2 H^*}{\partial \epsilon_i^2} S_{uu} H^T + 2 \frac{\partial H^*}{\partial \epsilon_i} S_{uu} \frac{\partial H^T}{\partial \epsilon_i} + H^* S_{uu} \frac{\partial^2 H^T}{\partial \epsilon_i^2} \right) |_{\epsilon=\bar{\epsilon}} \quad (23b)$$

where $\bar{\epsilon}$ is the mean value of ϵ , $\frac{\partial H}{\partial \epsilon_i}$ and $\frac{\partial^2 H}{\partial \epsilon_i^2}$ are the first and second order sensitivities of the frequency response function matrix H , respectively, and satisfy $\frac{\partial H^*}{\partial \epsilon_i} = \left(\frac{\partial H}{\partial \epsilon_i} \right)^*$, $\frac{\partial H^T}{\partial \epsilon_i} = \left(\frac{\partial H}{\partial \epsilon_i} \right)^T$, $\frac{\partial^2 H^*}{\partial \epsilon_i^2} = \left(\frac{\partial^2 H}{\partial \epsilon_i^2} \right)^*$, $\frac{\partial^2 H^T}{\partial \epsilon_i^2} = \left(\frac{\partial^2 H}{\partial \epsilon_i^2} \right)^T$. By differentiating equation (22) with respect to ϵ_i , it can be expressed as

$$\frac{\partial H}{\partial \epsilon_i} = \frac{\partial Q}{\partial \epsilon_i} T^{-1} Y - Q T^{-1} \frac{\partial T}{\partial \epsilon_i} T^{-1} Y \quad (24a)$$

$$\frac{\partial^2 H}{\partial \epsilon_i^2} = \frac{\partial^2 Q}{\partial \epsilon_i^2} T^{-1} Y - 2 \frac{\partial Q}{\partial \epsilon_i} T^{-1} \frac{\partial T}{\partial \epsilon_i} T^{-1} Y + Q T^{-1} \left(2 \frac{\partial T}{\partial \epsilon_i} T^{-1} \frac{\partial T}{\partial \epsilon_i} - \frac{\partial^2 T}{\partial \epsilon_i^2} \right) T^{-1} Y \quad (24b)$$

Since the expressions of Q and T are already known, the partial derivative terms in equation (24) can be determined easily.

It is not hard to prove that the mean square value and PSD of the random response have the same integral relationship.

4.2. Statistical moments of random responses

Taylor expanding the PSD of random response $S_{zz}(x, \omega)$ with respect to the random parameters vector ϵ up to the second order and ignoring the cross terms, it can be written as

$$S_{zz}(x, \omega) \cong \left\{ S_{zz} + \sum_{i=1}^r \frac{\partial S_{zz}}{\partial \epsilon_i} (\epsilon_i - \bar{\epsilon}_i) + \frac{1}{2} \sum_{i=1}^r \frac{\partial^2 S_{zz}}{\partial \epsilon_i^2} (\epsilon_i - \bar{\epsilon}_i)^2 \right\}_{\epsilon=\bar{\epsilon}} \quad (25)$$

where $\bar{\epsilon}_i$ is the mean value of ϵ_i . On the basis of equation (25), the mean value and variance of $S_{zz}(x, \omega)$ may be expressed as

$$E[S_{zz}(x, \omega)] \cong \left\{ S_{zz} + \frac{1}{2} \sum_{i=1}^r \frac{\partial^2 S_{zz}}{\partial \epsilon_i^2} \text{Cov}[\epsilon_i, \epsilon_i] \right\}_{\epsilon=\bar{\epsilon}} \quad (26a)$$

$$\begin{aligned} \text{Var}[S_{zz}(x, \omega)] \cong & \left\{ \sum_{i=1}^r \left(\frac{\partial S_{zz}}{\partial \epsilon_i} \right)^2 \text{Cov}[\epsilon_i, \epsilon_i] \right. \\ & \left. + \frac{1}{4} \sum_{i=1}^r \sum_{j=1}^r \frac{\partial^2 S_{zz}}{\partial \epsilon_i^2} \frac{\partial^2 S_{zz}}{\partial \epsilon_j^2} \text{Cov}[\epsilon_i, \epsilon_i] \text{Cov}[\epsilon_j, \epsilon_j] \right\}_{\epsilon=\bar{\epsilon}} \end{aligned} \quad (26b)$$

in which $\text{Var}[\]$ and $\text{Cov}[\]$ are the variance and covariance operators, respectively.

The mean value and variance of the mean square value $\sigma_z^2(x)$ can be similarly derived, the details will not be repeated here for simplicity.

5. Numerical examples

A simply supported four-span beam is adopted as an example. The lengths of the spans are 40, 50, 50 and 40m, respectively, and the moment of inertia of the cross-section is $I = 8.33 \times 10^{-2} \text{m}^4$. Young's modulus, the mass of unit length and damping per unit length are taken to be independent random variables, whose mean values are $E_0 = 2 \times 10^{11} \text{MPa}$, $m_0 = 7800 \text{kg/m}^2$, $c_0 = 1000 \text{N} \cdot \text{s/m}^2$, respectively. The dimensionless fluctuations are denoted as $\epsilon_E, \epsilon_m, \epsilon_c$.

The acceleration PSD used in this study was developed by Kanai and Tajimi, and further extended by Clough and Penzien [18]. The expression is

$$S_{\ddot{u}\ddot{u}}(\omega) = S_0 \frac{1+4\xi_{g1}^2(\omega/\omega_{g1})^2}{(1-(\omega/\omega_{g1})^2)^2 + 4\xi_{g1}^2(\omega/\omega_{g1})^2} \times \frac{(\omega/\omega_{g2})^4}{(1-(\omega/\omega_{g2})^2)^2 + 4\xi_{g1}^2(\omega/\omega_{g2})^2} \quad (27)$$

in which, $S_0 = 0.25 \text{m}^2 \text{s}^{-3} \text{rad}^{-1}$ is the amplitude of the white-noise bedrock acceleration, ω_{g1} is the resonant angular frequency of the first filter, which can be applied to characterize the site-response effect. In this study, the values of ω_{g1} for the supports are 15, 10, 5, 10 and 15 rad/s, respectively. ω_{g2} is the resonant frequency of the second filter, and satisfies $\omega_{g2} = 0.1\omega_{g1}$. $\xi_{g1} = \xi_{g2} = 0.6$ are the corresponding damping ratios. Since the ground motion is a stationary random process, PSDs of the displacement and acceleration yield

$$S_{uu}(\omega) = \frac{1}{\omega^4} S_{\ddot{u}\ddot{u}}(\omega) \quad (28)$$

Thus the diagonal terms of PSD matrix \mathbf{S}_{uu} in equation (3) can be obtained from equation (27) and (28), while the cross terms will be determined by using the relationship between the cross-PSD and auto-PSD.

The *apparent velocity* of seismic wave is $v_s = 1000$ m/s. The model developed by Loh and Yeh [19] is adopted to account for the incoherence effect, i.e.

$$\gamma_{ij}^{(i)}(\omega) = \exp\left[-\alpha \frac{\omega d_{ij}}{2\pi v_s}\right] \quad (29)$$

where d_{ij} is the x distance between the i th and j th supports, the constant α used is 0.125. The effective frequency region is taken as $\omega \in [0, 15]$ Hz and the frequency step-size as $\Delta\omega = 0.01$ Hz.

5.1. Validation of the present method

A Monte Carlo simulation is performed to assess the accuracy of the propose method. It is assumed that the standard deviations of the dimensionless fluctuations σ_{ϵ_E} , σ_{ϵ_m} and σ_{ϵ_c} are equal to 0.05. The number of samples for each random parameter is 50, which is considered sufficient for an accurate estimation of the mean value and the standard deviation; as a result, the total number of samples is $50^3 = 125000$. The mean values and standard deviations of the PSDs and mean square values are computed. As the PSDs of responses change with both the angular frequency and location, only the displacement PSD at the midpoint of the second span and the bending moment PSD at the third support location are shown in figures for convenience. Comparisons of the mean values and standard deviations of PSDs are presented in figure 3 and 4, where S_{YY} and S_{MM} are the PSDs of displacement and bending moment, respectively. Figure 5 and 6 show the comparisons of the mean values and standard deviations of σ_Y^2 and σ_M^2 , which are mean square values of the displacement and bending moment, respectively. It can be seen that the results of the present method and Monte Carlo simulation match very well, and the overall error is less than 10%. The results presented in figure 3 to 6 verify the accuracy of the present method numerically. Meanwhile, the computation time for the present method is much less than that for the Monte Carlo simulation.

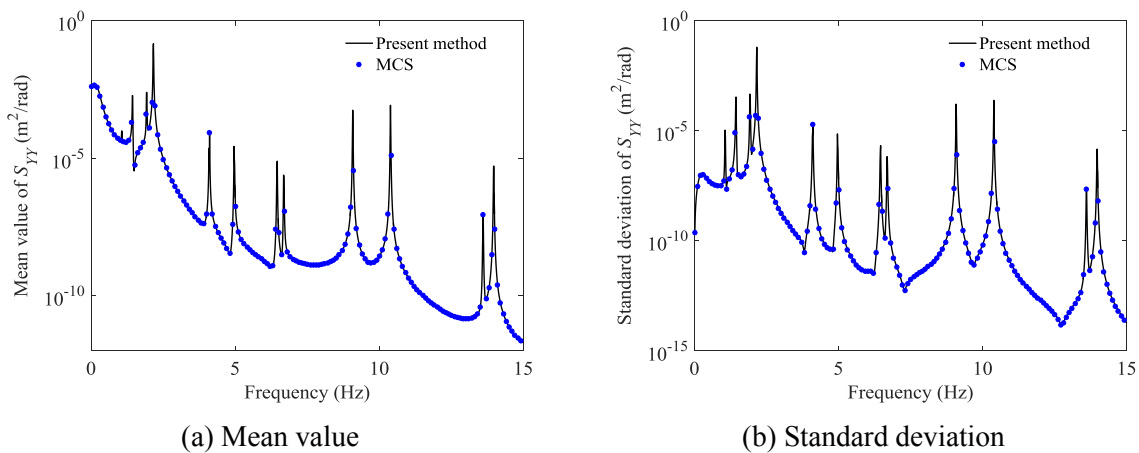


Figure 3. The mean value and standard deviation of S_{YY}

5.2. Influences of random parameters

Four representative cases with different random parameters are presented to investigate the influences of the randomness in parameters on the random responses. For Case 1 to 3, Young's modulus, the mass and damping of per unit length are selected to be random variables successively, while Case 4 is a reference case without any random variables. The details of the cases are shown in table 1. The mean value and standard deviation of the mean square of the displacement for the cases are plotted in figure 7, and the procedure is repeated for the bending moment in figure 8. As shown in figure 7, the results for Case 1 and 2 agree quite well, while the results for Case 3 and 4 match almost exactly. The same trend is observed in the results shown in figure 8. Since all structural parameters are deterministic, the standard deviations of random responses are equal to zero for Case 4. These results indicate that the

influences of randomness in Young's modulus are just as important as that of the mass per unit length. On the contrary, the influence of the randomness in the damping is negligible, which can be assumed to be a deterministic variable.

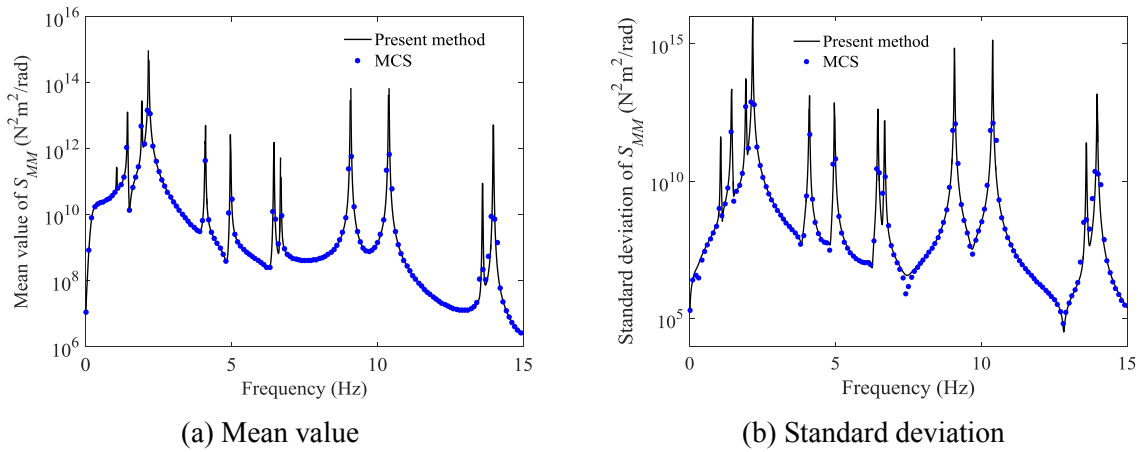


Figure 4. The mean value and standard deviation of S_{MM}

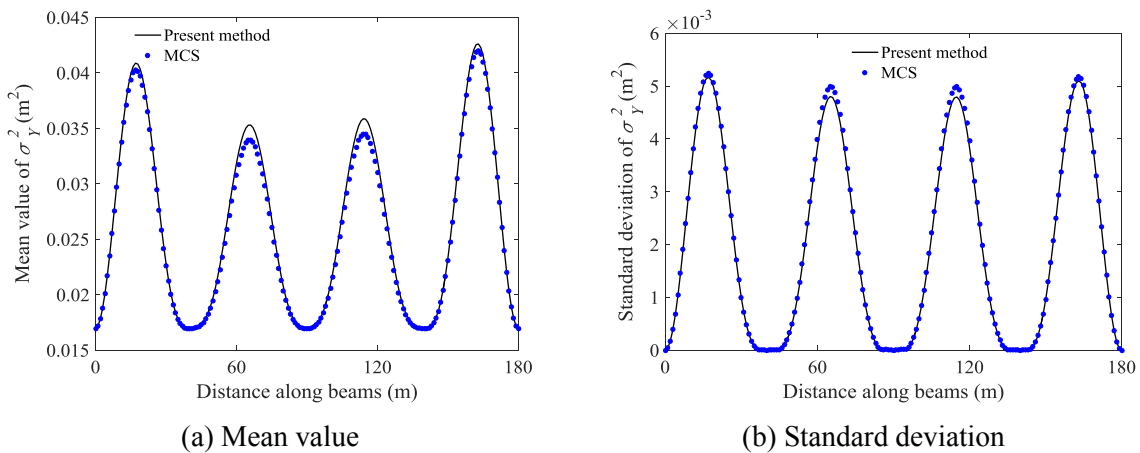


Figure 5. The mean value and standard deviation of σ_Y^2

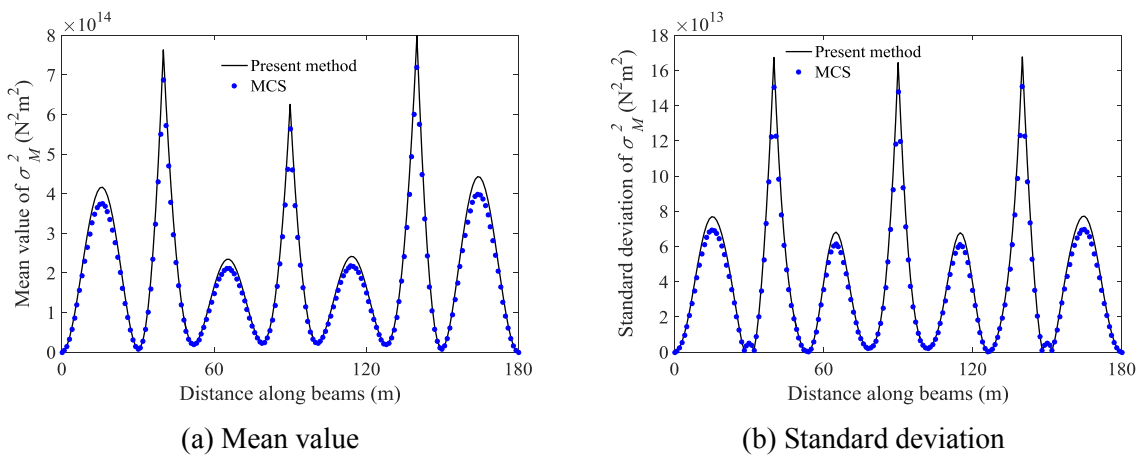
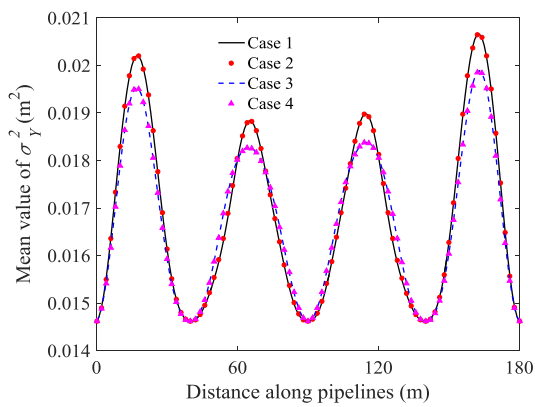


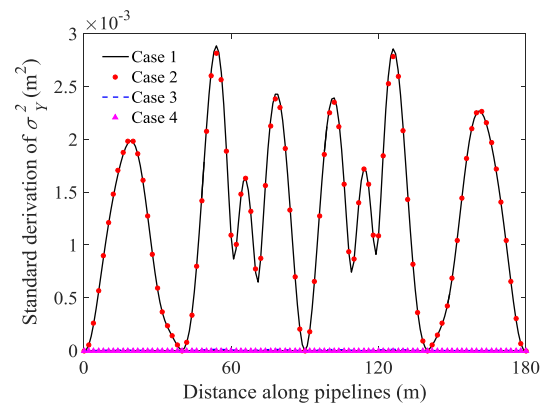
Figure 6. The mean value and standard deviation of σ_M^2

Table 1. Details of random parameters.

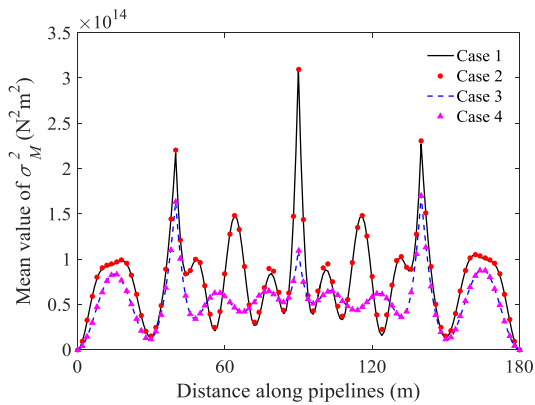
	σ_{ϵ_E}	σ_{ϵ_m}	σ_{ϵ_c}
Case 1	0.05	0	0
Case 2	0	0.05	0
Case 3	0	0	0.05
Case 4	0	0	0



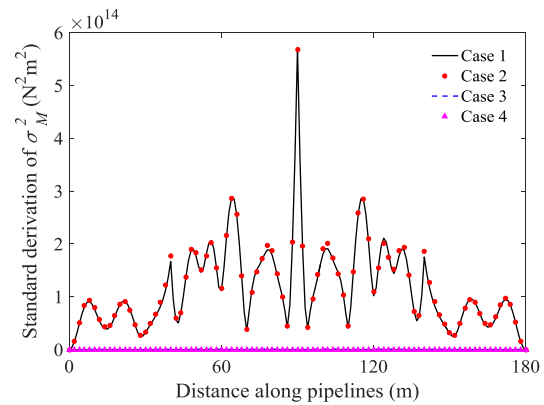
(a) Mean value



(b) Standard deviation

Figure 7. The mean value and standard deviation of σ_Y^2 

(a) Mean value



(b) Standard deviation

Figure 8. The mean value and standard deviation of σ_M^2

5.3. Sensitivity of random responses

The first and second sensitivities of the random responses to the random parameters ϵ_E , ϵ_m and ϵ_c are studied here. Figure 9 and 10 show the sensitivity results of σ_Y^2 and σ_M^2 , respectively. As shown in the figures, the first order sensitivities with respect to ϵ_E and ϵ_m are symmetrical about the horizontal axis, and the corresponding second order sensitivities match almost exactly. At most part of the beam, the first order sensitivities corresponding to Young's modulus are positive, while the ones corresponding to the mass are negative. Meanwhile, the second order sensitivities corresponding to both Young's modulus and the mass are negative. Moreover, both the first and second sensitivities with respect to ϵ_c are close to zero. This is because the ground motion is a kind of low frequency excitation, and the influence of the damping on the response is relatively small. Due to the reason that the motions of the

supports are equal to the ground motion, which is independent of the structural parameters, an interesting observation in figure 9 is that both the first and second sensitivities of σ_Y^2 are close to zero at the supports, i.e. $x = 0, 40, 90, 140$ and 180m .

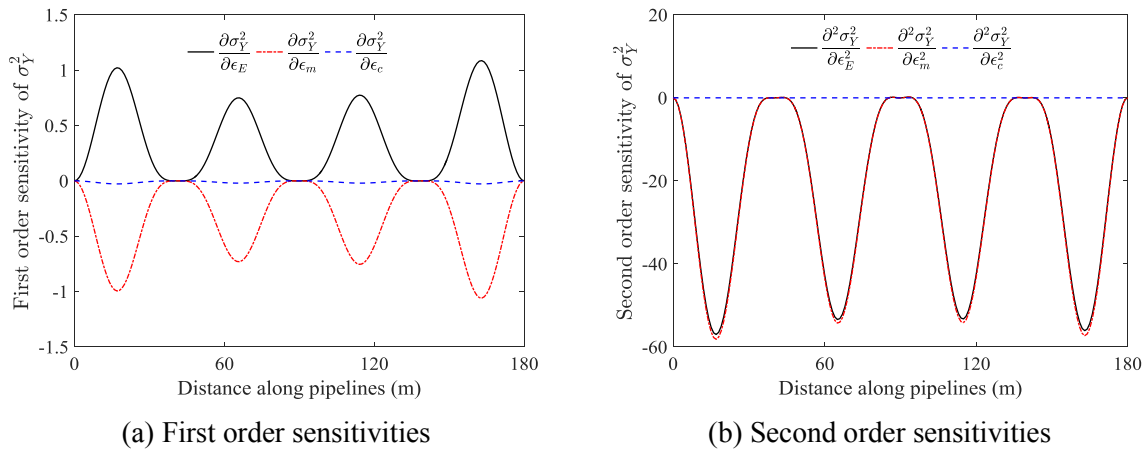


Figure 9. The sensitivities of σ_Y^2 to the random parameters

6. Conclusions

The seismic analysis of multi-span continuous beams with random structural parameters subjected to spatially varying ground motions is investigated in this paper. The explicit form of the random response is obtained by using an analytical method. Based on the Taylor expansion and sensitivity theory, analytical expressions of mean and standard deviations of the random response are presented. Excellent agreement between the results of the proposed method and the ones obtained from Monte Carlo simulations is observed. Furthermore, the influences of the randomness in Young's modulus, mass per unit length and damping on the random response are evaluated. The results indicate that the influence of randomness in Young's modulus and the mass are remarkable and approximately equal, while the one of the damping is negligible.

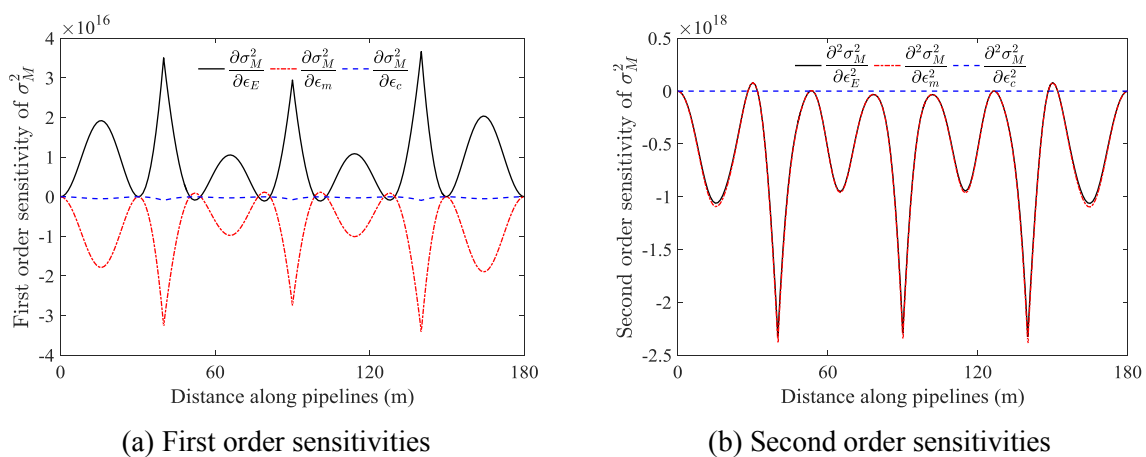


Figure 10. The sensitivities of σ_M^2 to the random parameters

Acknowledgments

The authors are grateful for support under grants from the National Science Foundation of China (11172056) and from the National Basic Research Program of China (2014CB046803).

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