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Inverse characterisation of frequency-dependent properties of adhesives

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Abstract. Traditional damping treatments are usually applied to the vibrating structure by means of adhesive layers. Environmental parameters, such as frequencies of excitation, may influence the behaviour of the bonding layer and modify the damping efficiency of the treatment. Therefore it is desired to take into account the viscoelastic behaviour of the adhesive layer in the finite element model.

The goal of this work is to present a procedure to characterise and model the adhesive layer. To that purpose, an experimental-numerical method for inverse characterisation of the frequency dependent properties of the adhesive layer is applied. The proposed inverse approach is based on a four-parameter fractional derivative model whose parameters are identified by minimising the difference between the simulated and the measured dynamic response of a multi-layered structure assembled by bonding. In the finite element model used for the optimisation, the adhesive layer is modelled by interface finite elements.

The influence of the adhesive layer on the efficiency of a damping treatment is evidenced by performing dynamic testing on a sandwich structure with a viscoelastic core, assembled by bonding. The proposed approach is applied to the characterisation of a pressure-sensitive adhesive.

1. Introduction

Noise and vibration control is a major concern in several industries and a lot of work has been dedicated to the design of efficient active or passive damping treatments. Such treatments are usually applied to the vibrating structure by means of an adhesive layer. Being generally made of polymers, adhesive layers may have their properties influenced by a number of environmental parameters, such as temperature or frequency. For instance, Figure 1 evidences the viscoelastic behaviour of a double coated tape used for the assembly of sandwich structures, while the epoxy adhesive has little influence on the dynamic behaviour of the assembled structure. A consequence of the viscoelastic behaviour of the adhesive layer is that it may modify the dynamics of the structure and affect the damping efficiency of the active or passive treatment applied (see Figure 2 and [1]).

In some cases, the adhesive layer must be modelled to have a predictive model of the treated structure [2]. Therefore, there is a need for a characterisation procedure to identify to frequency-dependent properties of the adhesive. Several characterisation techniques exist in the literature [3] to retrieve the viscoelastic properties of a material. Among them, the Dynamical Mechanical Analysis (DMA) is one of the most commonly used, and has already been applied to the

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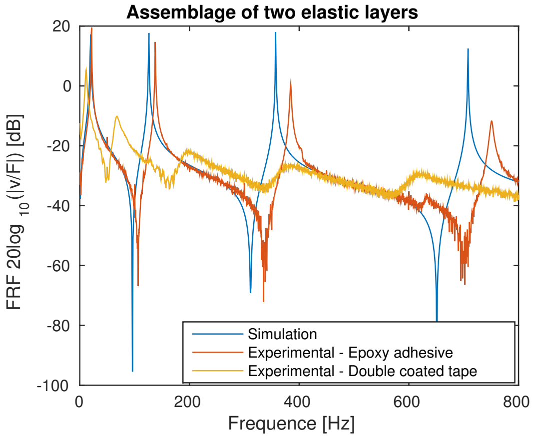


Figure 1: Measured frequency response functions of an assemblage of two 0.26m ×0.026m ×1mm steel beams realised by application of an epoxy adhesive (red) or a double coated tape (orange), compared to the simulated response computed by a finite element model neglecting the adhesive layer (blue).

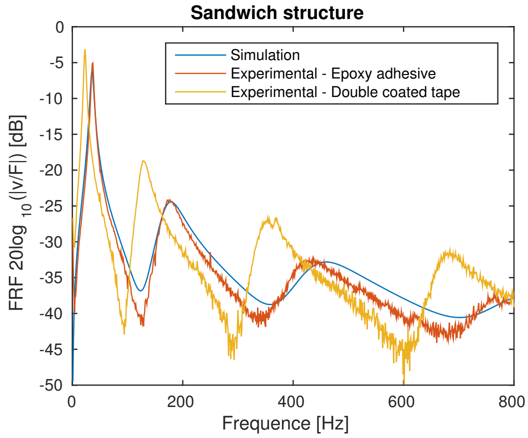


Figure 2: Measured frequency response functions of a sandwich beam with a viscoelastic core assembled by an epoxy adhesive (red) or a double coated tape (orange), compared with the simulated response computed by a finite element model neglecting the adhesive layer (blue).

characterisation of adhesives [4]. However, the bonding process generally has an important influence on the mechanical behaviour of the bonding layer, which makes inverse characterisation a more appropriate way of identifying the dynamic properties of the adhesive.

The goal of this work is to present a methodology to characterise and model the adhesive layer. To that purpose, an experimental-numerical method for inverse characterisation of the frequency dependent properties of the adhesive layer is applied. The proposed inverse approach is based on a fractional derivative model whose parameters are identified by minimising the difference between the simulated and the measured dynamic response of a multi-layered structure assembled by bonding. The fractional derivative model presents the advantage of describing accurately the viscoelastic behaviour of many polymers with only four parameters. In the finite element model used for the inverse method, the adhesive layer is modelled by interface finite elements, i.e. by bi-dimensional elements representative of the three-dimensional behaviour of the bonding layer.

The proposed characterisation and modelling procedure is applied to dynamic measurements of a structure assembled with a double coated tape.

2. Numerical model

The experimental-numerical method for inverse characterisation of the frequency dependent properties of the adhesive layer is based on a finite element model which is described in this section.

2.1. Viscoelastic model

Adhesives are generally made of polymers, which may exhibit a viscoelastic behaviour. The mechanical properties of viscoelastic materials depend strongly on several operational and environmental parameters, such as frequency and temperature. Many modelling approaches have been developed to account for the frequency- and temperature-dependency of viscoelastic properties [5]. In this work, a four-parameter fractional derivative model is used to describe the dependency of the complex shear modulus of the adhesive:

$$G^*(\omega) = \frac{G_0 + G_\infty (\mathrm{i}\omega\tau)^\alpha}{1 + (\mathrm{i}\omega\tau)^\alpha} \tag{1}$$

where G_0 and G_∞ are respectively the relaxed and unrelaxed moduli, τ is the relaxation time, and α is a fractional parameter comprised between 0 and 1 which corresponds to the non-integer order of derivation in the $\sigma(t) - \epsilon(t)$ relationships [6]. These four parameters must satisfy the following thermodynamical constraints:

$$G_\infty > G_0 > 0, \qquad \tau > 0 \quad \text{and} \quad 0 < \alpha < 1 \tag{2}$$

This viscoelastic model enables a good represention of the viscoelastic behaviour with only four parameters, which justifies its use in the proposed inverse method. It was already considered in [4] to characterise the frequency dependence of the elastic and dissipative properties of adhesives.

2.2. Modelling of the sandwich structure

Since the bonding layer is assumed to exhibit a viscoelastic behaviour, the assembled structure may be assimilated to a sandwich structure, the core being the adhesive layer. The modelling of sandwich structures is usually challenging chiefly because of the large transverse shear deformations undergone by the constrained viscoelastic layer. To overcome this difficulty, two main modelling approaches exist. The first one consists in modelling the whole structure by sandwich finite elements; the second one consists in modelling the viscoelastic layer by three-dimensional elements to represent properly its shear behaviour. In this work, quadratic hexahedra are used for the modelling of the elastic faces and interface finite elements, initially developped by the authors to model thin constrained viscoelastic layers [7], are used for the modelling of the bonding layer (see figure 3.a).

Interface finite elements are zero-thickness elements, defined by a mean surface and a fictive thickness, which are formulated as a three-dimensional finite element.From the coordinates of the nodes constituting the mean surface $\mathbf{X}_0^i$, the coordinates of the nodes forming the fictive volume $\mathbf{X}^i$ are calculating using the normal to the mean surface at each node $\mathbf{n}_i$:

$$\mathbf{X}^i = \mathbf{X}_0^i \pm \mathbf{n}_i \tag{3}$$

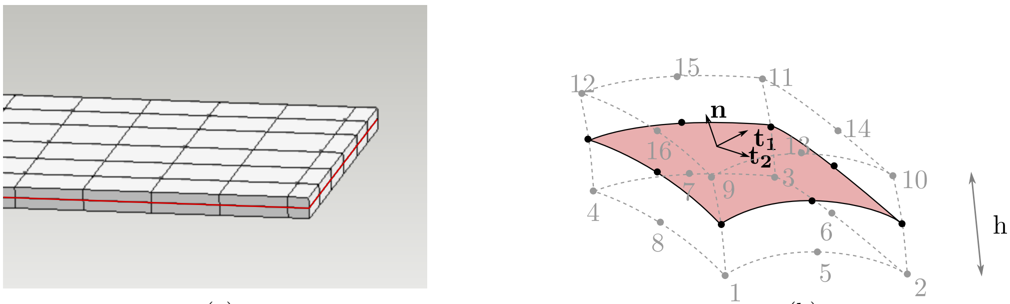
A three-dimensional behaviour law can then be implemented:

$$\mathbb{C}^* = G^*(\omega) \begin{bmatrix} \frac{2(1-\nu)}{1-2\nu} & \frac{2\nu}{1-2\nu} & \frac{2\nu}{1-2\nu} & 0 & 0 & 0 \\ \frac{2\nu}{1-2\nu} & \frac{2(1-\nu)}{1-2\nu} & \frac{2\nu}{1-2\nu} & 0 & 0 & 0 \\ \frac{2\nu}{1-2\nu} & \frac{2\nu}{1-2\nu} & \frac{2(1-\nu)}{1-2\nu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Here interface finite elements are formulated as 16-node prismatic elements (figure 3.b). This modelling approach provides a good representation of the shear behaviour in the adhesive layer and can be used in a second step to efficiently study the influence of the adhesive's thickness on the dynamic behaviour of a assembled structure.

The discretised equation of motion is of the form:

$$\left[\mathbf{K}_f + G^*(\omega)\mathbf{K}_c^0 - \omega^2\mathbf{M}\right]\mathbf{X} = \mathbf{F} \tag{5}$$



(a) (b)

Figure 3: Mesh of the assembled structure (a). Interface finite element (b): physical element (straight line) and fictive volume represented (dashed line).

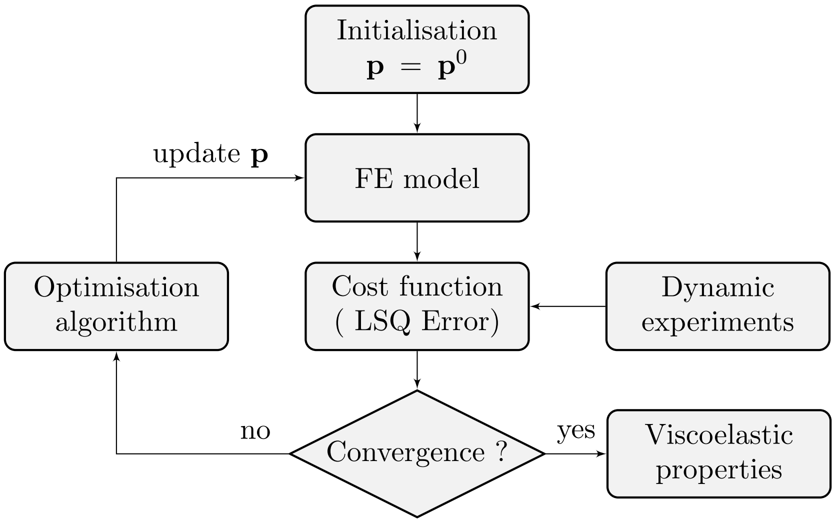


Figure 4: Flowchart of the optimisation procedure.

where $\mathbf{K}_f$ is the global stiffness matrix associated with the elastic faces, $\mathbf{K}_c$ is the global stiffness matrix associated with the core layer and evaluated for a unitary shear modulus, $\mathbf{M}$ is the global mass matrix, $G^*(\omega)$ is the complex modulus from Equation 1, and $\mathbf{X}$ and $\mathbf{F}$ are the displacement and force vectors respectively.

3. Inverse characterisation procedure

The goal of the inverse method is to identify the parameters of the viscoelastic model describing the frequency dependence of the material properties of the adhesive. This is achieved by applying a mixed numerical-experimental optimisation in which the design parameters $\mathbf{p}$ are the parameters of the viscoelastic model and the cost function represents the error between the measured structural response and the response simulated by the finite element model. Figure 4 provides a flowchart of the optimisation procedure.

At each step of the optimisation, the structural response is computed by the finite element model and compared to a measured response. The cost function is defined as the least square error

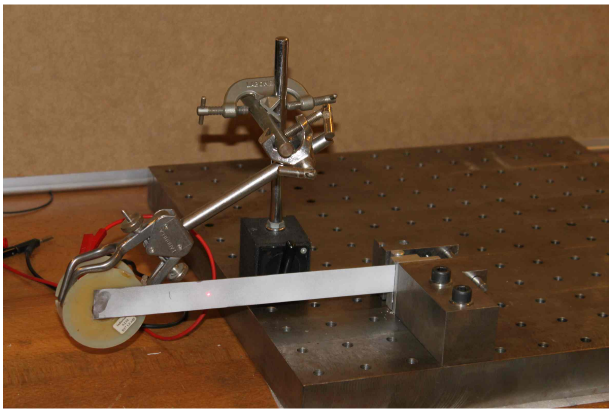


Figure 5: Photo of the experimental setup.

between the simulated and the measured frequency response at several points of the structure:

$$\epsilon(\mathbf{p}) = \sum_{j=1}^{N} \frac{\sum_{k=1}^{n_{\mathrm{freq}}} \left(T_{k,j}^{\mathrm{exp}} - T_{k,j}^{\mathrm{num}}(\mathbf{p})\right)^2}{\sum_{k=1}^{n_{\mathrm{freq}}} \left(T_{k,j}^{\mathrm{exp}}\right)^2} \tag{6}$$

where n_{freq} is the number of discrete frequencies at which the transfer function is computed, N is the number of points at which the frequency response is measured and computed, $T_{k,j}^{\mathrm{exp}}$ and $T_{k,j}^{\mathrm{num}}(\mathbf{p})$ are the measured and simulated transfer functions evaluated at a set of control frequencies ω_k and at a point j.
A gradient-based method (BFGS) is used to minimise the cost function and update the design parameters at each step of the optimisation. This optimisation method requires the calculation of the gradient of the cost function. In this work, the gradient is evaluated by a direct differentiation approach, described in [8].
The optimisation problem to be solved is not convex so the BFGS algorithm may converge towards a local minimum. Therefore, an initialisation step is introduced in the procedure. It consists in optimising the design parameters by minimising a cost function representing the difference between the measured and the simulated resonant frequencies of the structure. This approach has been used by several authors to minimise the risk of converging towards a local minima [9, 10].

4. Application and results

The previously described inverse identification method is applied to dynamic measurements of a steel assembly for the characterisation of a double coated tape (3M$^{\mathrm{TM}}$ 9040).
The assembled structure is composed of two strips made of steel, of dimensions 0.27 m $\times$ 0.026 m $\times$ 1 mm, and the double coated tape of thickness 100 μm. This structure was clamped at one end and excited at the opposite end by a coil/magnet system. The structural response was measured on a grid by a laser vibrometer (see Figure 5). The properties of the steel strips were determined by tensile tests and model updating: E = 170GPa for the Young modulus, $\nu = 0.3$ for the Poisson ratio, $\rho = 7450$ kg/m^3 for the density and 0.5% of structural damping. The density of the double coated tape was estimated to $\rho_v = 1100$ kg/m^3, and a Poisson ratio of $\nu_v = 0.3$ was considered in the finite element model.
The viscoelastic properties of the adhesive were identified from transfer functions measured on the frequency range $[0 - 800]$ Hz. Ten control frequencies are considered around the resonances

Parameters	G_0 [Pa]	G_∞ [Pa]	τ [s]	α
Initial parameters	10^6	10^8	10^{-8}	0.5
Optimised parameters	$2.54\ 10^4$	$1.51\ 10^8$	$1.32\ 10^{-6}$	0.88

Table 1: Initial and optimised design parameters of the viscoelastic model.

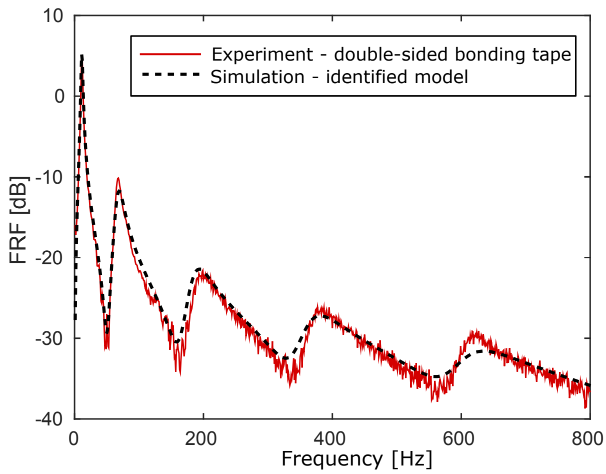


Figure 6: Frequency response of the assembled structure, measured by laser vibrometer (dashed line) and simulated by finite element using the optimised parameters of the viscoelastic model (straight line).

of the four excited modes. The initial and optimised parameters of the viscoelastic model are given in Table 1. Figure 6 shows the experimental frequency response of the structure at one measurement point, compared to the frequency response computed with the optimal parameters of the viscoelastic model. The properties of the double coated tape identified by the inverse procedure lead to a good representation of the dynamic behaviour of the assembled structure. The slight overestimation of the some modes (in particular the fifth excited mode) may be due to the fact that the four-parameter fractional derivative model may not be the most appropriate model to describe the frequency dependency of the adhesive's properties.

The master curves corresponding to the parameters of the viscoelastic model identified by the inverse procedure are plotted in Figure 7. The shear modulus of the adhesive is seen to vary significantly in the investigated frequency range, justifying the modelling of the bonding layer.

5. Conclusion

The inverse characterisation technique presented in this paper aims at determining the parameters of a fractional derivative model which describes the frequency-dependent mechanical properties of adhesives. The application of the method to a double coated tape showed that in some cases, the modelling of the viscoelastic properties of adhesives in the finite element model of the assembled structure is required to represent properly its dynamic behaviour. This approach can be used to improve the accuracy of a finite element model of a damped structure by taking into account the assembly process, and thus better predict the efficiency of a damping treatment.

In this work, a four-parameter fractional derivative model was considered as it generally describes

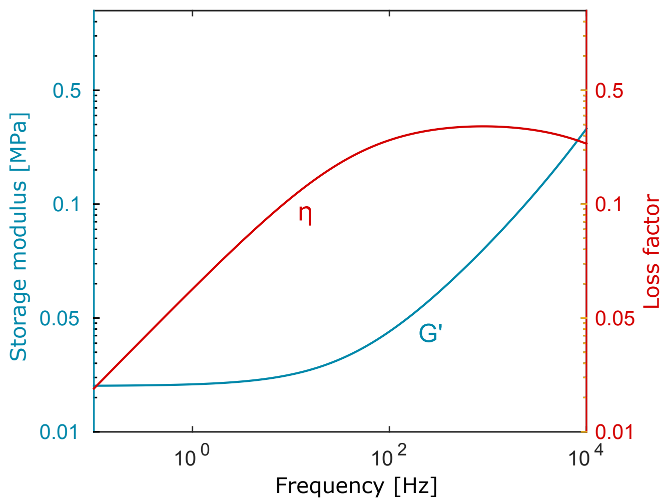


Figure 7: Master curves of the double coated tape using the parameters of the fractional derivative model identified by the inverse procedure.

accurately and with few parameters the frequency dependency of viscoelastic materials [12, 11, 4]. Nevertheless, other viscoelastic models such as generalised Maxwell model or GHM model should be investigated to better represent the viscoelastic behaviour of the adhesive. However, for these models, the number of parameters needed for a good description of the frequency dependence of the modulus is much higher and it raises difficulties in the optimisation [13].

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