# A numerical method for seeking the relationship between structural modes and acoustic radiation modes of complicated structures

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**Abstract**. Both structural modes and acoustic radiation modes play important roles in the investigation of structure-borne sound. However, little work has been done for inherent relations between these two kinds of modes. Previous work has mainly dealt with simple and regular structures such as rectangular plates and single-layer cylindrical shells. Therefore, the relationship between structural modes and acoustic radiation modes of complicated structures which has great theory significance and utility value is an important problem that must be studied. This paper presents a numerical method for seeking the relationship between structural modes and acoustic radiation modes of complicated structures. First, a governing equation for relating these two kinds of modes is given based on the characteristics of the modes. Then, substitute the normal structural mode shape matrix and the acoustic radiation mode shape matrix which are obtained by FEM into the governing equation, the modal participating coefficients can be solved, thus we can get the corresponding relations between these two kinds of modes. Using the model of a simply supported truncated conical shell, a numerical example is presented with the numerical method which this paper has proposed. And then, the radiated sound power is calculated to verify the effectiveness of this method and the correctness of this conclusion. The results show that the numerical method proposed in this paper is feasible.

## 1. Introduction

In recent years, the technique of Active Structural Acoustic Control (ASAC) as embodied in the research of Fuller and his co-workers [1], has become a research hotspot in the field of structural acoustics. This technique is to use controlling inputs applied directly to the structure to reduce or change the vibration distribution with the objective of reducing the overall sound radiation, putting forward more pressing requirement of the study on the relationship between structural vibration and acoustic radiation. For structural vibration problem, it can be put in the vibration modal space to study on. Vibration response (e.g. Displacement, velocity, etc) of the structure can be expressed as a linear superposition of each order structural mode. For structure-borne sound problem, the theory of acoustic radiation mode proposed by Borgiotti and Elliott [2-4] is normally used to analyze and control, for the acoustic radiation of each order structural mode is actually not independent and the coupling phenomenon exists. Thus, from the perspective of mode, study on the relationship of structural vibration and acoustic radiation comes down to seek corresponding relationship between structural mode and acoustic radiation mode.

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There is no doubt that, study on the corresponding relationship between the two modes is of great significance. For instance, in the research of active structural acoustic control with the theory of acoustic radiation mode, the dominant acoustic radiation mode can be exactly identified according to the corresponding relationship between structural mode and acoustic radiation mode. Thereby, control effect of broadband can be achieved, and any other effective control band can be freely selected with specific control objectives. Shuang Li and his co-workers [5] took simply supported rectangular plate as study object, derived the corresponding relationship between the two modes in terms of the symmetry or antisymmetry of mode shape and applied the relationship to ASAC. With same method, Weiping He et al. [6] derived the corresponding relationship between structural mode and acoustic radiation mode of a simply supported cylindrical shell. On the basis, Shaohu Ding and his partners [7] further analyzed the specific corresponding relationship from the features of acoustic radiation of cylinder shells and made study on the mechanism of active control.

However, the above study methods, which are on the basis of symmetry or antisymmetry of mode shapes, only applied to simple and regular structures. That is to say the shape of structural mode and acoustic radiation mode must be symmetric or antisymmetric. Moreover, Previous research objects only confined to some simple structures such as plates or single-layer cylindrical shells which can be dealt with analytic method, and the boundary conditions are also limited. While many modern engineering structures such as underwater vehicle shells and aircraft cabins were normally equivalent to truncated conical shells or coupled cylindrical-conical shells [8]. Because of complicated geometrical shapes and boundary conditions, it's difficult to solve with mathematical analysis method. Mode shapes are no more symmetric or antisymmetric. Thus, the corresponding relationship between structural mode and acoustic radiation mode of complicated structures is the problem needed to study on the next step, which is of great theoretical significance and practical value. This paper is on complicated structures and tries to put forward a numerical method for seeking corresponding relationships between modes combined with the theory of base vector in vector space and Finite Element Method (FEM). Firstly, from the concepts of structural mode and acoustic radiation mode, governing equation between the two modes would be derived according to the theory of base vector in vector space. And then numerical methods including Finite Element would be adopted to get normal structural mode shape matrix and acoustic radiation mode shape matrix of complicated structures in order to solve the modal participating coefficients and get corresponding relationship between the two modes. At last, taking truncated conical shell as an example, numerical example analysis with numerical method in this paper is to be presented. Furthermore, method and conclusion proposed in this paper would be tested and verified with radiated sound power of the structure.

## 2. Theory

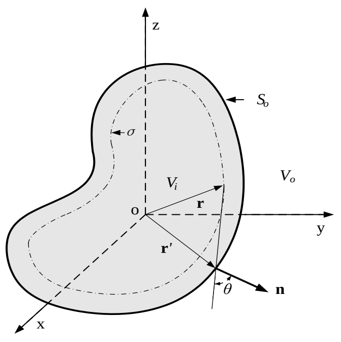


**Figure 1.** Schematic diagram of the vibrating structure

For any vibrating structure in the medium which velocity of sound is $c_0$ and average density is $\rho_0$, as is shown in figure 1. The structure is in situation of simple harmonic vibration with angular frequency $\omega$, and radiating sound energy to free field. The surface of the vibrating structure $S_o$ is uniformly discretized into a number of elements.

### 2.1. Structural mode

In terms of modal superposition principle of structural mode, vibration response of the structure can be expressed as a linear superposition of each order structural mode. Nodal velocity vector of finite elements $\boldsymbol{V}$ can be written as the following form

$$\boldsymbol{V} = \sum_{i=1}^{N} A_i \psi_i = \boldsymbol{\Psi A} \quad (1)$$

where $\boldsymbol{\Psi}$ is structural mode shape matrix, with the $i$th column $\psi_i$ representing the $i$th modal shape of the structure; $\boldsymbol{A}$ is the corresponding modal amplitude vector, which is determined by structure parameters and external excitation, its element $A_i$ represents the contribution of the $i$th structural mode to structural vibration response; $N$ is the number of elements on the surface of structure.

The structural normal velocity vector $\boldsymbol{V}_n$ and the nodal velocity vector of finite elements $\boldsymbol{V}$ are related with each other by

$$\boldsymbol{V}_n = \boldsymbol{GV} \quad (2)$$

$\boldsymbol{G}$ is transformation matrix, which can be calculated by nodal coordinates. Its function is to transfer nodal velocity vector of finite elements $\boldsymbol{V}$ to structural normal velocity vector $\boldsymbol{V}_n$. Substituting equation (1) into equation (2) gives

$$\boldsymbol{V}_n = \boldsymbol{G\Psi A} = \boldsymbol{\Phi A} \quad (3)$$

where $\boldsymbol{\Phi} = \boldsymbol{G\Psi}$ is normal structural mode shape matrix, with the $i$th column is denoted by $\phi_i$.

### 2.2. Acoustic radiation mode

Acoustic radiation mode is a kind of possible radiation forms on the surface of radiator and inherent nature of specific radiator. It is not determined by material features of radiator but the geometrical shape and vibrational frequency. The distribution of normal velocity on the surface of structure under any boundary conditions can usually be represented in terms of acoustic radiation modes as

$$\boldsymbol{V}_n = \sum_{k=1}^{N} y_k q_k = \boldsymbol{QY} \quad (4)$$

here $\boldsymbol{Q}$ is an orthogonal matrix which was got through eigen-decomposition of the structural radiation resistance matrix $\boldsymbol{R}$, defined as acoustic radiation mode shape matrix, with the $k$th column $q_k$ is then defined as the $k$th acoustic radiation mode; The structural radiation resistance matrix $\boldsymbol{R}$ can be calculated according to Ref. [4]. For convenience, we won't review it again here. For complicated structures, the problem is how to discretize the structural surface into a number of elements and acquire nodal coordinates. $\boldsymbol{Y}$ is modal amplitude vector corresponding to acoustic radiation mode, its element $y_k$ refers to the amplitude of the $k$th acoustic radiation mode.

According to the theoretical derivation introduced in Refs. [2-4], the sound power radiation from a vibrating structure $W$ can thus be written as

$$W = \sum_{k=1}^{N} \lambda_k |y_k|^2 \quad (5)$$

where $\lambda_k$ is the eigenvalue corresponding to the $k$th acoustic radiation mode. It is named as the coefficient of modal radiation efficiency, which is proportional to modal radiation efficiency.

### 2.3. Relationship between two modes

From the perspective of structural vibration and acoustic radiation, structural mode shows structural dynamic characteristic while acoustic radiation mode reflects acoustic radiation characteristic. There is a certain relationship between structural mode and acoustic radiation mode. From equations (3), (4) can be written as

$$\boldsymbol{V}_n = \boldsymbol{\Phi A} = \boldsymbol{QY} \quad (6)$$

It can be found that normal component of structural mode and acoustic radiation mode are both a group of base vectors of structural normal velocity, referred to as $\boldsymbol{\Phi}, \boldsymbol{Q}$ . Therefore, in terms of the theory of base vector in vector space, any base vector in $\boldsymbol{\Phi}$ can be expressed by linear combination of each order base vector in $\boldsymbol{Q}$ , that is

$$\phi_i = \sum_{k=1}^{N} c_k q_k = \boldsymbol{Q}\boldsymbol{C}_i \qquad (i = 1, \cdots, N) \tag{7}$$

where, $c_k$ is the participating coefficient of the $k$th acoustic radiation mode when the $i$th structural mode is expressed by linear combination of each order acoustic radiation modes; $\boldsymbol{C}_i$ is the column vector of modal participating coefficient.

Equation (7) is the governing equation between structural mode and acoustic radiation mode. Premultiply $\boldsymbol{Q}^{-1}$ (the inverse matrix of $\boldsymbol{Q}$ ) on both ends of equation, gives

$$\boldsymbol{C}_i = \boldsymbol{Q}^{-1}\phi_i \tag{8}$$

It is known that $\boldsymbol{Q}$ is an orthogonal matrix, here $\boldsymbol{Q}^{-1} = \boldsymbol{Q}^{T}$ .

## 3. Numerical method, example and verification

Through the above theoretical analysis, we can come to a conclusion: no matter how the structural shapes and boundary conditions change, whether mode shapes are symmetric or antisymmetric, the column vector of modal participating coefficient can be solved out through equation (8) as long as the normal structural mode shape matrix $\boldsymbol{\Phi}$ and the acoustic radiation mode shape matrix $\boldsymbol{Q}$ are known. Then, contrasting the participating coefficient $c_k$ of each order acoustic radiation mode, we can easily and visually get the dominant acoustic radiation mode which corresponding to the $i$th structural mode, and then we can sum up the corresponding relations between structural modes and acoustic radiation modes.

Therefore, for complicated structures whose corresponding relations between the two modes are difficult to seek by mathematical analysis method or by the method using symmetry/antisymmetry of modal shapes, a numerical method is proposed here. First, build the finite element model with the Finite Element Software, carry out modal analysis, then export the structural mode shape matrix $\boldsymbol{\Psi}$ , and then get the normal structural mode shape matrix $\boldsymbol{\Phi}$ by equation (3); Then, export nodal coordinates from Finite Element Software, calculate the structural radiation resistance matrix $\boldsymbol{R}$ according to Ref. [4], after that achieve acoustic radiation mode shape matrix $\boldsymbol{Q}$ through eigen-decomposition; Finally, substitute them into the governing equation (Equation (8)) to seek the corresponding relations between these two kinds of modes.

Next, use the model of truncated conical shell and carry out numerical analysis by the above numerical method. As shown in figure 2, the origin of coordinates is chosen in the circle center at the smaller end. The boundary condition is assumed to be simply supported at both ends, with material properties as steel. The surface of truncated conical shell is uniformly discretized into 480 elements (axial direction $\times$ circumferential direction = $16 \times 30$). Model data is given in table 1. The FE mesh and force impose on truncated conical shell is shown as figure 3.

**Table 1.** Model data

| Parameter | Value | Unit | Parameter | Value | Unit |
|---|---|---|---|---|---|
| $R_1$ | 0.3 | m | Poisson's ratio of structure | 0.3 | |
| $R_2$ | 0.5 | m | Density of structure | 7800 | kg/m$^3$ |
| $L$ | 1 | m | Speed of sound | 343 | m/s |
| Shell thickness | 0.002 | m | Exciting force | 10 | N |
| Young's modulus of structure | 210 | GPa | | | |

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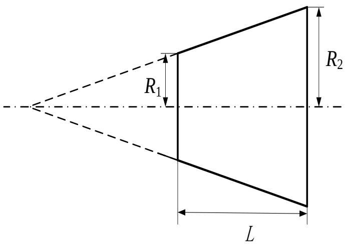



**Figure 2.** Model of truncated conical shell

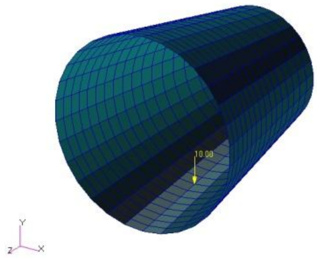


**Figure 3.** FE mesh and force impose on truncated conical shell

Derive nodal coordinates from Finite Element Software, calculate structural radiation resistance matrix $\boldsymbol{R}$, and achieve acoustic radiation mode shape matrix $\boldsymbol{Q}$ and the coefficient of modal radiation efficiency $\lambda_k$ through eigenvalue decomposition. Then, the radiation efficiencies of the first 10 acoustic radiation modes are shown in figure 4. It can be seen that one important feature of acoustic radiation mode is that the radiation efficiencies reduce rapidly with the increase of modal order at low frequency. This paper will choose the low frequency range ($10Hz \le f \le 300Hz$) which active control concerned to analyze. For more precision, we extend the scope of discussion to the first 20 acoustic radiation modes. For truncated conical shell, the shapes of radiation modes are different in each order. Figure 5 shows the first 6 acoustic radiation modes of the model at $f = 151Hz$.

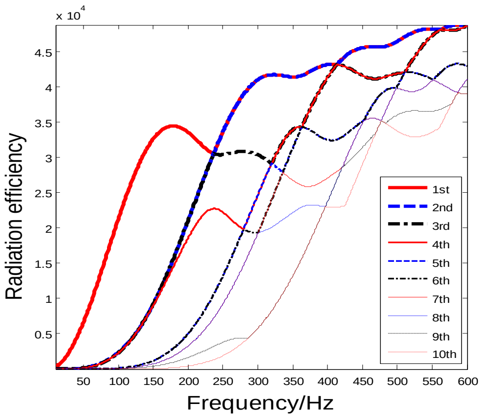
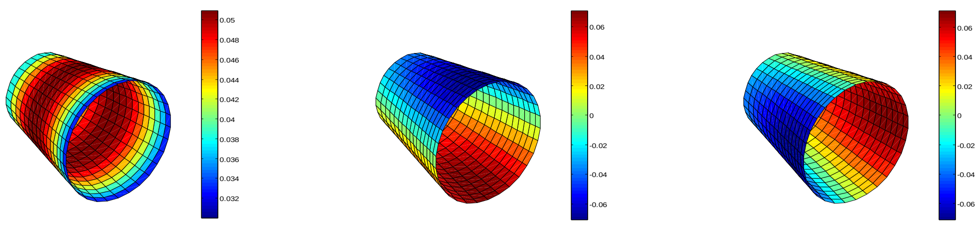
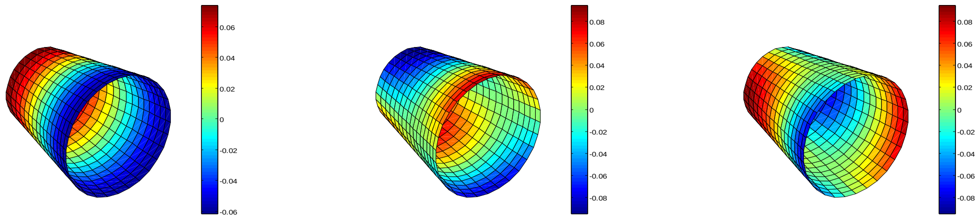



**Figure 4.** Radiation efficiencies of the first 10 acoustic radiation modes of the structure



( i ) ( ii ) (iii)



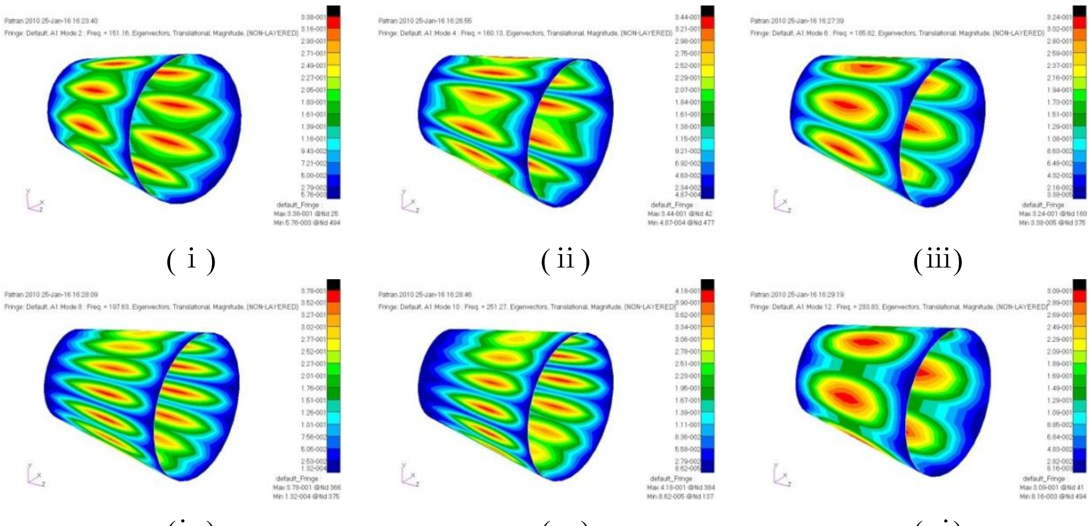
(iv) (v) (vi)

**Figure 5.** The first 6 acoustic radiation modes of the structure at $f = 151Hz$

Through modal analysis of finite element model, structural mode shape matrix $\boldsymbol{\Psi}$ can be got. The normal structural mode shape matrix $\boldsymbol{\Phi}$ can thus be obtained by transformation based on equation (3). This model has 6 structural modes below $300Hz$. The distribution of its characteristic frequency is shown in table 2. The first 6 structural modes are shown in figure 6.

**Table 2.** Characteristic frequency distribution of truncated conical shell (m and n correspond to axial mode number and circumferential mode number)

| Mode order $i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Mode index (m,n) | (1,5) | (1,6) | (1,4) | (1,7) | (1,8) | (1,3) |
| Frequency (Hz) | 151.16 | 160.13 | 185.82 | 197.63 | 251.27 | 283.83 |



(i) (ii) (iii)
(iv) (v) (vi)

**Figure 6.** The first 6 structural modes of the structure

Both figures 5 and 6 show that, compared with the simple and regular structures such as rectangular plates and single-layer cylindrical shells, the geometrical shape of truncated conical shell is more complicated. Its mode shapes are no more symmetric or antisymmetric, especially in its axial direction. Hence, previous method proposed in Refs. [5-7] is no longer applicable here.

Use the numerical method proposed in this paper, and then substitute the acoustic radiation mode shape matrix $\boldsymbol{Q}$ and the normal structural mode shape matrix $\boldsymbol{\Phi}$ into equation (8). The vector of modal participating coefficients $\boldsymbol{C}_i$ can be solved. Figure 7 shows the column vector of modal participating coefficients $\boldsymbol{C}_i\left(i=1,2,\cdots 6\right)$, while the first 6 structural modes are expressed as a linear superposition of acoustic radiation modes, respectively. It is seen that, in figure 7(a), the values of the 1st and 9th modal participating coefficients (i.e., $c_1$ , $c_9$ ) are significantly higher than the other orders. It indicates that when the 1st structural mode is expressed by linear combination of each order acoustic radiation modes, the 1st and 9th acoustic radiation modes are bigger contributors. Thus we can consider that the 1st and 9th acoustic radiation modes are the dominant acoustic radiation modes which corresponding to the 1st structural mode. Using the same analytic method, figures 7(b-f) indicate that: the 2nd , 3rd and 4th structural modes, different from the 1st structural mode, all correspond to the 1st , 4th and 9th acoustic radiation modes; the 5th structural mode corresponds to the 3rd and 4th acoustic radiation modes; the 6th structural mode corresponds to the 3rd, 6th and 9th acoustic radiation modes. The corresponding relations of higher order structural modes can be sought by this method as well. There is no need to enumerate all the discussions here. Furthermore, comparing these conclusions with previous works in Refs. [5-7], we can find that the relationships between structural modes and acoustic radiation modes of truncated conical shell do not meet those modal corresponding laws of simple structures any more.

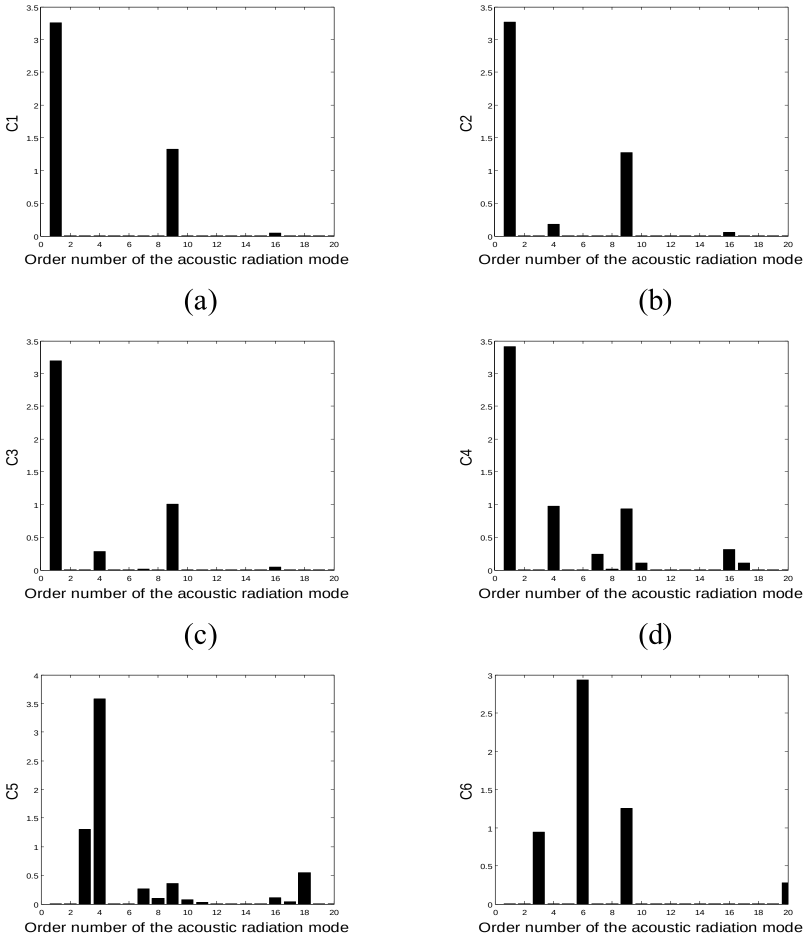



**Figure 7.** The corresponding relations between structural modes and acoustic radiation modes

Finally, the radiated sound power is calculated to verify the effectiveness of this method and the correctness of this conclusion. Use point force excitation with amplitude of $10N$ . The location is chose to ensure all major structural modes within the analysis frequency band will be motivated, as shown in Fig. 3. According to equation (5), figure 8 shows the total radiated sound power of the truncated conical shell. We can find that these peaks in the curve always occur at the characteristic frequencies of truncated conical shell. The radiated sound power from the 1st, 3rd, 4th, 6th, and 9th acoustic radiation modes are shown in figure 9. Comparing figure 8 with figure 9, we can find that, the total radiated sound power tends to be dominated by the 1st and the 9th acoustic radiation modes while the truncated conical shell vibrates mainly by the 1st structural mode at its natural frequency(i.e.,$151Hz$ ). Therefore, we can get in line with the above conclusion, the 1st and the 9th acoustic radiation modes are the dominant acoustic radiation modes which correspond to the 1st structural mode. Similar conclusions can be achieved at other natural frequencies. It indicates that the corresponding relations between these two kinds of modes of truncated conical shell we have gotten from last paragraph are correct, and then it demonstrates that the numerical method for seeking the relationship between structural modes and acoustic radiation modes of complicated structures proposed in this paper is feasible.

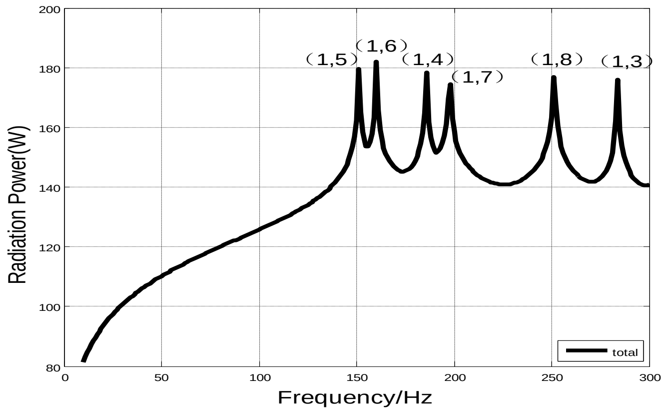



**Figure 8.** The total radiated sound power of the structure

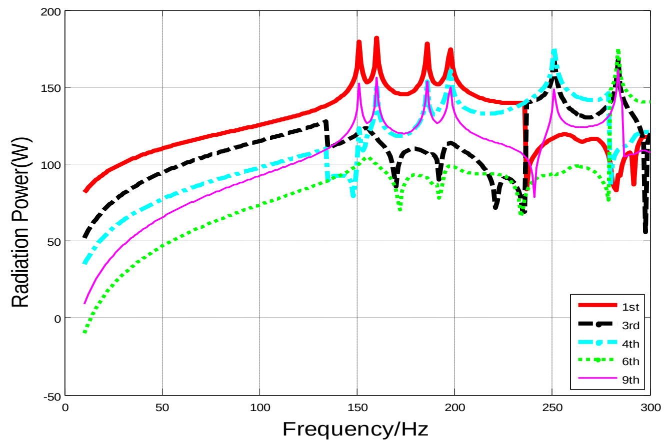



**Figure 9.** Radiated sound power from each order acoustic radiation mode

## 4. Conclusions

In view of complicated structures, this paper presents a numerical method for seeking the relationship between structural modes and acoustic radiation modes. First, based on the theory of base vector in vector space, a governing equation for relating the two kinds of modes is given from the consideration of characteristics of the modes. Then, substitute the normal structural mode shape matrix and the acoustic radiation mode shape matrix which are obtained with the aid of FEM into the governing equation, the modal participating coefficients can be solved, thus we can get the corresponding relations between these two kinds of modes. Using the model of a simply supported truncated conical shell, a numerical example is presented with the numerical method which is proposed in this paper, and the corresponding relations of truncated conical shell are achieved. Finally, the radiated sound power is calculated to verify the effectiveness of this method and the correctness of this conclusion.

In comparison with previous works, the research object of seeking the relationship between these two kinds of modes was extended to complicated structures in this paper. That is to say, no matter how the structural shapes and boundary conditions change, or whether modal shapes have symmetry or antisymmetry or not, we can use the numerical method proposed in this paper to obtain the normal structural mode shape matrix $\boldsymbol{\Phi}$ and the acoustic radiation mode matrix $\boldsymbol{Q}$ ,and then seek the relationship between structural modes and acoustic radiation modes. This method can be used to further investigate and interpret a series of sound radiation problems of complicated structures. Especially it can play a guidance role in ASAC of complicated structures.

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