Analysis of Dynamic Stiffness Effect of Primary Suspension Helical Springs on Railway Vehicle Vibration

W Sun[1,2], D J Thompson[3], J Zhou[2], D Gong[2]

[1]School of Mechanical Engineering, Tongji University, Shanghai 201804, People's Republic of China

[2]Institute of Rail Transit, Tongji University, Shanghai 201804, People's Republic of China

[3]Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, UK

Corresponding author: sunwenjing19@gmail.com

Abstract. Helical springs within the primary suspension are critical components for isolating the whole vehicle system from vibration generated at the wheel/rail contact. As train speeds increase, the frequency region of excitation becomes larger, and a simplified static stiffness can no longer represent the real stiffness property in a vehicle dynamic model. Coil springs in particular exhibit strong internal resonances, which lead to high vibration amplitudes within the spring itself as well as degradation of the vibration isolation. In this paper, the dynamic stiffness matrix method is used to determine the dynamic stiffness of a helical spring from a vehicle primary suspension. Results are confirmed with a finite element analysis. Then the spring dynamic stiffness is included within a vehicle-track coupled dynamic model of a high speed train and the effect of the dynamic stiffening of the spring on the vehicle vibration is investigated. It is shown that, for frequencies above about 50 Hz, the dynamic stiffness of the helical spring changes sharply. Due to this effect, the vibration transmissibility increases considerably which results in poor vibration isolation of the primary suspension. Introducing a rubber layer in series with the coil spring can attenuate this effect.

1. Introduction

When a train is running on a track, vibration is transmitted from the wheel/rail contacts to the vehicle structure through the primary and secondary suspensions. It is widely accepted that the suspension components can isolate and reduce most of the high frequency vibration transmission from the wheel/rail interaction. Helical springs are widely used in the primary suspension of high-speed trains. However, most methods of railway vehicle dynamic modelling ignore the structure and inertia of the springs. However, as the train speed increases, the excitation from the wheel/rail interaction extends to higher frequencies, which means that internal resonances of the spring itself can no longer be neglected. A number of fractures of helical springs of high speed trains and locomotives have occurred recently in China which may be linked to this phenomenon [1]. Moreover, due to the internal resonances, the considerable dynamic stiffening of the spring occurs. Normally, the suspension spring is considered as a simplified static stiffness, which is valid within the frequency range up to 20 Hz [2],

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

but in fact the dynamic stiffness varies a lot at higher frequencies. The dynamic stiffening effect of the suspension system is considerably enhanced with the increase of vehicle speed. It should therefore be introduced into railway vehicle vibration simulation, and the effect of the dynamic stiffness on the vibration transmission property of the whole vehicle should be investigated.

There is much previous research about the wave effect and dynamic stiffness property for a single coil springs. For example, Wittrick [3] obtained an approximate solution by regarding the spring as a Timoshenko beam to establish linear differential equations. Mottershead [4] obtained the exact solution of the static problem by using finite element method. Lee and Thompson [5] used the dynamic stiffness matrix approach to obtain the natural frequencies and dynamic stiffness of coil springs. The results show the significant dynamic stiffening effect in helical springs. For the studies of coil springs as suspension components, Fu and Wang [6] used shock wave transmission theory to establish the dynamic model of a coil spring suspension of an automobile in order to obtain the vibration responses at high frequencies more accurately. Zhang and Yu [7] applied the method of four-end parameters to study the influence of coil spring standing wave effect on suspension vibration transmission property. The wave effect increased vibration peaks of the car body considerably. These applications are mostly automotive but so far, no studies have been carried out on the influence of the dynamic stiffness of helical springs within the primary suspension on the high frequency vibration of railway vehicle systems.

For this purpose, this paper aims to establish a reasonable dynamic model to represent the dynamic stiffening property of a coil spring within the primary suspension of a railway vehicle. The dynamic stiffness matrix method based on [5] is used to determine the stiffness values of helical springs as a function of excitation frequency. Its influence on the vibration transmission from the wheelsets to the bogie and vehicle body is studied. The present work includes the coil spring model into a high frequency dynamic model of a high speed train vehicle system. A track irregularity spectrum is considered as the input and the influence of dynamic stiffness of the helical springs within the primary suspension on the dynamic behaviour of the railway vehicle is investigated. The vibration responses of the bogie and flexible car body are both obtained.

2. Dynamic stiffness model of helical spring

2.1. Dynamic stiffness matrix of helical spring

The dynamic stiffness model of the helical spring is based on [5]. The relation between the local coordinates and global coordinates of the coil spring is shown in Figure 1. ϕ is the conversion angle between the local and global coordinates. s is the coordinate along the coil spring length. α is the helical angle of the spring.

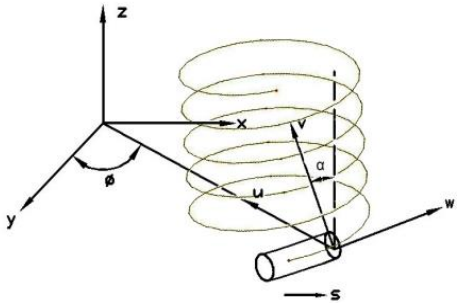


Figure 1. Co-ordinate system of a coil spring

The global coordinates can be expressed as follows:

$$x = \frac{D\phi \tan\alpha}{2}, y = \frac{D\cos\phi}{2}, \mathrm{z} = \frac{D\tan\phi}{2} \qquad (1)$$

where D is the mean diameter of the spring.

When the dynamic loads act on the coil spring, they will generate internal forces and torques in three directions and displacements and rotations in three directions. According to the static equilibrium equation of D'Alembert's principle combined with system inertial force, the partial differential equation of the dynamic system can be obtained as follows:

$$\frac{\partial}{\partial s}\begin{Bmatrix}\delta\\ p\end{Bmatrix}=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{Bmatrix}\delta\\ p\end{Bmatrix}+\begin{bmatrix}0 & 0\\ B_{21} & 0\end{bmatrix}\frac{\partial^2}{\partial t^2}\begin{Bmatrix}\delta\\ p\end{Bmatrix} \tag{2}$$

where *A, B* are coefficient matrices related with the structural and material properties of the coil spring, δ is the 6x1 vector containing displacements and rotations; *p* is the 6x1vector of internal forces and moments; both are functions of *s*.

Equation (2) can be simplified to get a relation between the loads and displacements of the spring as follows:

$$\{p\}=-[A_{12}]^{-1}[A_{11}]\{\delta\}+[A_{12}]^{-1}\frac{\partial}{\partial s}\{\delta\} \tag{3}$$

A wave in the spring consists of temporal and spatial variation. According to the wave motion theory, if the responses of the wire element are harmonic in time, the displacements and forces for a free wave can be written as:

$$\begin{Bmatrix}\delta\\ p\end{Bmatrix}=\begin{Bmatrix}\Delta\\ \mathrm{P}\end{Bmatrix}e^{ks}e^{i\omega t} \tag{4}$$

where Δ and P are the vectors of complex amplitudes. Substituting equation (4) into equation (2), the equation can be written as follows:

$$(k[I]-\begin{bmatrix}A_{11} & A_{12}\\ A_{21}-\omega^2 B_{21} & A_{22}\end{bmatrix})\begin{Bmatrix}\Delta\\ \mathrm{P}\end{Bmatrix}=0 \tag{5}$$

At a given frequency ω, equation (5) can be solved as an eigenvalue problem to find 12 eigenvalues k_i and the corresponding eigenvector matrix [Φ].

In order to get the point stiffness and transfer stiffness of the helical spring, the force and displacement vector of the two ends are given as follows:

$$\begin{cases}F=[p(0) \quad p(L)]^T\\ U=[\delta(0) \quad \delta(L)]^T\end{cases} \tag{6}$$

where *s*=0 and *s*=L are the helical spring positions of the two ends. The dynamic stiffness vector can be obtained through the relation between the force vector *F* and the displacement vector *U* of the two ends.

According to the linear superposition method, the displacement vector of the spring can be expressed as a sum of 12 different waves:

$$\{\delta\}=\sum_{i=1}^{12}a_i\{\phi_i\}e^{k_i s}e^{i\omega t}=[\Phi]E\{a_i\}e^{i\omega t} \tag{7}$$

where, a_i is the amplitude of each wave and *E* is a diagonal matrix containing the exponential terms of the form e^{ks}.

According to equation (7) and equation (3), the force vector can be written as:

$$\{p\} = [A_{12}]^{-1} \left\{ [\Phi] \begin{bmatrix} k_1 & & \\ & \cdots & \\ & & k_{12} \end{bmatrix} E - [A_{11}][\Phi] E \right\} \{a_i\} \quad (8)$$

It is assumed that one end is fixed and the other end is subject to displacement excitation in the axial direction which means U only has one non-zero component. The force vector F at the two ends can be calculated with equations (6) and (8) and hence the point and transfer dynamic stiffnesses can be obtained. The spring point and transfer dynamic stiffnesses K_p and K_t in the axial direction are defined as follows:

$$K_p = \frac{F_b}{z_b}, K_t = \frac{F_t}{z_b} \quad (9)$$

where F_b, F_t are the axial forces at the bottom and top of the spring respectively and z_b is the excitation displacement amplitude at the bottom. These stiffness values will be used for the railway vehicle dynamic analysis in the next section; the components in other directions will be neglected.

2.2. Dynamic stiffness results and comparison with FE model

2.2.1 Spring parameters

The primary suspension studied here is installed with a spring set (two springs) which can take a larger load than a single one. To avoid interference between the different springs and over-torsion of the interacting surfaces, the spring set is normally made up of one left-handed spring and one right-handed spring. The equivalent combined stiffness is the sum of the two spring stiffnesses. The deformations of the two springs are equal. So the total load is the sum of the loads on the two springs which are related to their own stiffness.

Normally, it is assumed that the helical spring only takes axial loads and the influence of its helical angle is ignored. The vertical simplified static spring stiffness [8] can be calculated as follows:

$$K_s = \frac{F}{f} = \frac{Gd^4}{8nD^3} \quad (10)$$

where f is the vertical deflection of the spring, G is the shear modulus, d is the diameter of spring wire, n is the effective number of spring circles. The parameters of the two springs within the primary suspension of a railway vehicle are listed in Table 1.

Table 1. Parameters of coil springs set in axle box

Parameter	Outer	Inner
Orientation	right	left
Number of active coils n	3.2	5.0
Wire diameter d (mm)	39.5	28.5
Spring diameter D (mm)	239.4	156.0
Free height H (mm)	306.0	304.6
Static height H_0 (mm)	240	240
Shear modulus G (N/mm^2)	78500	78500
Damping loss factor	0.001	0.001
Poisson ratio λ	0.3	0.3
Shear correction factor κ_s	0.75	0.75
Simplified vertical stiffness k_s (N/mm)	537.2	348.7
Mass m (kg)	30	12

2.2.2 *Finite element model*

According to the geometric model of the springs shown in Figure 2, finite element models of the outer and inner springs have been established with beam elements to confirm the results of dynamic stiffness matrix method. There are 1235 nodes and 1234 elements in the outer spring model and 1239 nodes and 1238 elements in the inner spring model in total. In this model, six degrees of freedom at one end are all constrained, and a displacement with amplitude 1 mm in the axial direction is input at the other end with excitation frequencies from 0 to 1000 Hz. The reaction forces at both ends can be obtained and then point and transfer stiffnesses are derived through the ratio of the forces and the input displacement.

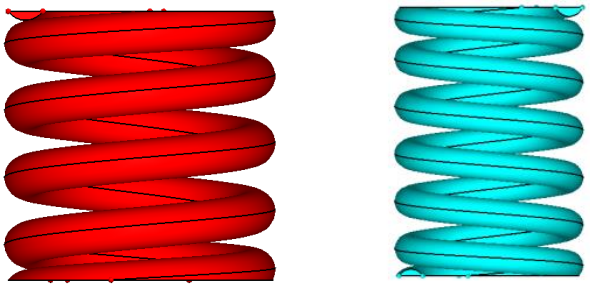


Figure 2. Geometric model of helical springs

Figure 3 shows the modeshapes of the first five modes of inner spring with fixed-free boundary condition. Modes 1 and 2 are predominantly lateral and modes 3 and 4 are extensional. Mode 5 is torsional. 42.05 Hz is the modal frequency of mode 3 which is a vertical extensional mode. Similar results are observed for outer spring. The first vertical extensional modal frequency of outer spring is 38.96Hz. It can be seen clearly that each spring has its internal resonance characteristics.

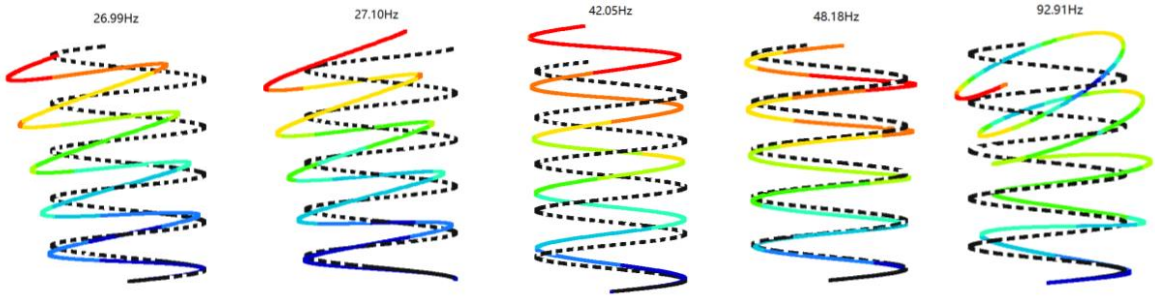
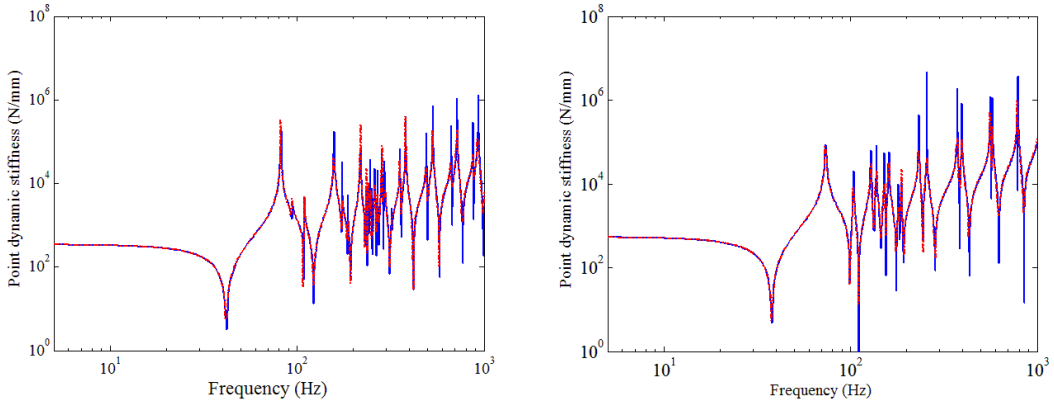


Figure 3. Mode shapes of inner spring with fixed-free boundary condition

2.2.3 *Dynamic stiffness results*

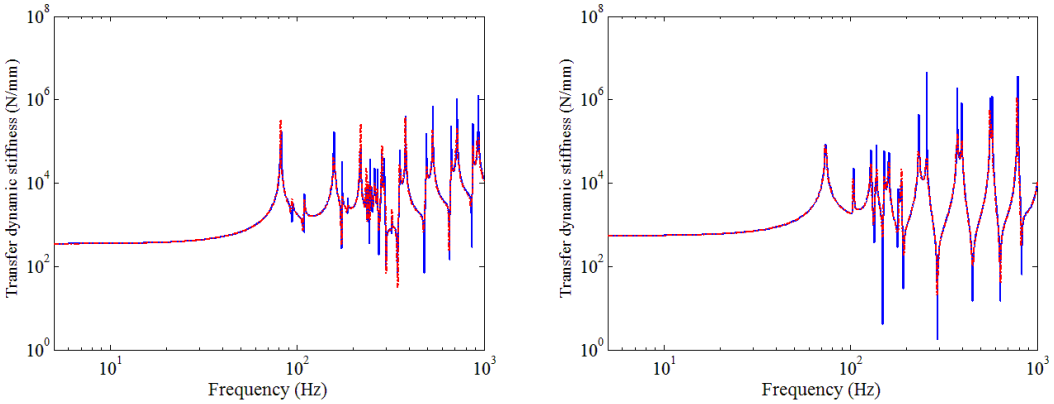
Figure 4 shows point dynamic stiffness results of the two individual springs obtained with the dynamic stiffness matrix method and FE simulation. The two results are almost identical. The stiffness has a constant quasi-static value at low frequencies, 348.7 N/mm for the inner spring and 537.2 N/mm for the outer spring. Then above about 10 Hz the stiffness decreases. The first dips of the point stiffness of the inner and outer springs occur at about 42 Hz and 39 Hz, which are the first vertical extensional frequency of each spring. Because many internal resonances of the two springs are excited with the increase of frequency, a series of peaks and dips occur at high frequencies. The stiffness values change in a very large range between 1 N/mm and 10^6 N/mm.



(a) Inner spring (b) Outer spring

Figure 4. Point dynamic stiffness of helical springs ——FE model; -·-·-dynamic stiffness matrix.

The transfer dynamic stiffness is shown in Figure 5. This generally increases at high frequencies, similar to the point dynamic stiffness. The results of the FE model and dynamic matrix method again show good agreement. Comparing with the outer spring, the inner spring has a smaller diameter and more circles. There are more resonances of the inner spring below 1000 Hz than for the outer spring.



(a) Inner spring (b) Outer spring

Figure 5. Transfer dynamic stiffness of helical springs ——FE model; -·-·-dynamic stiffness matrix.

3. Vibration responses of railway vehicle

Besides the coil spring set, when installed in the vehicle there is a rubber layer beneath the steel spring set. This is used for absorbing shocks and decreasing the stress of the steel material. The structure of the rubber layer spring within the primary suspension is shown in Figure 6. These two parts are installed in series. The upper end of the suspension is connected to the bogie frame by bolts, while the bottom one is located on the upper surface of axle box. The vertical stiffness of the rubber layer used here is 10 kN/mm and the damping loss factor is 0.25. The stiffness value of the primary suspension should be calculated from the superposition of the coil spring set and rubber layer in series.

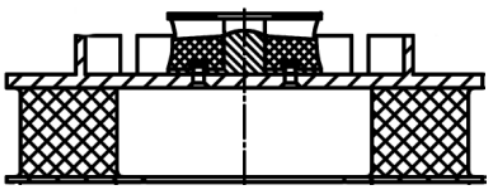
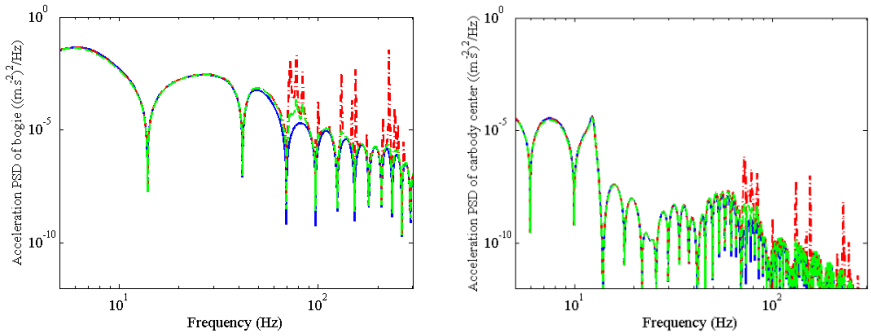


Figure 6. Rubber layer spring of primary suspension

A vehicle-track coupled dynamic model is established to obtain the vibration responses of the system using the Green's function method to solve the flexible car body dynamic model [9]. A high-speed railway track irregularity spectrum corresponding to the speed of 250km/h is considered as the input [10]. All vibration responses of the railway vehicle are calculated with different suspension models: the static spring stiffness, the dynamic stiffness model of the coil spring unit and the combined dynamic stiffness of this unit and the rubber layer.

Figure 7 shows the vibration power spectral densities (PSDs) of bogie and car body with the different models. Along with the change of dynamic stiffness of the helical springs above 38 Hz, high frequency vibrations of both bogie and car body significantly increase. Compared with the static stiffness spring model, there are many peaks with the dynamic spring model. The vibration energies of the vehicle structures at high frequencies become greater due to the dynamic stiffening of the primary suspension. For the dynamic stiffness spring model with rubber layer, the value of each peak decreases because of the damping and stiffness of the rubber layer, but the vibrations are still stronger than those of the static spring model.



(a) Bogie (b) Flexible car body centre

(c) Flexible car body above bogie 1 (d) Flexible car body above bogie 2

Figure 7. Vibration responses of three models ——static spring; -·-·- dynamic spring;---- dynamic spring with rubber layer.

From the above, it can be seen the dynamic stiffening effect of the primary springs causes the high frequency vibrations of the car body and bogie increase. When wave motions in the spring are included in the dynamic model, the primary suspension acts differently from when it is considered as a simplified static stiffness. The rubber layer applied in series with the coil spring can suppress the worst vibration peaks at high frequencies. In order to reduce the vibration of the railway vehicle, the dynamic stiffness of the helical springs within the primary suspension should be controlled. The design of the rubber layer is very important to achieve this.

4. Conclusions

In this paper, the dynamic stiffness matrix method is used to determine the dynamic stiffness of a helical spring from a railway vehicle primary suspension. Coil springs in particular exhibit strong

internal resonances, which lead to high vibration amplitudes within the spring itself as well as degradation of the vibration isolation. Then the spring dynamic stiffness is introduced into a vehicle-track coupled dynamic model of a high speed train and the effect of the dynamic stiffening of the spring on the vehicle vibration is investigated. It is shown that, for frequencies above the first natural frequency of the helical spring, the dynamic stiffness changes dramatically. The vibrations of both bogies and flexible car body increase considerably at high frequencies. Introducing a rubber layer in series with the coil spring can attenuate this effect.

Acknowledgments
This work was sponsored by the National Science and Technology Support Program of China (no. 2015BAG19B02) and Project funded by China Postdoctoral Science Foundation (no. 2015M571597).

References
[1] Wang W, Li G, Tang W, et al. Research on mechanism of fatigue crack of high speed train axle box spring, Journal of the China Railway Society, 2015, 37(6):41-47.
[2] Bruni S, Vinolas J, Berg M, et al. Modelling of suspension components in a rail vehicle dynamics context. Vehicle System Dynamics, 2011, 49(7): 1021-1072.
[3] Wittrick W H. On elastic wave propagation in helical springs [J]. International Journal of Mechanical Sciences, 1966, 8(1): 25-47.
[4] Mottershead J E. The large displacements and dynamic stability of springs using helical finite elements. International Journal of Mechanical Sciences, 1982, 24(9): 547-558.
[5] Lee J, Thompson D J. Dynamic stiffness formulation, free vibration and wave motion of helical springs. Journal of Sound and Vibration, 2001, 239(2): 297-320.
[6] Fu C, Wang X. Wave theory modeling of vehicle vibration and its frequency response analysis of helical spring suspension. Chinese Journal of Mechanical Engineering, 2005, 41(5): 54-59.
[7] Zhang L, Yu Z. The influence of spiral spring's standing wave effect on the automotive suspension vibration isolation characteristics. Journal of Vibration and Shock, 2002, 21(2): 45-47.
[8] Zhang Y, Liu H, Wang D. Spring manual. Beijing: China Machine Press, 2008.
[9] Sun W, Zhou J, Thompson D, et al. Vertical random vibration analysis of vehicle–track coupled system using Green's function method. Vehicle System Dynamics, 2014, 52(3): 362-389.
[10] Wang F, Zhou J, Ren L. Analysis on track spectrum density for dynamic simulations of high speed vehicles. Journal of the China Railway Society. 2002, 24(5):21-27.