

FEM Techniques for High Stress Detection in Accelerated Fatigue Simulation

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Abstract. This work presents the theory and a numerical validation study in support to a novel method for a priori identification of fatigue critical regions, with the aim to accelerate durability design in large FEM problems. The investigation is placed in the context of modern full-body structural durability analysis, where a computationally intensive dynamic solution could be required to identify areas with potential for fatigue damage initiation. The early detection of fatigue critical areas can drive a simplification of the problem size, leading to sensible improvement in solution time and model handling while allowing processing of the critical areas in higher detail. The proposed technique is applied to a real life industrial case in a comparative assessment with established practices. Synthetic damage prediction quantification and visualization techniques allow for a quick and efficient comparison between methods, outlining potential application benefits and boundaries.

1. Introduction

The proposition to analyze a full engineering assembly (car, airplane, wind turbine etc.) for its dynamic behavior can be a daunting task, with the size of the numerical model and the length of the integration time histories requiring ever-increasing CPU, data storage and access time. A dynamic solution could be required for subsequent fatigue assessment to especially address areas of high stress concentration. Fatigue damage is microstructure degradation accumulated under cyclic variable amplitude (VA) loading, typically with maximum stress considerably lower than material yield [1]. The primary fatigue analysis methods, the Stress-Life (S-N) and the Strain-Life (ϵ -N), estimate the expected fatigue life by comparing stress or strain levels with cycles obtained during material testing. In FE based simulation fatigue analysis can be seen as a post-processing step to quasi-static and dynamic simulations whose objective is the resolution of a stress or strain time history for a given component[2].

Figure 1 depicts the options in durability design cycle with integrated finite elements, FE, multibody dynamics, MBD, and fatigue solvers. The amount of data that is generated, stored and post-processed and the ensuing fatigue solution time constitute an ever increasing burden and is a common requirement to reduce the problem size through model or load filtering.

Accelerated fatigue simulation refers to the techniques generally aimed at improving the speed and efficiency of the fatigue simulation process. Traditionally, academic research has focused on time reduction techniques to compress long service loads into reduced length tests producing equivalent damage. A common time editing method is the Small Load Omission Criteria (SLOC) also commonly termed cycle gating or peak and valley extraction (PVX). Heuler and Seeger first proposed an allowable gating level at 50% of the endurance limit of the material under constant amplitude loading [3]. Farrar



et al. [4] classified the accelerated vibration techniques according to testing for fewer cycles by scaling the loads, deletion of cycles with little damage potential and increasing the forcing frequency to reduce test time.

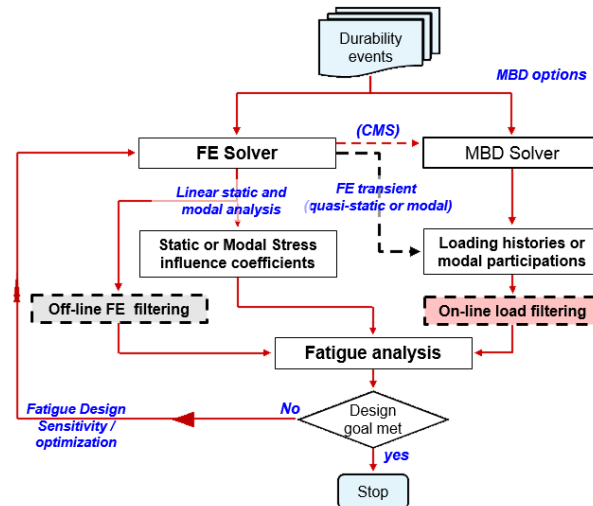


Figure 1. Durability design cycle based on Finite Element (quasi-static and modal transient), or multibody dynamics (with option for flex bodies integration via CMS).

Later Yan et al [5] investigated a more general SLOC based on a fatigue crack initiation threshold. Fatigue data editing, FDE, the elimination of less-damaging cycles, was recently investigated by Kadhim et al. [6] in FE durability assessment. As previously noted by Farrar, Kadhim remarked that despite the wide acceptability of time editing practices, very limited research is available, especially when dedicated to FE simulation simplification.

In the meantime during the 90s, along with much evolved computational and software capabilities and under the impulse of transport industry, the combination of FE and Multi Body Dynamics, MBD, led to important advancements in dynamic durability simulation. Conle and Mousseau [7] reported an analytical study of the fatigue life of chassis components using MBD in combination with proving ground experimental data. The initial rigid-body quasi-static approach was improved with the integration of flexible bodies, mode-acceleration corrections and integrated fatigue solutions via modal stress superposition [8]. In 1997 Huang et al. [9] presented an accelerated process for durability analysis based on FE methods; they used short peak-load events to prioritize entities based on Von Mises stress level distribution; only the top 100 elements were selected as critical and fully addressed in a subsequent fatigue analysis. Dietz et al.[10] combined FE, MBD and fatigue using the concept of stress load matrix to isolate selected critical regions. Later, Huang and Agrawal [11] patented a method of identifying critical elements in fatigue analysis based on Von Mises stress bounding and filtering modal coordinates' history.

In 2002 Haiba et al.[12] reviewed the life assessment techniques applied to dynamically loaded automotive components applicable to optimization algorithms including fatigue. Due to complexity of dynamic and fatigue solutions, they recommended the use of quasi-static techniques when components operate substantially below their natural frequency. In 2007 Wannenbunrg [13] proposed a framework for mapping numerical methods in durability assessment of transport structures. Aiming for fatigue simulation simplification Wannenbunrg et al.[14] demonstrated possible applications for fatigue equivalent static loading in heavy vehicle design. Recently Rentainen et al.[15] presented a novel fatigue approach based on sub-modelling and FE-MBD analysis. A detailed sub-model is defined *a priori* for selected critical details (e.g. welded connections) and stress recovery is performed quasi-statically using displacements obtained from dynamic simulation as boundary conditions.

The majority of the research so far has focused on simulation acceleration by reduction of fatigue solution time, or *time editing*, with only a handful of references dedicated to prioritization and filtering of FE entities [9–11,16]. This is failing to tackle a major aspect in modern computational methods that is the trend towards ever increasing model sizes, especially since FE modelling has moved durability investigation upfront in the design cycle [2]. The typical 500k degrees of freedom of a vehicle full-frame of the late 90's [9], is currently upwards of 10M [17] and in order to meet time and resources constraints, the industry often adopts simplification methods that are mostly driven by empirical knowledge [18].

This paper describes an accelerated fatigue simulation process through the identification of a subset of critical regions, henceforward termed hotspots, applied before, *a priori*, and independently, *off-line*, to the dynamic solution process. The investigation aims to validate the hypothesis that, under certain conditions, an augmented modal reduction base (e.g. Craig-Bampton or equivalent) can be efficiently used to provide a means of filtering non-critical regions of the FE model.

The technique is first developed through simple analytical beam model and hotspot search algorithm implemented in MATLAB®. The method is then applied to a real life industrial model in a comparative assessment with selected established simulation acceleration practices. Probabilistic prediction quality assessment and damage visualization techniques allow for a quick and efficient comparison between the methods, leading to the potential application benefits and boundaries of the proposed technique.

2. Accelerated fatigue simulation by reduction of FE entities

As indicated by Huang [9], only a small fraction of any durability model, around 1% in vehicle industry, is expected to represent a fatigue concern. The main object of this work is to seek the reduction of the total number of FE entities in a durability simulation. A useful classification can be made between *on-line* and *off-line* filtering. *On-line* methods are applied during the FE or FE-MBD dynamic solutions (Figure 1) by virtue of time based stress recovery, the filtering process being repeated with each durability event. Conversely, *Off-line* methods don't use the time-history loads and can be applied independently, *a priori*, to the dynamic solution, with the filtering method executed only once across multiple events.

2.1. On-line two-pass elimination

On-line methods are delivered via a two-step process, also referred to as *two-pass elimination* [19]. A fast *first pass* fatigue calculation uses strong time editing (PVX) of the loading histories (e.g. 90% from peak-load) to compress the time data. Due to the omission of cumulative damage from edited smaller cycles, the simplified first pass solution is not valid for fatigue structural integrity assessment and is used only to provide a prioritization of the likely critical entities. In a *second pass* the most critical entities alone are submitted to a refined fatigue simulation using weakly filtered loading histories.

The *limits-only* elimination is an extreme first pass simplification using only the peak (Max_i) and trough (Min_i) in each individual loading *channel*. A loading channel is the time evolution at either a loading point (unit loading scaling in quasi-static method) or modal participation factor (modal superposition in modal transient). A simple limits-only approach is based on an *assessed Von Mises stress* defined at each FE entity in the model (element or node) as:

$$\sigma_{Assessed} = \sum_i (Max_i - Min_i) \sigma_{VM,i} \quad i=1 \text{ to } c, \quad (1)$$

where c is the total number of loading channels and $\sigma_{VM,i}$ is the Von Mises stress at channel i obtained from static or modal influence coefficients. The assessed stress is not a true stress occurrence, given that the max and min values are not expected to happen simultaneously, but represents a stress range upper boundary inherent in the real load data.

2.2. A priori, off-line elimination

A priori, off-line, methods are essentially a one-off upfront filtering and sorting capability that is applied before running the quasi-static or modal transient events. The filtering could again be based on stress influence coefficients but without the weighting from actual loads.

A simple implementation considers the element *assessed stress*, $\sigma_{Assessed}$, as the summation of the Von Mises stress in each available load case:

$$\sigma_{Assessed} = \sum_i \sigma_{VM,i} \quad \text{for } i=1 \text{ to } c. \quad (2)$$

During the filtering process, if $\sigma_{Assessed}$ falls below a predetermined ratio the corresponding element/node is eliminated from the subsequent fatigue calculation. Aside the inherent approximation, such a simple procedure is only applicable to a quasi-static method. In modal methods the Von Mises stress in each mode is not in a comparable scale and definition (2) loses significance. A variant of this approach, that is applicable equally to quasi-static and modal methods, constitutes the basis for the novel simulation acceleration technique presented in the next section.

3. Development of *a priori* hotspot prediction method

The novel technique is based on two guiding principles:

- In a structure characterized by many stress risers, concentrated loads and boundary conditions, only a small fraction of the model is expected to represent a fatigue concern (a true hotspot)
- A higher stress, strain or strain energy density corresponds to a higher fatigue damage potential (a candidate fatigue hotspot).

Given the above, the further position is taken that, within certain measurable approximations, a sufficiently broad set of static correction and component modes can anticipate *many* areas of potential fatigue damage, the candidate hotspots, to be identified using threshold levels proportional to a peak parameter (e.g. Von Mises stress, VM , or strain energy density, SED). A sufficiently broad set of modes is one that considers all the load application points, constraints and the inertial modes that are likely to be excited based on load position and frequency.

A parametric analytical beam is the baseline model to develop and demonstrate the method.

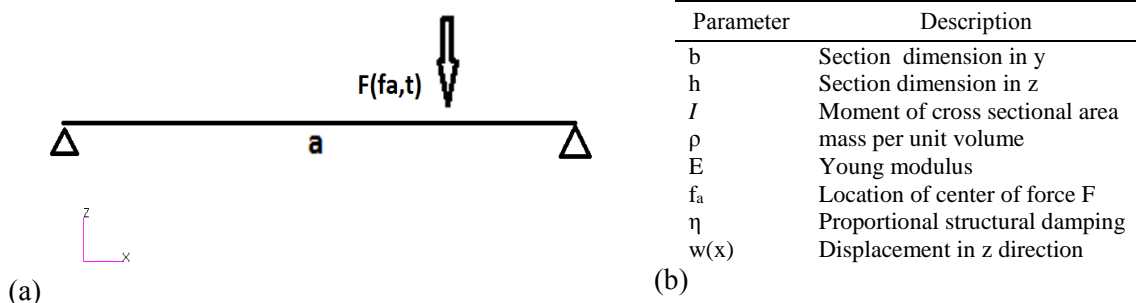


Figure 2. (a) Simply supported bending beam with point force F at $x=f_a$; (b) beam parameters.

3.1. Hotspots prediction for an analytical model of a beam in bending with a concentrated harmonic point force

The simple case in Figure 2 is used to present the concepts and steps for *a priori* hotspot prediction. The true hotspots distributions obtained using Bernoulli-Euler theory [20] are compared with the predicted hotspots. A simple Bayesian framework is proposed as a means to assess the prediction quality.

3.1.1. True hotspots calculation. The beam is solved in the frequency domain for a harmonic concentrated load for any location (refer to Appendix). The subsequent distributions of stress and strain energy density, SED , are obtained for a unit value of the force applied at a fixed location and as a function of the frequency. The aim is to locate the true hotspots movements as the frequency increases and not to solve or compare the load severity at each frequency.

The surface plot in Figure 3 (a) and (b) shows the beam bending stress distribution as a function of the beam x coordinate (frontal axis) and excitation frequency from 0 to 200Hz (remaining parameters: $a=250$ mm, $b=h=0.5$ mm, $f_a=25$ mm and $\eta=.05$). For added clarity, standing stems topped by circle

markers trace the position of the absolute maxima at each solution frequency, providing an indication of the hotspot approximate locations. The emerging pattern is visible as top-view in figure 3(b).

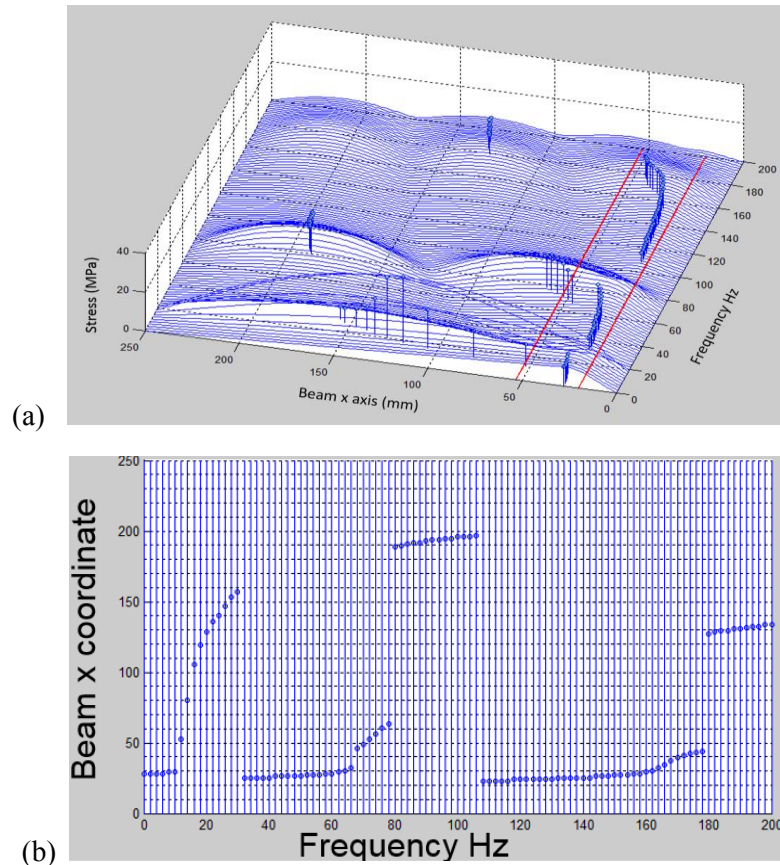


Figure 3. (a) Surface plot of the bending beam stress response. (b) Top-view of surface plot, with the marker showing the peak response pattern.

As anticipated, the maxima move from the initial static position at 25 mm to the resonant locations as we approach a normal mode frequency, respectively at 19Hz, 75Hz and 169Hz. Whenever there's a sufficient frequency distance from a natural frequency the maxima tend to fall back to the static position, approximately indicated by the parallel lines at $x=20$ and $x=55$.

3.1.2. Candidate beam Hotspots with Predictive Algorithm (HPA). The prediction algorithm performs the following 4 main steps:

1. Obtain m modes of interest (frequency cut-off depends on max loading frequency).
2. Calculate s static solution modes in response to F_s loads.
3. For each $m+s$ (mode and static) solution retain areas of SED value above a pre-set thresholds.
4. Consolidate predicted $m+s$ hotspot regions in a reduced analysis group.

Figure 4 shows the normalized bending beam $SED_m(x)$ distribution for the first 3 modes. The candidate areas to be retained, red markers, correspond to beam locations with a SED above a pre-set limit, in this case 90%, with respect to the normalized peak energy (blue markers).

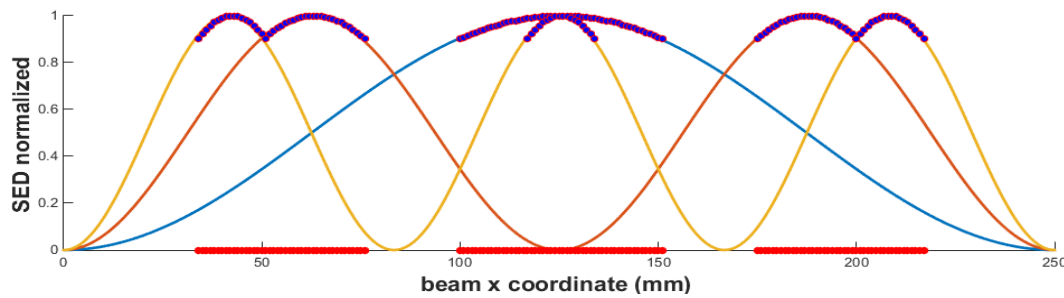


Figure 4 Bending beam modal strain energy density (SED_m) and related hotspots for first three modes.

Evidently the hotspot candidates so far identified are not sufficient, as they don't take into account the loading position. The critical areas resulting from a force statically applied at any position f_a are determined from the truncated modal summation with null frequency (eq. (a.5) (a.6) in Appendix)

$$\sigma_x = \sum_{m=1}^{m=m_{tf}} -\frac{zE}{2} \sqrt{\frac{2}{\rho abh}} \left(\frac{m\pi}{a}\right)^2 \sin \frac{m\pi x}{a} \sin \frac{m\pi f_a}{a} \frac{F}{\omega_m^2} \quad , \quad (3)$$

with the modal truncation value m_{tf} obtained from convergence studies (for this paper $m_{tf}=200$).

Figure 5 shows the static response to a single point force arbitrarily applied on the beam. The resulting blue filled markers indicate SED levels above the 90% threshold.

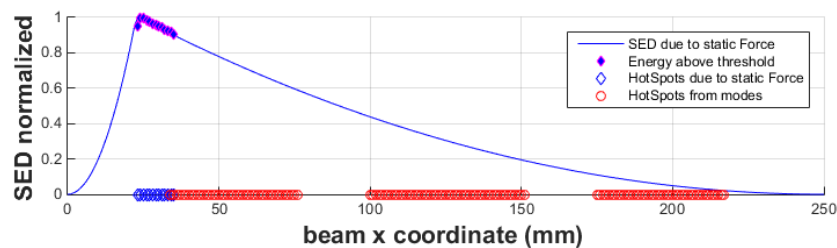


Figure 5. Static response to a single point force applied at 1/11 of the beam dimension.

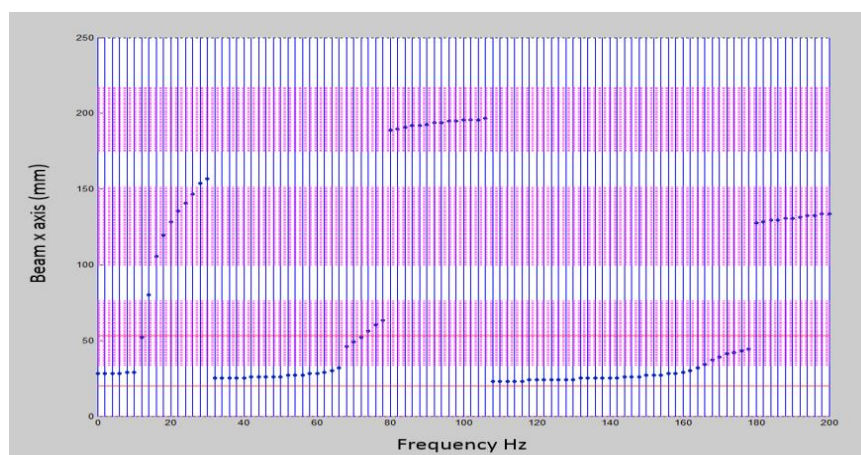


Figure 6. Frequency response peaks (dots) against bending beam consolidated hotspots represented by the shaded magenta area and the static response band.

In Figure 6 the calculated hotspots as a function of frequency are superimposed to the consolidated modal hotspots (shaded magenta areas) and to the static response band (area between parallel red lines), suggesting that the true hotspot areas could be well predicted for this simple example. The next section provides a means for a rigorous quantification of the prediction quality.

3.2. Assessing Hotspot prediction quality

Considering the inherent approximations of the proposed methodology, a quantification of the prediction quality is only attainable in probabilistic terms. For this purpose a discreet set of solution locations, each defined by approximately the same value, is evenly distributed across the component.

$P(H)$ is defined as the prior probability of any given solution location to develop into a true hotspot under the given durability schedule. Here $P(H)$ is intended as a pre-set value indicating the portion of the model to be considered of fatigue interest in any durability process.

$P(D)$ is the probability of any location to be an hotspot candidate according to the prediction algorithm and is the combined effect of the SED thresholds, the number and distribution of the influence coefficients.

A simple application of Bayes theorem provides a measurable indication of both the accuracy and efficiency of the prediction:

$$P(H/D) = \frac{P(H)P(D/H)}{P(D)}, \quad (4)$$

where $P(D)$ can be further decomposed in

$$P(D) = P(H)P(D/H) + P(H')P(D/H'). \quad (5)$$

In equation (4) $P(H/D)$ is the posterior probability of correctly predicting the hotspot locations and is a measure of the prediction quality of the algorithm. $P(D/H)$ is the hit-rate of the predictive method and represents the locations that are correctly predicted as critical (i.e. true positive). Finally, $P(D/H')$ in equation (5) is the probability of a false positive identification and is a measure of the efficiency (or cost) of the prediction.

The definitions above are tested with the beam example described in the previous paragraph. The beam comprises of 250 locations and Figure 7 shows the Bayesian parameters as a function of loading frequency. For this test $P(H)$ is set to 10% of the model at each solution frequency, while $P(D)$ results in 59% based on the chosen threshold at 90% of peak SED.

The reliability of the prediction is directly expressed by the hit rate, which is typically high except for selected frequencies falling in between modes. The amount of false positive gives an indication of the efficiency of the calculation. The ratio of hit-rate and false positive rate, the *likelihood ratio* (LR), combines both reliability and efficiency in a single parameter and represents the usefulness or *diagnostic capacity* of the prediction method. A likelihood ratio close to unity indicates no prediction capability, implying an equal probability to detect a true hotspot and to make a false prediction.

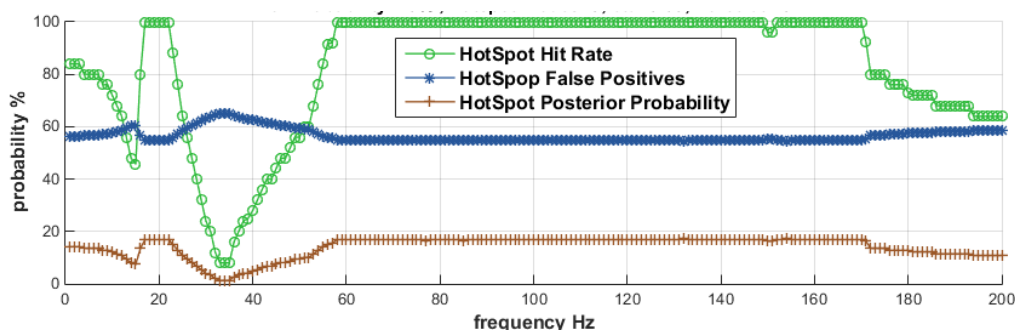


Figure 7. Bayesian probability parameter as a function of frequency for the beam in 3.1.2

3.3. Discussion of preliminary results

The experiment shown in 3.2 can be repeated for different loading points, beam parameters and with more complex loading combinations. This constitutes a useful exercise to investigate trends and develop sensitivities, however a useful mathematical interpretation of the static stress solution (3) can readily clarify the limits of the prediction. Equation (3) represents the static stress contributions of force F to the terms of a Fourier series with infinite arbitrary coefficients, the resulting summation capable to represent any shape compatible with the boundary conditions. This puts into mathematical terms the intuition that with multiple arbitrarily distributed loads the predictive algorithm could prove to be incorrect or inefficient. Such hypothetic conditions might not constitute a practical occurrence but have to be considered as a possibility.

The purpose of the method is to provide an early stage and automated model simplification tool that can tolerate a level of uncertainty, for example during heavy duty optimization runs, when loads information is partial and in general during all initial or intermediate design phases. The high concentration of element SE_{ni} is taken as indicative of potential hotspot danger but certainly not a sufficient condition (false positive) nor a necessary one (missed hotspot).

A reliable prediction is one where all critical hotspots have been identified, regardless of the amount of false positives, the latter being relevant to the efficiency and practical value of the method. The driving principles are evidently an approximation and the energy threshold filtering directly affects both the reliability and the efficiency of the prediction. Even for the simple case of single applied force shown in 3.1, Figure 7 demonstrates that certain spurious combinations of loading frequency and forcing location can significantly decrease the reliability of the method.

In the next sections the hotspot method is presented in the context of a FE driven durability analysis. In order to assess the realistic practical value, the concepts and methods so far developed are applied to a real-life industrial case of representative complexity.

4. Hotspot prediction applied to FE processes

Figure 1 describes the schematic workflow of quasi-static or modal transient analysis typical of industrial durability design cycles. The a priori filtering algorithm can be placed as an off-line parallel step to the generation of stress influence coefficients and using the same 4 steps presented in 3.1.2 for an analytical case.

The concept of a general *augmented modal basis* is used to indicate the combination of static and component modes, with the aim to consider all possible sources of stress concentration via multiple modes of energy transfer. Just as seen for the analytical case, a sufficiently broad set of modes is one that considers all load application points, constraints and inertial modes that are likely to be excited based on the position and frequency of the excitation. In the limit case of quasi-static analysis, the method could be based on the influence coefficients obtained under unit loads and inertia relief conditions [7]. For the general case Craig and Kurdila [20] listed several alternative mode shapes representations under the collective definition of *component modes*, including normal modes, fixed and free boundary modes and attachment modes.

In this work Component Mode Synthesis with Craig-Bampton method, C-B, [20] is proposed as a primary choice for hotspot prediction in FE and in FE/MBD simulation, due to the widespread acceptability and to the capacity to easily combine energy transfer patterns from applied loads (constraint modes) with the internal dynamics (fixed boundary modes). In order to drive the entity filtering process the method uses the levels of strain energy density, SED, as a simple scalar parameter capable to represents the local deformation driving fatigue damage.

4.1.1. Modal Strain Energy Density in FE modelling. For the FE derivation of the element modal strain energy density, considering \mathbf{K}_i the stiffness matrix of i^{th} element and assuming an application of C-B or equivalent leading to $[\boldsymbol{\phi}]_s$ augmented component modes, the element modal strain energy for i^{th} element, n^{th} mode is given by

$$SE_{ni} = \frac{1}{2} \{\boldsymbol{\phi}_n\}^T [\mathbf{K}_i] \{\boldsymbol{\phi}_n\}. \quad (6)$$

Where \mathbf{K}_i in (6) is opportunely zero-padded in order to match the rows of $\{\Phi_n\}$.

The implicit dependency of SE_{ni} on the element size, can be readily eliminated dividing by the element volume V_i ,

$$SED_{ni} = \frac{SE_{ni}}{V_i}. \quad (7)$$

Unlike the time dependent strain energy density, $SE(t)$, obtained after the full solution of the dynamic problem, SED_{ni} can be obtained *a priori* by solving an eigenvector problem.

5. Industrial application: Honda All-Terrain Vehicle durability duty cycle

The reduction technique is tested in the common restrictive conditions typical of vehicle durability, characterized by multiple stress risers, localized loads and constraints and with only few, low frequency, low modal density, natural modes that are dynamically excited.

An All-Terrain Vehicle (ATV) is modelled in the MBD solver MSC Adams® MBD, **Figure 8(a)**, using an FE flexible mainframe obtained via C-B reduction with 10 fixed boundary modes and 16 attachment points, **Figure 8(b)**.

The tubular mainframe, comprising 28433 shell elements, is the object of a detailed fatigue analysis using established S-N critical plane theory and linear damage summation rule [1]. The Hotspot Prediction Algorithm (HPA) is delivered via MATLAB® scripts parsing standard FE output in MSC Nastran® formats. Alternative simulation acceleration practices are tested and compared; dedicated hotspot visualization assessment tools are proposed as a way to quickly and efficiently compare the different methods.

5.1. MBD-FE model and duty-cycle

The Honda ATV in **Figure 8** had previously been the subject of a number of experimental and numerical investigations [21] [22] [23] with the aim to develop simulation methodologies to predict structural fatigue life of automotive components.

A fatigue durability schedule defines a number of events and repetitions representative of customer usage, also alternatively termed fatigue *duty-cycle*. The fatigue structural integrity of any proposed design is measured in terms of the fatigue damage accumulated through the whole duty-cycle.

The durability schedule for the ATV model had been acquired through successive test track runs using instrumented wheel force transducers. The rotating wheel forces time histories are converted into the wheels spindle loads that can be applied directly onto the MBD model.

This work focuses on a single event comprising 600 seconds of Aggressive Hillwork run, AH600, figure 8 (c).

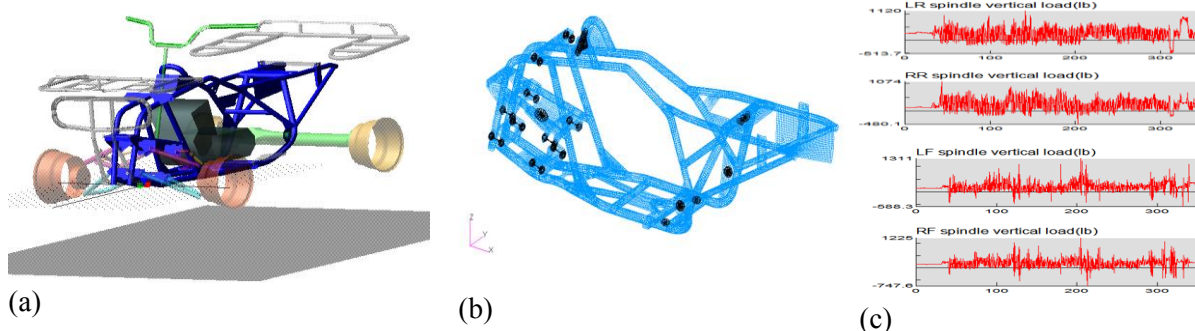


Figure 8 (a) MSC Adams MBD model with flexible frame. (b) Detail of the flexible frame showing 16 attachment point. (c) Vertical spindle forces for event AH600 (from top Left-Rear, Right-Rear, Left-Front and Right-Front respectively).

5.2. Solution speed and accuracy comparison

The speed and accuracy performances of the hotspot algorithm are compared with the common techniques introduced in section 2.1 and 2.2. In Figure 9 the solution run time is normalized with a

baseline run obtained in absence of any entity filtering. The hotspot acceleration factor is demonstrated for SED threshold values at 65% and 80%, resulting in 5.3% and 2.7% model fractions. For the chosen single event the solution acceleration of the a priori filtering is just behind the top performers, however with multiple events the efficiency of the two-step approach is expected to decrease as each event requires its own incremental first pass search. By contrast the priori filtering, adopting a one-off hotspot search, is insensitive to the number of events to be processed and becomes gradually more advantageous as the number of the events increases.

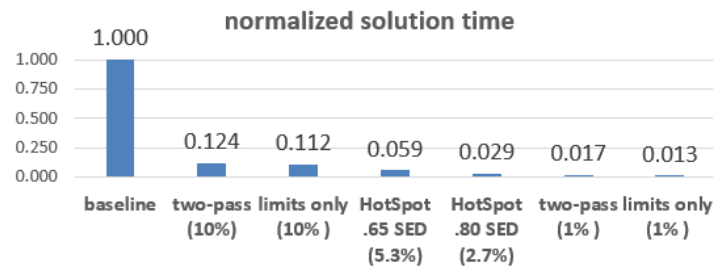


Figure 9 Solution acceleration with various techniques normalized with unfiltered baseline runtime.

Figure 10 represents the fatigue life as function of model fraction in order of increasing fatigue life; the unfiltered baseline run is compared with various acceleration techniques. The hotspot prediction (continuous red line) shows an improved capacity to identify critical elements compared to the faster *limits only* with 1% model retention, but less accuracy than all other methods.

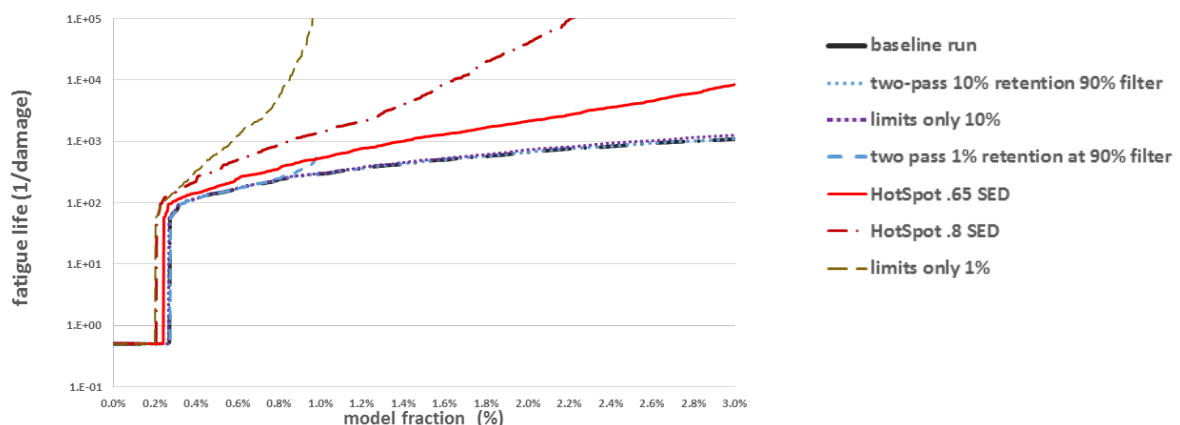


Figure 10 Life increase rate as function of model fraction for different acceleration techniques.

A close-up look at figure 10 shows that a few top damaged locations have been missed by the hotspot algorithm, suggesting the need for a closer investigation with regard to their nature and location.

In Figure 11 the fatigue life of the most damaged elements is presented in order of element ID number. Due to the customary contiguity of FE elements with progressive numbering, the image represents the hotspots areas as vertical clusters. Comparing the unfiltered baseline and HPA fatigue results indicates one critical area (red box in figure 11) that is clearly missed. The missed area can be retraced on the FE geometrical model where the baseline calculated critical areas (red filled elements) are presented alongside the predicted hotspots (dark wireframe). The insert close-up image details the missed hotspot, an area remote from the attachment points and therefore possibly escaped by the approximation of the hotspot filtering criterion.



Figure 11 fatigue life for top damaged elements represented in order of element ID. Boxed area points to missed hotspots.

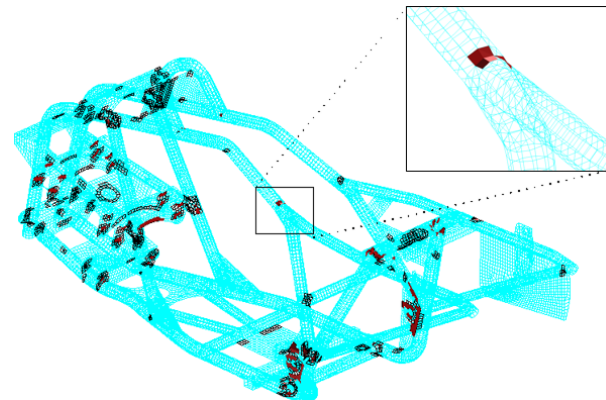


Figure 12 Calculated critical areas (red filled elements) over predicted hotspots (dark wireframe), the insert close-up detailing the missed hotspot

5.2.1. Hotspot quality assessment. The life increase rate in Figure 10 indicates that the top 3% of the model already contains in excess of three orders of magnitude of life difference, implying that the rest of the model could have been safely excluded from fatigue calculations. The true hotspot posterior evidence can be used for an initial quantification according to the Bayesian parameters introduced in paragraph 3.2.

In Table 1 two SED thresholds are tested against a hotspot prior probability of 1%, 2% and 3%. For all cases the increased posterior probability and high likelihood ratio are indicative of the positive diagnostic capacity of the prediction method. Solutions with greater hit rate are preferable, however the higher hit rate is reached at the expenses of the efficiency (false positive rate). For comparable levels of hit rate, the likelihood ratio provides an indicative measure of the computational costs associated to the prediction thus providing a useful term for quality comparison.

Table 1. Hotspot prediction quality in Bayesian probability assessment

| Case | $P(H)$ | SED | $P(D)$ | $P(D/H)$ | $P(H/D)$ | $P(D/H')$ | $\frac{P(D/H)}{P(D/H')}$ |
|---------|---------------|---------|--------|----------|-------------|-----------------|--------------------------|
| | (Prior Prob.) | Thresh. | | Hit rate | Post. Prob. | False pos. rate | Likelihood ratio |
| AH600-A | .01 | .65 | .053 | .70 | .13 | .046 | 15.03 |
| AH600-B | .02 | .65 | .053 | .52 | .20 | .043 | 12.00 |
| AH600-C | .03 | .65 | .053 | .49 | .28 | .039 | 12.61 |
| AH600-D | .01 | .80 | .027 | .45 | .17 | .023 | 19.98 |
| AH600-E | .02 | .80 | .027 | .35 | .26 | .020 | 17.33 |
| AH600-F | .03 | .80 | .027 | .29 | .32 | .019 | 15.48 |

6. Conclusions

The acceleration of fatigue simulation cycles is a key driver in modern industrial design pressed by time and resources constraints. Relatively small academic research is available on the subject and industrial practice is often based on empirical knowledge.

An initial distinction is made between acceleration methods aimed at temporal or spatial compression. The spatial compression methods are further classified in on-line or off-line, whether or not the dynamic solution constitutes a requirement for the filtering process.

The on-line methods are based on actual stress data and are less likely to miss critical areas compared to *a priori* methods, however for the same reason, with large models and multiple long events they might not offer the same level of acceleration potential.

A proposed off-line method is aimed at *a priori* hotspot identification based on common modal reduction techniques. The analytical model of a vibrating beam in bending is used to develop and demonstrate the method, including a probabilistic quantification of the algorithm performances. The use of fast-deployment parametric analysis allows to test hypotheses and uncover limitations.

The Honda ATV simulation test case provides the necessary complexity needed to evaluate the performances of the method in a realistic industrial environment and in comparison with established practices. This initial application demonstrates good potential of the *a priori* approach along with areas for improvements.

The technique is inherently approximate but could lead to potential time saving benefits especially in early phases of the durability design cycle.

Ongoing investigation is oriented at further establishing the foreseeable benefits and limits of the proposed *a priori* hotspot prediction methodology.

Finally, the methodological framework, including acceleration method classification, probabilistic quality assessment and dedicated comparative charts, could be similarly deployed to new or existing variants of simulation acceleration techniques to improve the accuracy and efficiency of the empirical approach often adopted.

Appendix: Bernoulli-Euler Beam formulation and modal energy density definition

For a simply supported beam in bending (figure 2) the normal modes given by

$$w_m(x) = \sqrt{\frac{2}{\rho abh}} \sin \frac{m\pi x}{a} \quad m = 1, 2, \dots, \quad (a.1)$$

with the corresponding natural frequencies

$$\omega_m = \sqrt{\frac{EI}{\rho b h}} \left[\left(\frac{m\pi}{a} \right)^2 \right] \quad m = 1, 2, \dots \quad (a.2)$$

Considering u the displacements parallel to the un-stretched middle axis, the strain x component is

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z \frac{\partial^2 w}{\partial x^2}, \quad (a.3)$$

The modal strain energy density (per unit length) simplifies to

$$e_m(x) = \frac{1}{2} E \varepsilon_x^2 = \frac{EI}{\rho abh} \left(\frac{m\pi}{a} \right)^4 \left(\sin \frac{m\pi x}{a} \right)^2 \quad (a.4)$$

Integrating (a.5) along the length of the beam leads to the expected total modal energy for mode m , assuming mass normalized mode of natural frequency (a.2), that is:

$$MSE_m = \frac{1}{2} \left\{ \pi^2 \sqrt{\frac{EI}{\rho b h}} \left[\left(\frac{m}{a} \right)^2 \right] \right\}^2 = \frac{1}{2} \omega_m^2 \quad (a.5)$$

The bending stresses and strains for a concentrated harmonic force applied at f_a are given by

$$\sigma_x = E \varepsilon_x, \quad (a.6)$$

$$\varepsilon_x = \sum_{m=1}^{\infty} -\frac{2z}{\rho abh} \left(\frac{m\pi}{a} \right)^2 \sin \frac{m\pi x}{a} \sin \frac{m\pi f_a}{a} F e^{i\omega t} \frac{1}{\omega_m^2(1+i\eta)-\omega^2}, \quad (a.7)$$

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