A Method for Stable Deployment of an Electrodynamic Tethered Satellite in Three-Dimensional Space

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Abstract. The paper presents an asymptotic stabilization strategy for the deployment of a controlled tethered satellite system in three-dimensional space, in which the tether length rate is taken as the control variable. Firstly, a rigid-rod tether model is employed to establish the nonlinear dynamic equations of in-plane and out-of-plane motions of the system. Then, by stability analysis of the linearized system at a preassigned direction to deploy, the control law and asymptotic stability condition for the deployment are obtained. The electrodynamic tethered satellite in equatorial plane is discussed. As a result, the large swing motions during deployment are stabilized asymptotically through reliance on the electrodynamic force and the tether length rate. The case studies in the paper well demonstrate the proposed stabilization control strategy.

1. Introduction

The inherent Coriolis effect of tethered satellite systems makes their deployment/ retrieval unstable [1]. And, during state-keeping phase, there have always been unstable periodic motions in the electrodynamic tethered satellite system [2-4].

A large number of intensive studies have been made for the deployment/retrieval process of tethered satellite systems. For example, Barkow et al [5] investigated the deployment of a tethered satellite system in a circular orbit by using the optimal control with the free- and force-braked deployment and the Kissel's law, respectively. Jin and Hu [6] studied the deployment and the retrieval of a tethered sub-satellite of three degrees of freedom via nonlinear optimal control. Based on a 3-D model, Mantri et al [7] identified five dimensional parameters that affect the percentage of the total length to which the tethered satellite system to be deployed, including initial separation velocity, tether tension force, orbital height, effective mass, and final desired tether length. Kumar and Patel [8] developed control laws for multi-connected satellites aligned along local horizontal configuration. Liu et al [9] designed a variable structure control for the deployment/retrieval of tethered satellite system.

With electrodynamic tethered satellite systems, Wen et al [10,11] presented two nonlinear optimal control schemes for the retrieval of a tethered satellite system in inclined orbit by adjusting electrodynamic force and tether tension. Williams [12] designed a time-delayed predictive control law for the librations of electrodynamic tether. Kojima et al [13] showed that a 4-periodic motion was successfully synchronized among electrodynamic tether systems. Zhong and Zhu [14] achieved an optimal balance between a fast deorbit and a libration stability of a short electrodynamic tethered nano-satellite with the minimum control efforts. Iki et al [15] represented an electrodynamic tether deployment from a spool-type reel using thrusters, in which key parameters are estimated on the

ground-based experiments. Based on the time-delayed auto-synchronization, Iñarrea et al [16] applied two feedback control methods in suppressing the unstable periodic motions of an electrodynamic tether.

Most of the published works are devoted to control of the unstable periodic motions caused by electrodynamic forces during state-keeping phase, and have paid less attention to the use of electrodynamic forces in the deployment process of tethered satellite systems. This paper utilizes the tether length rate to stabilize the deployment of tethered satellite systems with three dimensional attitude motion. In Section 2, the study here, begins with establishing a rigid rod model of a tethered satellite system in three-dimensional space, and then in Section 3, based on Lyapunov stability theory, gives the deployment control law and stability condition. The deployment process of electrodynamic tethered satellite in equatorial plane is discussed. Finally, in Section 4, case studies are carried out numerically.

2. Mechanics model

A tethered satellite in inclined circular orbit is shown in figure 1. The system consists of a mother satellite M of mass m_M and a sub-satellite S of mass m_S. The two satellites are connected by a tether of deployed length l that is viewed as a massless rigid rod. An inertial geocentric frame O-XYZ is established such that the X-axis points the direction of ascending node from the center of the Earth O, the Z-axis is aligned with the Earth's axis, and the Y-axis is determined by right-hand rule. An orbital frame o-xyz is on the center of mass of the system, where the y-axis points the direction of motion, the z-axis is perpendicular to the orbital plane, and the x-axis is given by right-hand rule. In figure 1, δ represents the angle of inclination between the orbital plane and the equatorial plane. The in-plane pitch angle θ and out-of-plane roll angle ϕ are selected as the generalized coordinates, respectively.
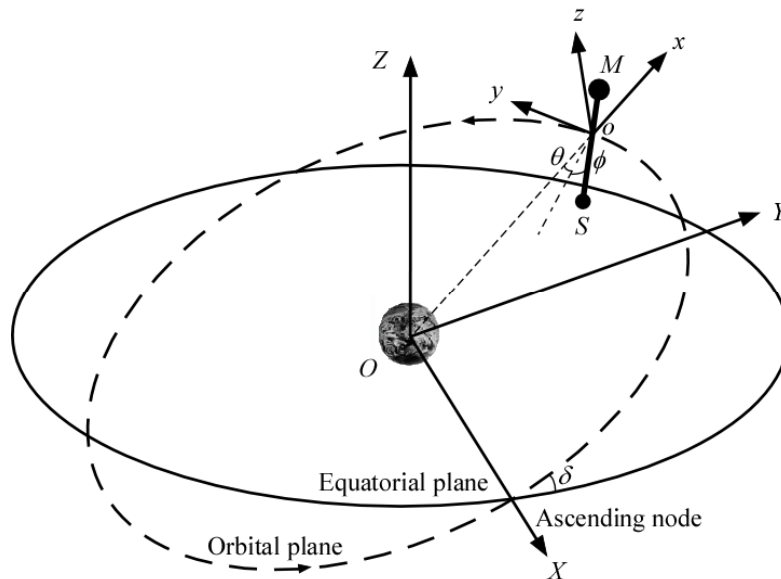

Figure 1. The simplified representation of tethered satellite system.

Application of Lagrange's equations leads to the following dynamic equations of the system [17]

$$
\begin{cases}
\tilde{m}l^2[(\theta''+v'')\cos^2\phi - 2(\theta'+v')\dot{\phi}\cos\phi\sin\phi] + 2\tilde{m}ll'(\theta'+v')\cos^2\phi + \dfrac{3\mu_E\tilde{m}l^2}{R^3}\sin\theta\cos\theta\cos^2\phi = Q_\theta \\[4mm]
\tilde{m}l^2[\phi''+(\theta'+v')^2\sin\phi\cos\phi] + 2\tilde{m}ll'\phi' + \dfrac{3\mu_E\tilde{m}l^2}{R^3}\sin\phi\cos\phi\cos^2\theta = Q_\phi
\end{cases}
\tag{1}
$$

where $()' = \mathrm{d}/\mathrm{d}t$ represents the derivative with respect to time t, v the orbit true anomaly, μ_E the Earth's gravitational constant, R the radius of the running orbit, the non-dimensional parameter

$\tilde{m} = m_M m_S / (m_M + m_S)$, Q_θ and Q_ϕ the generalized forces in the pitch and roll generalized coordinates. If they are produced by electrodynamic forces, one can get the following form

$$Q_\theta = \frac{Il^2(m_M - m_S)}{2(m_M + m_S)} \cos\phi [\sin\phi(B_x \cos\theta + B_y \sin\theta) - B_z \cos\phi] \tag{2}$$

and

$$Q_\phi = -\frac{Il^2(m_M - m_S)}{2(m_M + m_S)}(B_y \cos\theta - B_x \sin\theta) \tag{3}$$

where I is the current in tether. It is set as positive if the current flows to the sub-satellite from the mother satellite. Based on a non-tilted dipole model, the Earth magnetic field established in the orbital frame o-xyz can be expressed as

$$\mathbf{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} -2(\mu_m/R^3)\sin\nu\sin\delta \\ (\mu_m/R^3)\cos\nu\sin\delta \\ (\mu_m/R^3)\cos\delta \end{bmatrix} \tag{4}$$

and μ_m is the magnetic moment of the Earth's dipole. Using the non-dimensional transformations

$$\frac{d()}{dt} = \frac{d()}{d\nu} \cdot \frac{d\nu}{dt}, \quad \xi = \frac{l}{l_{max}} \tag{5}$$

the original dynamic equations of the system become

$$\begin{cases} \ddot{\theta} + 2(\dot{\theta}+1)(\dfrac{\dot{\xi}}{\xi} - \dot{\phi}\tan\phi) + 3\sin\theta\cos\theta = \dfrac{Q_\theta}{\tilde{m}\xi^2 l_{max}^2 \omega^2 \cos^2\phi} \\[4mm] \ddot{\phi} + 2\dot{\phi}(\dfrac{\dot{\xi}}{\xi}) + [(\dot{\theta}+1)^2 + 3\cos^2\theta]\sin\phi\cos\phi = \dfrac{Q_\phi}{\tilde{m}\xi^2 l_{max}^2 \omega^2} \end{cases} \tag{6}$$

in which the dot represents the derivative with respect to ν, ξ denotes non-dimensional tether length, l_{max} the maximum value of the deployed tether length, $\omega = \sqrt{\mu_E/R^3}$ orbit angular velocity around the Earth. The simplified non-dimensional form of equation (6) is

$$\begin{cases} \ddot{\theta} + 2(\dot{\theta}+1)(\dfrac{\dot{\xi}}{\xi} - \dot{\phi}\tan\phi) + 3\sin\theta\cos\theta = H_\theta \\[4mm] \ddot{\phi} + 2\dot{\phi}(\dfrac{\dot{\xi}}{\xi}) + [(\dot{\theta}+1)^2 + 3\cos^2\theta]\sin\phi\cos\phi = H_\phi \end{cases} \tag{7}$$

where

$$H_\theta = \frac{IR^3(m_M - m_S)}{2\mu_E m_M m_S \cos\phi}[\sin\phi(B_x \cos\theta + B_y \sin\theta) - B_z \cos\phi] \tag{8}$$

and

$$H_\phi = -\frac{IR^3(m_M - m_S)}{2\mu_E m_M m_S}(B_y \cos\theta - B_x \sin\theta) \tag{9}$$

Note that, H_θ would vary with the current I only, say,

$$H_\theta = -\frac{\mu_E I(m_M - m_S)}{2\mu_E m_M m_S} \tag{10}$$

and

$$H_\phi = 0 \tag{11}$$

if the system moves in an equatorial plane. Here, other parameters of the system keep unchanged. It is the motivation for utilizing the electrodynamic force to control the deployment.

3. Stabilization strategy

Clearly, the electrodynamic tethered satellite is a nonautonomous system. Therefore, Lyapunov stability theory is not suitable to the electrodynamic tethered satellite system. However, when the system moves in an equatorial plane, the expression of H_θ and H_ϕ can be rewritten as equation (10) and (11). Now, the electrodynamic tethered satellite system is an autonomous one. The linearization of equation (7) gives [2]

$$\begin{cases} \ddot{\theta} + 2(\dot{\theta}+1)(\dfrac{\dot{\xi}}{\xi} - \phi\dot{\phi}) + 3\theta = H_\theta \\[4mm] \ddot{\phi} + 2\dot{\phi}(\dfrac{\dot{\xi}}{\xi}) + [(\dot{\theta}+1)^2 + 3]\phi = H_\phi \end{cases} \tag{12}$$

By setting $\varphi_1 = \theta$, $\varphi_2 = \dot{\theta}$, $\varphi_3 = \phi$, and $\varphi_4 = \dot{\phi}$, equation (12) can be recast as a set of state equations

$$\begin{cases} \dot{\varphi}_1 = \varphi_2 \\[2mm] \dot{\varphi}_2 = H_\theta - 2(\varphi_2+1)(\dfrac{\dot{\xi}}{\xi} - \varphi_3\varphi_4) - 3\varphi_1 \\[2mm] \dot{\varphi}_3 = \varphi_4 \\[2mm] \dot{\varphi}_4 = H_\phi - 2\varphi_4(\dfrac{\dot{\xi}}{\xi}) - [(\varphi_2+1)^2 + 3]\varphi_3 \end{cases} \tag{13}$$

It is easy to get the equilibrium positions

$$\begin{cases} \varphi_{10} = \dfrac{\xi H_\theta - 2\dot{\xi}}{3\xi} \\[3mm] \varphi_{20} = 0 \\[3mm] \varphi_{30} = -\dfrac{H_\phi}{4} \\[3mm] \varphi_{40} = 0 \end{cases} \tag{14}$$

It is evident that the equilibrium position of out-of-plane roll angle always equals to 0. On the other hand, for a deployment with a specified direction, say, $\varphi_{10} = \theta_e$, we have

$$\dot{\xi} = \frac{\xi H_\theta - 3\xi\theta_e}{2} \tag{15}$$

According to the tether length rate, the specified direction would be maintained over the deployment process. Next, let's analyze the stability of the system at the specified direction. The Jacobian matrix of equation (13) is

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3 & -2(\dot{\xi}/\xi - \varphi_3\varphi_4) & 2(\varphi_2+1)\varphi_4 & 2(\varphi_2+1)\varphi_3 \\ 0 & 0 & 0 & 1 \\ 0 & -2(\varphi_2+1)\varphi_3 & -[(\varphi_2+1)^2+3] & -2(\dot{\xi}/\xi) \end{bmatrix} \tag{16}$$

After Substituting equation (14) of equilibrium positions into equation (16), four eigenvalues of which are

$$\begin{cases} \lambda_{1,2} = -\dfrac{\dot{\xi}}{\xi} \pm \left[(-\dfrac{\dot{\xi}}{\xi})^2 - 3\right]^{1/2} \\[4mm] \lambda_{3,4} = -\dfrac{\dot{\xi}}{\xi} \pm \left[(-\dfrac{\dot{\xi}}{\xi})^2 - 4\right]^{1/2} \end{cases} \tag{17}$$

Obviously, if

$$-\frac{\dot{\xi}}{\xi} < 0 \tag{18}$$

the system becomes asymptotically stable on the basis of Lyapunov stability theory. Substitution of equation (15) into equation (18) gives

$$\theta_e < \frac{H_\theta}{3} \tag{19}$$

The same result can be obtained by substituting equation (15) into the deployment condition, i.e., $\dot{\xi} > 0$. Note that, if $H_\theta = 0$ for $I = 0$, the magnitude of θ_e must be less than zero. With the help of the electrodynamic force, hence, the linearized system in an equatorial plane of equation (12) can be deployed along a specified direction for $\theta_e < 0$ or $\theta_e > 0$.

In addition, the uniform deployment with an expected tether length rate $\dot{\xi}_e$ can also be achieved through the following form

$$I(\xi) = -\frac{2\mu_E m_M m_S [2\dot{\xi}_e + 3\xi\theta_e]}{\mu_m (m_M - m_S)} \tag{20}$$

which is obtained by substituting equation (10) into equation (15). It is easy to find that tether deployment rate of equation (15) always satisfies equation (19) even if current I change continuously.

4. Case studies

In order to verify the stabilization strategies, a set of parameters was taken as follows. The masses of mother satellite and sub-satellite were $m_M = 500\text{kg}$ and $m_S = 20\text{kg}$, respectively. The system moved in an equatorial plane. The current strength was taken as $I = -1\text{A}$. The initial true anomaly $v_0 = 0$. The initial non-dimensional tether length was set as $\xi_0 = 0.05$. The initial in-plane pitch angle and its angle velocity were $\theta_0 = -0.02\text{rad}$ and $\dot{\theta}_0 = 0.05\,\text{rad/s}$, respectively. The initial out-of-plane roll angle and its angle velocity were $\phi_0 = -0.1\text{rad}$ and $\dot{\phi}_0 = 0.05\,\text{rad/s}$, respectively. Based on above the initial system parameters and Runge-Kutta integration method, the following numerical simulation is carried out by the Matlab software.

According to equation (19), the expected pitch angle reads $\theta_e < 0.16\text{rad}$ such that $\theta_e = 0.05\text{rad}$. In term of the tether length rate in equation (15), the stable deployment can be achieved within $v = 18\text{rad}$, as shown in figure 2. From figure 2(a) and 2(b), the time histories of the pitch and roll motions in true anomaly, one can see that the amplitudes of the motions decrease gradually within $v = 18\text{rad}$, and approach to $\theta = 0.05\text{rad}$ and $\phi = 0$ in the end. figure 2(c) shows the trajectory of the sub-satellite during deployment in a non-dimensional orbital plane frame $o - \chi\eta$, where the χ-axis points the opposite direction of motion, η-axis points the direction of the mass center of the system from the center of the Earth O. The non-dimensional tether length versus the true anomaly is depictured in figure 2(d).
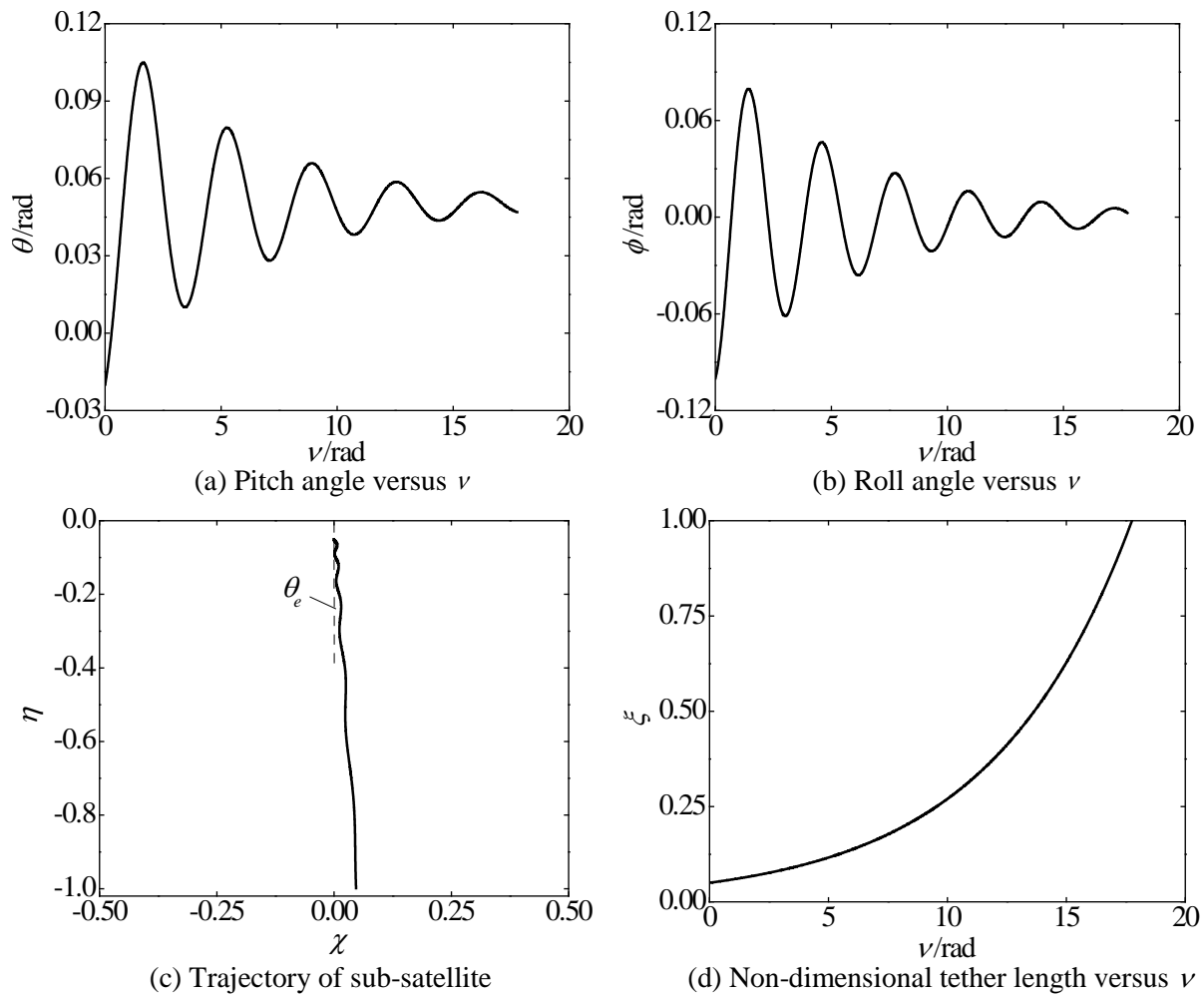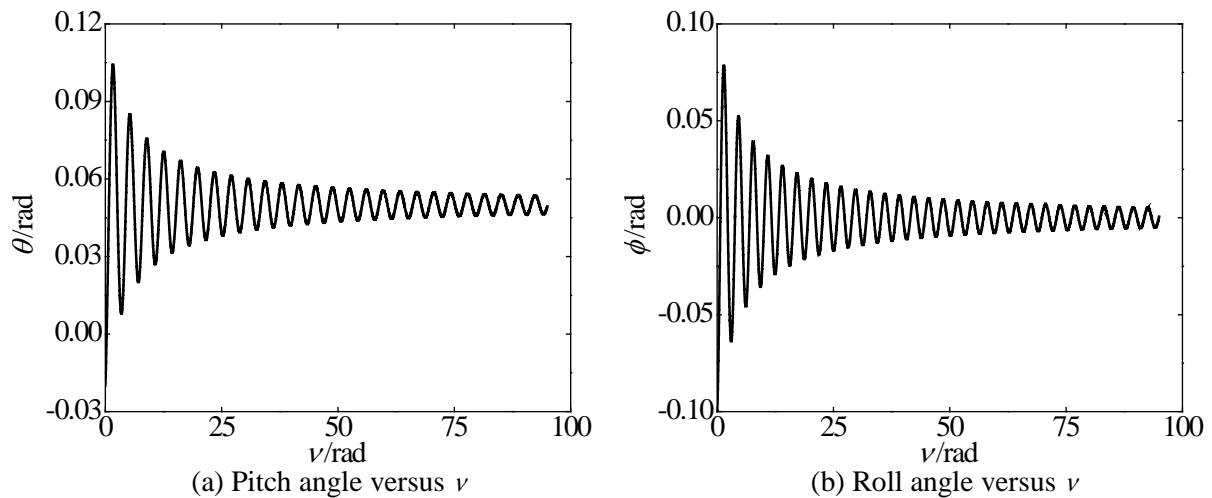
(a) Pitch angle versus v

(b) Roll angle versus v

(c) Trajectory of sub-satellite

(d) Non-dimensional tether length versus v

Figure 2. Stabilization strategy with constant current.

Next, setting the expected pitch angle $\theta_e = 0.05\text{rad}$, an uniform deployment at $\dot{\xi}_e = 0.01$ is shown in figure 3. Based on the current change in equation (20), the duration of the deployment for $\xi = 0.95$ lasted for $v = 95\text{rad}$, wherein the absolute value of the current in tether decreases gradually from -1.15A to -0.35A with the increase of the true anomaly.

(a) Pitch angle versus v

(b) Roll angle versus v

(c) Trajectory of sub-satellite   (d) Current in tether versus v
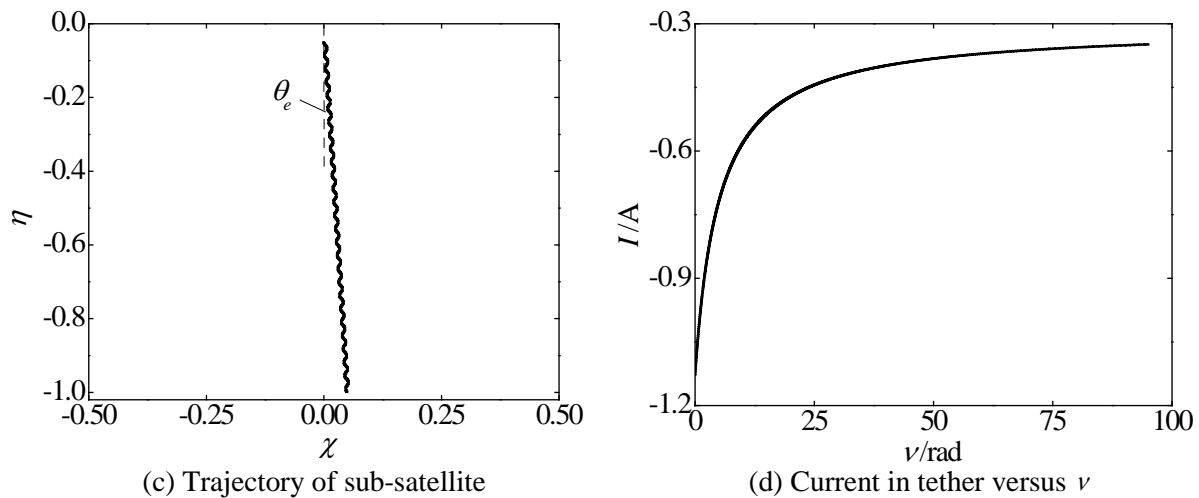
Figure 3. Stable deployment with constant deployment rate.

5. Conclusions

In three-dimensional space, the asymptotic stabilization of deployment of a tethered satellite system along a preassigned direction can be achieved via tether length rates. In equatorial plane, the tether can be deployed along the desired pitch angle through reliance on electrodynamic forces in conjunction with tether length rates. In this situation, the deployment control with a constant tether length rate can also be performed by electrodynamic forces. Next, the proposed method would be extended to elliptical orbits in which tethered satellite systems can be also asymptotically stable deployed.

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