

# Research on damping properties optimization of variable-stiffness plate

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**Abstract :** This paper investigates damping optimization design of variable-stiffness composite laminated plate, which means fibre paths can be continuously curved and fibre angles are distinct for different regions. First, damping prediction model is developed based on modal dissipative energy principle and verified by comparing with modal testing results. Then, instead of fibre angles, the element stiffness and damping matrixes are translated to be design variables on the basis of novel Discrete Material Optimization (DMO) formulation, thus reducing the computation time greatly. Finally, the modal damping capacity of arbitrary order is optimized using MMA (Method of Moving Asymptotes) method. Meanwhile, mode tracking technique is employed to investigate the variation of modal shape. The convergent performance of interpolation function, first order specific damping capacity (SDC) optimization results and variation of modal shape in different penalty factor are discussed. The results show that the damping properties of the variable-stiffness plate can be increased by 50%-70% after optimization.

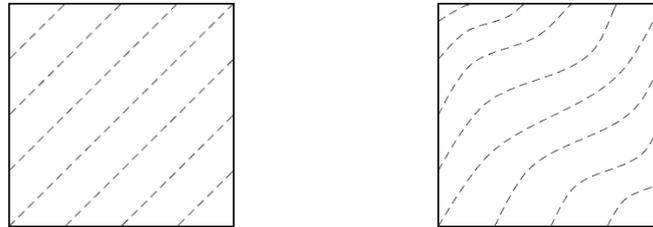
## 1. Introduction

Fibre-reinforced materials are prominent for their high specific strength and high specific stiffness, which can effectively reduce the weight of the aviation spacecraft. More importantly, composite laminates, which belong to a kind of polymer with viscoelastic properties, excel metals in damping performance. Therefore, composite laminates have been widely used in aviation, aerospace and other fields.

In recent years, in order to further excavating the excellent performance of composite materials, variable stiffness design concept[1~2] was proposed. This concept implies that fibre paths are not necessarily straight but can be continuously curved( see Fig.1(a) and Fig.1(b)). Thus, the properties at every point of the panel can be varied independently leading to optimal tailoring of the composite panel to design requirements. Two methodologies could be applied to optimize variable stiffness laminates[3]: 1) by using specific curves to define the fibre orientations, the parameters of the curves are then optimized; 2) the methodology originally proposed by Zehnder and Ermanni[4] involves a parameterization scheme for the purpose of minimizing design variables and accommodating



production focuses on zoning the smallest, inseparable material entities called "Patches", where the orientation angles cohere to build laminated composite structures.



**Fig 1(a).** constant-stiffness plate      **Fig 1(b)** variable-stiffness plate

Aiming at different performance parameters of composite optimization, Stegmann and Lund[5] are pioneers for putting forward Discrete Material Optimization (DMO) formulation. Prof Bruyneel[6] improved the material characteristics interpolation function of DMO, which requires fewer design variables and better convergence rate comparing to classical DMO. A composite cylinder was optimized by Blom[7] for maximum buckling under pure bending by varying the stiffness in circumferential direction. Ferreira et.al[8] applied the hierarchical optimization in the design of laminated fiber composites with the goal of simultaneously designing macrostructural and microstructural levels of structures in terms of plies orientations, fiber volume fractions and fiber cross-sectional shapes, respectively.

For fibre reinforced composite laminates with same material properties of each ply, the stiffness properties can be denoted by 12 lamination parameters which represent the layup configuration of laminates (fiber orientation angles, ply thickness and number of plies). Liu et.al[11] found that the feasible region based on only a few ply groups is very close to the overall one lamination parameters determined by infinite plies, and they established a two step optimization scheme for simultaneous optimization of layup configuration and in-plane fiber distribution for maximum stiffness design of laminated plates. Setoodeh[11] investigated the optimal design of fibre reinforced rectangular composite plates for minimum compliance by varying the lamination parameters of plates. Khani[12] also introduced lamination parameters as design variables to maximize the strength of perforated laminates with axial tensile.

While most of previous studies mainly focus on optimizing the laminates' stiffness, strength, and natural frequency, few works have been concerned with the optimal design for damping structures. The superior damping performance of composites plays a vital role in vibration control, noise reduction and the ability to increase the structural life under cyclic loading and impact loading in various engineering applications[13]. Therefore, this paper employ Hencky's theory of shells and elastic-viscoelastic assumption to solve the laminates SDC based on finite element method (FEM), whose results are then compared with experimental data from other open literatures. By selecting the stiffness matrix and damping matrix as optimization variables, and applying discrete material optimization method (DMO), single plate damping optimization model is established, where MMA is utilized to solve the problem. The paper concludes with a discussion on the modal shape change in the iteration under different penalty factors alongside their eventual optimization results, and damping optimization results under different boundary conditions and different number of patches zoned.

## 2. Damping model and verification

### 2.1. Geometric matrix

By applying the plate/shell theory of Hencky, displacements of an arbitrary point in the plate can be expressed as:

$$\begin{cases} u(x, y, z, t) = z\theta_y(x, y, t) \\ v(x, y, z, t) = -z\theta_x(x, y, t) \\ \omega(x, y, z, t) = \omega(x, y, t) \end{cases} \quad (1)$$

wherein,  $\theta_y(x, y)$  and  $\theta_x(x, y, t)$  are rotation angles of middle surface. Strain matrix can be expressed as:

$$\{\epsilon_i\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = z \begin{Bmatrix} \frac{\partial \theta_y}{\partial x} \\ -\frac{\partial \theta_x}{\partial y} \\ \left( \frac{\partial \theta_y}{\partial y} - \frac{\partial \theta_x}{\partial x} \right) \\ \frac{\partial \omega}{\partial y} - \theta_x \\ \frac{\partial \omega}{\partial x} + \theta_y \end{Bmatrix} = [B]\{\delta\}^e \quad (2)$$

where  $\{\delta\}^e = [w_1 \ \theta_{x1} \ \theta_{y1} \ w_2 \ \theta_{x2} \ \theta_{y2} \ \dots \ \theta_{x8} \ \theta_{y8}]^T$  represents the displacement values of nodal point of element. Geometric matrix can be defined as:  $[B] = [B_1 \ B_2 \ \dots \ B_8]$ . When quadrilateral 8 node shell elements are applied, shape function can be written as:

$$N_i = \begin{cases} \frac{1}{4}(1 + \xi_i \xi)(1 + \eta_i \eta)(\xi_i \xi + \eta_i \eta - 1) & (i = 1, 2, 3, 4) \\ \frac{1}{2}(1 - \xi_i^2)(1 + \eta_i \eta) & (i = 5, 7) \\ \frac{1}{2}(1 + \xi_i \xi)(1 - \eta_i^2) & (i = 6, 8) \end{cases} \quad (3)$$

Then, the geometric matrix can be obtained as:

$$[B_i] = \begin{Bmatrix} \begin{bmatrix} 0 & 0 & \frac{\partial N_i}{\partial x} \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & -\frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \end{bmatrix} \\ \begin{bmatrix} \frac{\partial N_i}{\partial y} & -N_i & 0 \\ \frac{\partial N_i}{\partial x} & 0 & N_i \end{bmatrix} \end{Bmatrix} z \quad (i = 1, 2, \dots, 8) \quad (4)$$

The shape function  $N_i(\xi, \eta)$  is in their local coordinates while the derivation of Eq.(4) at global coordinates, coordinate transformation relation can be expressed as:

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} \quad (5)$$

where [J] is a 2-order Jacobi matrix. By correlating Eq.(4) and Eq.(5), the shape function of coordinate point  $(\xi, \eta)$  and first order partial derivative of shape function can be obtained.

2.2. *Specific damping capacity*

At present, there are two kinds of damping prediction model of composite materials: strain energy method and complex modulus method. The complex modulus method is pronounced for its clearness in concept definition, which shares a similarity with dynamics problems of undamped system in their convenience to describe the dynamic properties of viscoelastic materials. However, the disadvantage is that the process involves complex computations, which leads to a lower speed as a result.

According to the elastic-viscoelastic assumption, specific damping capacity (SDC) can be defined as a ratio of dissipated energy and total strain energy of the system in one period:

$$\varphi = \frac{\Delta U}{U} \tag{6}$$

The total strain energy of composite laminates can be expressed as

$$U = \frac{1}{2} \{\delta\}^T [K] \{\delta\} \tag{7}$$

where  $\delta$  is model shape, which can be obtained by solving the characteristic equation and  $K$  is total stiffness matrix of laminated plates.

The dissipated energy in the structures can be assumed as the sum of dissipated energy formed stresses from every direction.

$$\Delta U = \frac{1}{2} \left[ \int_V \psi_{11} \sigma_1 \varepsilon_1 dV + \int_V \psi_{22} \sigma_2 \varepsilon_2 dV + \int_V \psi_{13} \sigma_4 \varepsilon_4 dV + \int_V \psi_{23} \sigma_5 \varepsilon_5 dV + \int_V \psi_{12} \sigma_6 \varepsilon_6 dV \right] \tag{8}$$

Or it can be expressed simply as:

$$\Delta U = \frac{1}{2} \int_V \{\varepsilon_i\}^T [\psi] \{\sigma_i\} dV \tag{9}$$

where:

$$[\psi] = \begin{bmatrix} \psi_{11} & & & & & \\ & \psi_{22} & & 0 & & \\ & & \psi_{12} & & & \\ & & & \psi_{23} & & \\ & & & & \psi_{13} & \\ & & & & & \psi_{13} \end{bmatrix} \tag{10}$$

$\psi_{ij}$  is loss factor of every direction, which can be obtained by improvement of bridging model proposed earlier[14]. Eq.(8) can be written as:

$$\int_{\Omega} \int_h \{\varepsilon\}^T [D_d] \{\varepsilon\} dz d\Omega = \sum_{k=1}^N \int_{\Omega} \int_{h_{(k-1)}}^{h_k} \{\bar{\varepsilon}_i\}^T [R_{ij}^k] \{\bar{\varepsilon}_j\} dz d\Omega \tag{11}$$

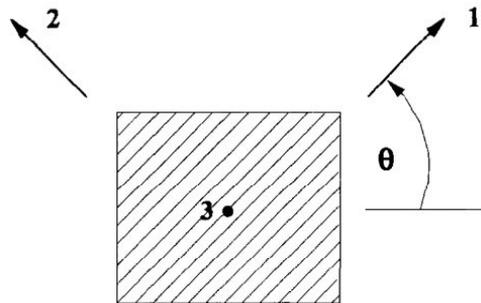
where:

$$[R_{ij}^k] = [T]^{-1} [\psi] [Q_{ij}^k] [T]^{-T} \tag{12}$$

$T$  is coordinate transformation relation expressed as

$$T = \begin{bmatrix} l^2 & m^2 & 2lm & 0 & 0 \\ m^2 & l^2 & -2lm & 0 & 0 \\ -lm & lm & l^2 - m^2 & 0 & 0 \\ 0 & 0 & 0 & l & -m \\ 0 & 0 & 0 & m & l \end{bmatrix} \tag{13}$$

where,  $l = \cos \theta, m = \sin \theta$ .  $\theta$  is a rotation angle between global coordinate system and local coordinate system(Fig 2.).



**Fig 2.** A lamina with local and global coordinate system

Therefore, the equivalent damping coefficient matrix  $D_d$  can be expressed as:

$$D_{dij} = \frac{4}{h^3} \sum_{k=1}^N R_{ij}^k (h_k^3 - h_{k-1}^3) \quad (i, j = 1, 2, 6)$$

$$D_{dij} = \frac{1}{h} \sum_{k=1}^N R_{ij}^k (h_k - h_{k-1}) \quad (i, j = 4, 5)$$
(14)

The total dissipated energy of composite laminates can be expressed in the form of finite element:

$$\Delta U = \frac{1}{2} \{\delta\}^T [K_d] \{\delta\}$$
(15)

where  $K_d$  is total damping matrix:

$$[K_d] = \int_{V^e} [B]^T [D_d] [B] dV$$
(16)

The total strain energy  $U$  and dissipated energy  $\Delta U$  can be obtained by Eq.(7) and(15). The SDC of any order can be obtained by substituting  $U$  and  $\Delta U$  into Eq.(6).

### 2.3. Model verification

In order to verify the validity of the model, the present SDC results, Rayleigh-Ritz analysis and others' experiments[15] are compared here. From Table1, the results of this paper agree well with the result of Rayleigh-Ritz analysis and experiments[15]. Specific material parameters can be found in the references[15]. Therefore, this example can prove the validity of the model of this paper preliminary.

**Table 1.** The[45°/-45°/0°/0°/0°/45°/-45°/0°/0°]s 913C-TS plate, (a=254(mm), b=254(mm), h=2.320(mm))

	Rayleigh - Ritz	1.98	2.37	1.18	1.79	1.83	2.10
SDC (%)	model	2.09	2.36	1.23	1.80	1.88	2.15
	experiment[15]	2.00	2.14	1.28	1.79	1.82	—
	error (%)	4.5	10.2	-3.90	0.558	3.29	—

## 3. Optimization models and sensitivity analysis

### 3.1. Optimization models

Discrete Material Optimization (DMO) can be considered as groundbreaking work of multi-phase topology optimization[16]. Its core idea is to treat different laying angles as different materials. Thus the optimization problem of composite laying angle is converted into the optimal selection of materials, where the angle corresponding to optimal material is equivalent to the optimal angle for the optimization problem.

According to the DMO method, the material stiffness  $D_k$  of an arbitrary "patch" is defined as a weighted sum of the candidate material properties:

$$D_k = \sum_{i=1}^n w_i D_i = w_1 D_1 + w_2 D_2 + w_3 D_3 + w_4 D_4 \tag{17}$$

where  $n$  is the number of candidate angle,  $D_i$  is the material stiffness corresponding to a candidate material ( $0^\circ, \pm 45^\circ, 90^\circ$ ). The value of weight coefficient  $w_i$  is set between 0 and 1. Besides, with the convergence of objective function, every "patch" must have one single weight of value 1 which stands for the optimal angle, and all other weights of value 0. In this paper, Shape Function based Penalization (SFP)<sup>[6]</sup> is applied, where weight coefficient is written as:

$$w_i = \left[ \frac{1}{4} (1 \pm x_{i1})(1 \pm x_{i2}) \right]^p \tag{18}$$

Compared with the scheme in DMO<sup>[5]</sup>, the advantages of weight coefficient in SFP are that it can effectively reduce design variables and lead to a convergence more easily.

In order to force the selection of only one material at the solution, intermediate values of the weight coefficient is penalized, and  $p$  in equation (18) is the penalty factor, thus limiting the occurrence of any blending of materials in a given physical ply at the optimum. For measuring the convergence condition of the interpolation function in optimization results, define:

$$R = \frac{Wc}{Ws} \tag{19}$$

where  $Wc$  is the number of convergence weight coefficient, and  $Ws$  is the total number. If the eventual result of interpolation function is not completely convergent, the intermediate values will be artificially adjusted to 0/1, whereas the values are rounded to get the final optimization results.

Based on finite element method, this paper converts the optimal ply angle select problem to the problem of selecting the optimal element stiffness/damping matrix:

$$K_k = \sum_{j=1}^n w_j K_j, \quad C_k = \sum_{j=1}^n w_j C_j \tag{20}$$

where  $K_i$  and  $C_i$  represent the element stiffness matrix and damping matrix of each material element,  $K_j$  and  $C_j$  represent the element stiffness matrix and damping matrix under each ply angle, inasmuch as the element stiffness matrix and the element damping matrix corresponding to each ply angle can be calculated before the iteration, then stored for the later calling in the iteration, which greatly saves the integral time.

In some engineering applications, in order to obtain damping as large as possible,  $\lambda = 1/\varphi_r$  is chosen as the objective function, whose minimum value represents the maximum of SDC. Whereas the models of the optimization problem with arbitrary order modal damping ratio as the objective function is defined:

$$\begin{aligned} &\text{find} \quad : \{x_{ik}\} (i=1, 2 \dots \quad 2) \\ &\text{minimize} \quad : \lambda \\ &\text{subject to} : ([K] - \omega^2 [M]) \{\delta\} = 0 \\ &\quad \quad \quad : -1 \leq x_{ik} \leq 1 (i=1, 2 \dots \quad 2) \end{aligned} \tag{21}$$

### 3.2. Sensitivity analysis

For composite laminates, it is assumed that ply angle has little impact on modal shape. Then, the first and second derivative of  $\lambda$  with respect to any variable can be written as:

$$\frac{\partial \lambda}{\partial x_{ik}} = \frac{(\delta_i^T \frac{\partial K}{\partial x_{ik}} \delta_i) * (\delta_i^T K_d \delta_i) - (\delta_i^T \frac{\partial K_d}{\partial x_{ik}} \delta_i) (\delta_i^T K \delta_i)}{(\delta_i^T K_d \delta_i)^2} \tag{22}$$

In order to prove the above hypothesis in numerical examples, the model tracking technique is employed to estimate the change of modal shape during the course of iterations.

$$MAC(\phi_{i-1}, \phi_i) = \frac{|\phi_{i-1}^T \phi_i|^2}{|\phi_{i-1}^T \phi_{i-1}| |\phi_i^T \phi_i|} \tag{23}$$

where  $\phi_{i-1}, \phi_i$  represent the acquired modal shape after previous and present iteration respectively. The closer that the value of MAC is to 1, the smaller the change will be.

### 3.3. Optimization algorithm

After the optimization model is built, an appropriate optimization algorithm will be another important problem. In the various optimization problems, the optimization algorithms based on derivatives are most efficient with minimum computational cost. In 1987, Prof. Svanberg[17] developed MMA algorithm based on explicitly near convexity. During each iteration, the original objective and constraint function are turned into explicit near iteration point, and the expressions are shown as:

$$\begin{aligned} \text{minimize : } f_0^{(k)}(x) &= \sum_{j=1}^n \left( \frac{p_{0j}^{(k)}}{U_j^{(k)} - x_j^{(k)}} + \frac{q_{0j}^{(k)}}{x_j^{(k)} - L_j^{(k)}} \right) + r_0^{(k)} \\ \text{subject to : } f_i^{(k)}(x) &= \sum_{j=1}^n \left( \frac{p_{ij}^{(k)}}{U_j^{(k)} - x_j^{(k)}} + \frac{q_{ij}^{(k)}}{x_j^{(k)} - L_j^{(k)}} \right) + r_i^{(k)} \quad \text{for } i = 1, \dots \end{aligned} \tag{24}$$

where  $x_j^{(k)}$  denote the  $j$ th variable in the  $k$ th iteration, and the range of value satisfies:

$$\max\{x_j^{\min}, \alpha_j^{(k)}\} \leq x_j^{(k)} \leq \min\{x_j^{\max}, \beta_j^{(k)}\} \tag{25}$$

Among them,  $L_j^{(k)}$  and  $U_j^{(k)}$  are equations of moving asymptotes adjusting the convexity of optimization problem, and corresponding expressions can be written as:

$$\begin{aligned} L_j^{(k)} &= x_j^{(k)} - 0.1(x_j^{\max} - x_j^{\min}) \\ U_j^{(k)} &= x_j^{(k)} + 0.1(x_j^{\max} - x_j^{\min}) \end{aligned} \tag{26}$$

where  $\alpha_j^{(k)}$  and  $\beta_j^{(k)}$  are ‘‘moving limits’’, which satisfies:

$$L_j^{(k)} < \alpha_j^{(k)} < x_j^{(k)} < \beta_j^{(k)} < U_j^{(k)} \tag{27}$$

$p_{ij}^{(k)}$  and  $q_{ij}^{(k)}$  are the first order Talyor series expansion of objective function and constraint equation at present iteration point for variables respectively.

$$\begin{aligned} p_{ij}^k &= \max\{0, (U_j^{(k)} - x_j^{(k)})^2 \frac{\partial f_i}{\partial x_j}(x_j^{(k)})\} \\ q_{ij}^k &= \max\{0, (x_j^{(k)} - L_j^{(k)})^2 \frac{\partial f_i}{\partial x_j}(x_j^{(k)})\} \end{aligned} \tag{28}$$

For Eq.(24),  $f_i^{(k)}(x) = f_i(x^{(k)})$  at  $x = x_j^{(k)}$ , therefore,  $r_i^{(k)}$  can be expressed as:

$$r_i^{(k)} = f_i(x^{(k)}) - \sum_{j=1}^n \left( \frac{p_{ij}^{(k)}}{U_j^{(k)} - x_j^{(k)}} + \frac{q_{ij}^{(k)}}{x_j^{(k)} - L_j^{(k)}} \right) \tag{29}$$

The approximate solution of original problem can be obtained using the sub-problems of Eq.(24) based on duality theory . For MMA, only the function and derivative at present design point are need during the iteration computations, so it is suitable for large scale optimization problems with few constraints.

**4. Numerical results**

This paper takes first order SDC as optimum objective, the candidate angles are  $0^\circ, \pm 45^\circ$  and  $90^\circ$ . The laminates are made of T300/BMP316, and material parameters are given in Table 2.

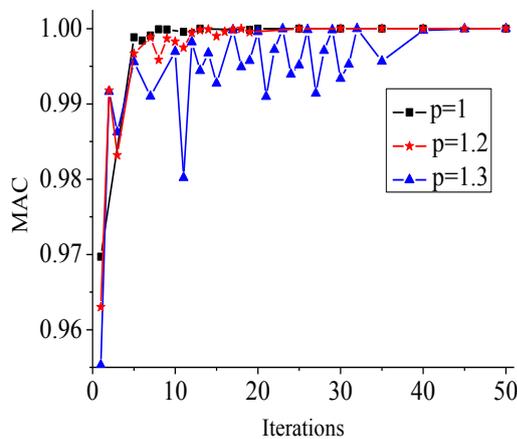
**Table 2.** Material parameter of T300/BMP316

$E_1$ (GPa)	$E_2, E_3$ (GPa)	$G_{12}, G_{23}, G_{13}$ (GPa)	$\mu$	$\rho$ (Kg/m <sup>3</sup> )	$\Psi_{11}$ (%)	$\Psi_{22}$ (%)	$\Psi_{12}$ (%)	$\Psi_{23}$ (%)	$\Psi_{13}$ (%)
128.8	8.3	4.1	0.355	1578	0.55	4.96	5.92	5.63	0.49

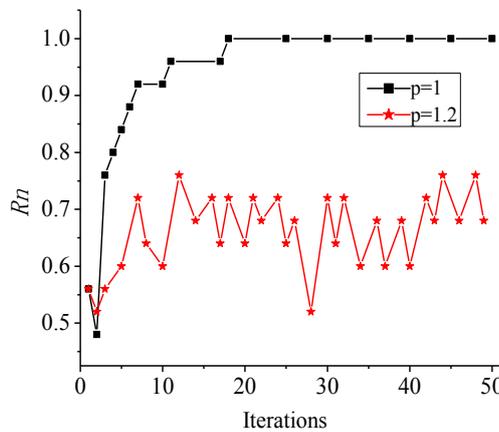
Taking square plate as example, the size is 100\*100mm, 8mm thick.

**Case 1** Variation of modal shape and convergence of interpolation function in different penalty factors

In this case, the plate has 10\*10 patches, and divided into 10\*10 elements. Respectively penalty factor take 1, 1.2 and 1.5, the variation of modal shape was depicted in Fig 3, the convergence of interpolation function was depicted in Fig 4.



**Fig 3.** variation of modal shape in different penalty factors



**Fig 4.** convergence of interpolation function in different penalty factors

In fig 3, it is seen that penalty factor is equal to 1, the change of model shape is gently; and then p is equal to 1.3, the model shape varied intensely, besides, the variation range of MAC is bigger. We can conclude that with the increase of p, the variation of model shape more drastic and the variation range of MAC will expand. In fig 4, it is seen that along with the increase of p, the quantity of middle value will increase. In conclusion, the value of p is closer to 1, the smaller the model shape change will be and the better the convergence of interpolation function.

**Case 2:** Comparison of optimization results in different penalty factor

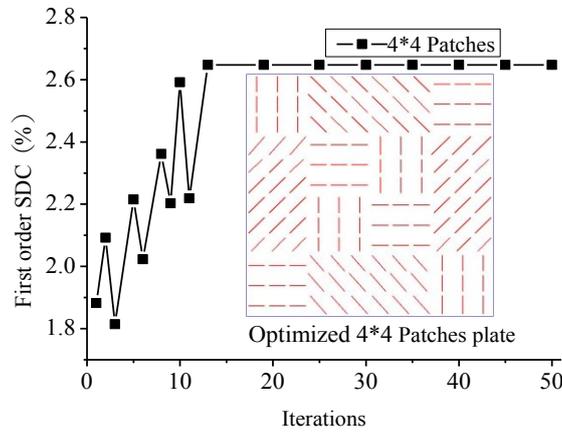
**Table 3.** comparison of optimization results in different penalty factor

p	iterations	objective
1	27	2.84%
1.2	21	3.19%
1.5	18	3.19%
1.8	31	3.19%
2	38	3.03%

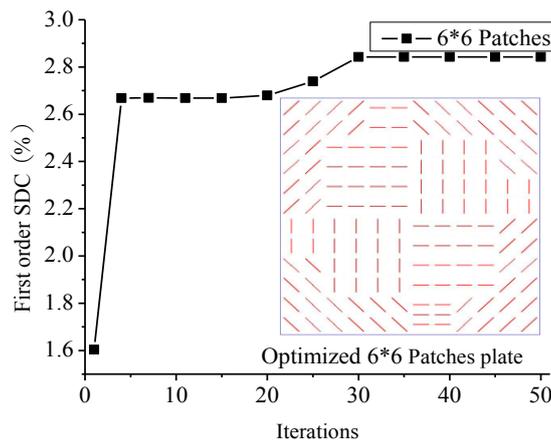
The results are presented in Table 4. The value of penalty factor has effect on value and convergence speed of objective function. When the value of penalty factor is smaller or larger, convergence speed is slow, and the solution is not optimal solution. When the penalty factor value between 1.2 and 1.8 can get the optimal solution, but the convergence speed is different. Therefore,

penalty factor values should be chosen carefully to ensure good optimization results and reduce the amount of calculation.

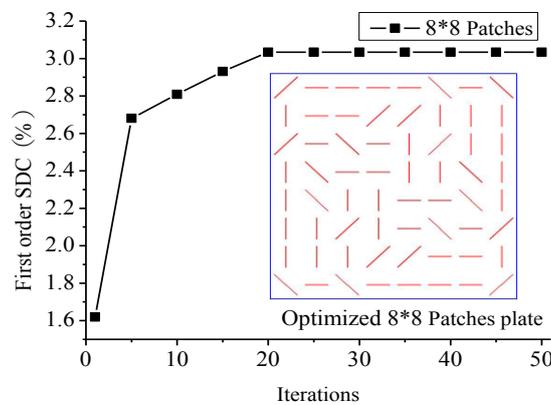
**Case 3:** Comparison of first order SDC optimization results in different number patches  
 For comparison, penalty factor take 1 in this case.



**Fig 5.** the result of 4\*4 patches plate



**Fig 6.** the result of 6\*6 patches plate



**Fig 7.** the result of 8\*8 patches plate

**Table 4.** Comparison of first order SDC optimization results in different number patches

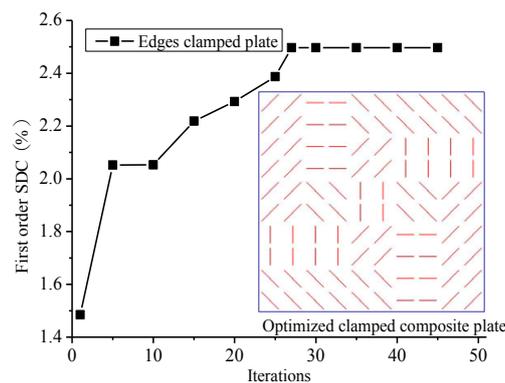
patches	objective (%)	iterations	R	Promotion
1*1	1.79	3	1	-
4*4	2.65	13	1	48%
6*6	2.84	28	1	58%
8*8	3.03	20	1	69%

The results are presented in Fig 5-7. The 4\*4 patches plate has lesser design variables, therefore it has highest convergence rate; the 8\*8 patches plate convergence rate is higher than 6\*6 patches plate, but the ply orientation of 8\*8 patches plate is disordered, we can deduce the 8\*8 block may haven't get the optimal value, this may be the reason why its convergence speed is faster than 6\*6 patches plate. By the way, for ordinary desktop with 6G memory and 2.8GHz clock speed, each iteration will take 0.70s, therefore, by introducing the element stiffness and damping matrices as optimization variables, a lot of integral operation can be saved, thus reducing the computation time greatly.

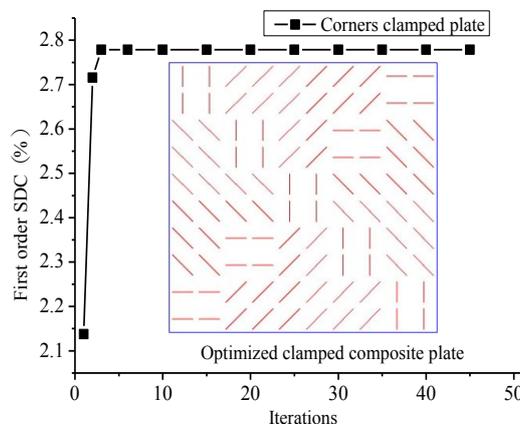
As seen in Eq.(6), the essence of SDC optimization problem is the multi-objective optimization problem with two objectives including maximize dissipated energy  $\Delta U$  and minimize strain energy  $U$ . Therefore, the ply orientations of plates have two laws, On the edge of the plate, the fibre should be surround by the center of plate, thus, this can make the strain energy of the plate minimal; in the center of the plate, fibre direction is given as  $0^\circ$  and  $90^\circ$  to get maximized dissipated energy.

**Case 4:** Comparison of first order SDC optimization results in different boundary conditions

Penalty factor take 1 in this case. Comparison of first order SDC optimization results in four corners clamped and four edges clamped plate.



**Fig 8.** results of 5\*5 patches with four edges clamped plate



**Fig 9.** the result of 5\*5 patches with four corners clamped plate

As shown in Fig. 8 and 9, owing to the higher stiffness of four edges clamped plate, the optimization result of four edges clamped plate is lower than four corners clamped plate. On the other

hand, the convergence rate of four corners clamped plate is faster than four edges clamped plate. This suggests that the convergence rate is associated with boundary conditions. Comparison of the two layer angle result can be found, on the edge of the plate, the fibre orientation is different, but in center of plate the ply orientation results are basically the same. The reason for this is that the boundary conditions have a little impact on center of plate.

## 5. Conclusions

Damping prediction model is developed based on modal dissipative energy principle in this paper, the first order SDC was solved by FEM. Then, instead of fibre angles, the element stiffness and damping matrixes are translated to be design variables on the basis of novel Discrete Material Optimization (DMO) formulation, thus reducing the computation time greatly.

The numerical example shows: 1. The value of penalty factor is closer to 1, the smaller the model shape change and the better the convergence of interpolation function will be, when penalty factor is equal to 1, the model shape can be considered as a constant; 2. Compared with classic laminates, with the increase of patches number, the SDC will increase 48%—69% by optimization design; 3. By introducing the element stiffness and damping matrices as optimization variables, a lot of integral operations can be saved, thus reducing the computation time greatly; 4. With four edges clamped boundary conditions, the fibre should be surround by the center of plate to maximize the first order SDC, On the edge of the plate, in the center of the plate, fibre direction is given as  $0^\circ$  and  $90^\circ$ ; 5. Owing to resulting higher stiffness, the four corners clamped optimum solution is bigger than four edges clamped plate.

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552

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