# Combustion Control System Design of Diesel Engine via ASPR based Output Feedback Control Strategy with a PFC

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**Abstract.** In this paper, a design method of an output feedback control system with a simple feedforward input for a combustion model of diesel engine will be proposed based on the almost strictly positive real-ness (ASPR-ness) of the controlled system for a combustion control of diesel engines. A parallel feedforward compensator (PFC) design scheme which renders the resulting augmented controlled system ASPR will also be proposed in order to design a stable output feedback control system for the considered combustion model. The effectiveness of our proposed method will be confirmed through numerical simulations.

## 1. Introduction

Regulation of exhaust gas of motor vehicles has tightened due to increase of public eco-awareness including energy conservation and emission-reduction of $CO_2$, and thus technical renovation in the field of automotive technology has been advancing. Especially developments of HV vehicles and electric vehicles have been actively done. However, since it is predicted that the demand of automobiles with an engine (internal combustion engine) will not quickly reduce for a while, the further development of engines is also continuously expected. Therefore, the innovative technical developments of engine for automobiles still attract a great deal of attention, and one of the innovations could be control technologies for internal combustion engines.

As is well known, conventional engine control was based on a model based control with a feedforward control input only. However, developing the model based control, a number of experiments were required in order to obtain in-depth data for making an accurate engine model and this requires a lot of time and effort. Furthermore, since engines have been highly developed during recent decade, more advanced and innovative control strategy with a feedback loop, which is robust with respect to changes of environment and disturbances during the operation, has been strongly expected in order to achieve high-performance engine control. With this in mind, a simple model, so called 'University of Tokyo Discrete Model (or TD Model)', which can be developed through a few experimental data, has been provided for designing a feedback control of engine combustion by a research group in University of Tokyo [1]. This model was proposed for on-board control of engine.

In this paper, we propose an output feedback control system design scheme for engine combustion control. Unlike the conventional engine control issue, which deals with the control
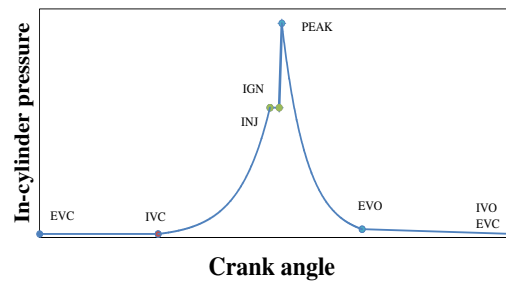
**Figure 1.** Representative points of the discrete-time model

problem of engine intake and exhaust systems [2–5], the problem dealt with in this paper is a direct combustion control of engine. An output feedback control based on system's ASPR-ness will be proposed for a TD Model based SISO system by considering the fuel injection timing as a control input and the peak pressure as an output of the system. The system is said to be ASPR (Almost Strictly Positive Real) if there exists a static output feedback such that the resulting closed loop system is strictly positive real (SPR) [6–8]. The engine model based on TD Model is ASPR. Unfortunately however, since the discrete ASPR model has to have a direct feedthrough term of the input, one cannot directly design a control system for original ASPR engine model due to causality problem. We will introduce a pre-compensator and a parallel feedforward compensator (PFC) [9, 10] to solve the causality problem and making an ASPR augmented controlled system for designing a stable output feedback control of engine combustion. A simple feedforward control design strategy will also be proposed for eliminating the affect from the changes of external inputs including fuel injection quantity, boost pressure and EGR changes. Finally, the effectiveness of the proposed method will be confirmed through numerical simulations based on the TD Model.

## 2. Engine combustion model

We first introduce a discrete dynamics model of diesel combustion (University of Tokyo Discrete model: TD Model) which has been provided by Yamasaki et al [1]. TD Model is a simplified combustion model which is developed for a model-based control and thus it is a model for on-board application of control methods.

The TD Model dealt with in this paper is the one to a single-stage injection. This model is a nonlinear model with 2 states, 4 external inputs and 3 measurable outputs. The concept to develop TD Model is to discretize a single diesel cycle at several specific points. In the TDM considered in this paper, the following six specific points: EVC (exhaust valve closed timing), IVC (intake valve closed timing), INJ (injection timing), IGN (ignition timing), PEAK (peak pressure timing) and EVO (exhaust valve opened timing) are considered as shown in Fig. 2, and calculating states on each points, a discrete combustion model of diesel engine is obtained with the single cycle period as a sampling period.

$$
\begin{bmatrix} n_{o_2,RG,k+1} \\ T_{RG,k+1} \end{bmatrix} = \boldsymbol{f}(T_{RG,k}, n_{o_2,k}, Q_{fuel,k}, \theta_{INJ,k}, P_{boost,k}, \psi_k)
$$
$$
\begin{bmatrix} W_k \\ P_{PEAK,k} \\ \theta_{PEAK,k} \end{bmatrix} = \boldsymbol{g}(T_{RG,k}, n_{o_2,k}, Q_{fuel,k}, \theta_{INJ,k}, P_{boost,k}, \psi_k)
\tag{1}
$$

Where, $T_{RG,k}$: temperature of residual gas at EVC [K] and $n_{o_2,RG,k}$: Oxgen mole of residual gas at EVC [mol] are states of the engine combustion system, and $W_k$: indicated output [kW],

**Table 1.** State variables, external inputs and outputs of the discrete-time model

| | | |
|---|---|---|
| $x_k$: State variable | $T_{RG,k}$ | Temperature of residual gas at EVC [K] |
| | $n_{O_2,RG,k}$ | Oxygen mole of residual gas at EVC [mol] |
| Input | $Q_{fuel,k}$ | Fuel injection quantity [mm³] |
| | $\theta_{INJ,k}$ | Fuel injection timing [degATDC] |
| | $P_{boost,k}$ | Boost pressure [kPa] |
| | $\psi_k$ | External EGR ratio [-] |
| $y_k$: Output | $W_k$ | Indicated output [kW] |
| | $P_{PEAK,k}$ | Peak pressure [MPa] |
| | $\theta_{PEAK,k}$ | Peak pressure timing [degATDC] |

$P_{PEAK,k}$: peak pressure [MPa] and $\theta_{PEAK,k}$: peak pressure timing [deg ATDC] are the output of the system. As external inputs, $Q_{fuel,k}$: Fuel injection quantity [mm³], $\theta_{INJ,k}$: fuel injection timing [deg ATDC], $P_{boost,k}$: boost pressure [kPa] and $\psi_k$: external EGR ratio are considered (See Table 1).

## 3. Problem statement

In order to refer the TD Model for controller design, we first consider a linear approximation of the model in (1) at a general operating point as follows:

$$
\begin{aligned}
\left[ \begin{array}{c} n_{o_2,RG,k+1} \\ T_{RG,k+1} \end{array} \right] &= A \left[ \begin{array}{c} n_{o_2,RG,k} \\ T_{RG,k} \end{array} \right] + B \left[ \begin{array}{c} Q_{fuel,k} \\ \theta_{INJ,k} \\ P_{boost,k} \end{array} \right] \\
\left[ \begin{array}{c} W_k \\ P_{PEAK,k} \\ \theta_{PEAK,k} \end{array} \right] &= C \left[ \begin{array}{c} n_{o_2,RG,k} \\ T_{RG,k} \end{array} \right] + D \left[ \begin{array}{c} Q_{fuel,k} \\ \theta_{INJ,k} \\ P_{boost,k} \end{array} \right]
\end{aligned}
\tag{2}
$$

where, $(A, B, C, D)$ are appropriate system matrices. Due to higher nonlinearities of EGR, we omitted the EGR ratio in the linear approximation. Therefor, we have a 3-inputs/3-outputs system.

In this paper, we regard this engine combustion system as a SISO system with $\theta_{INJ,k}$ (fuel injection timing) as the control input and $P_{PEAK,k}$ (peak pressure) as the output, and thus we have the following representation of the system.

$$
\begin{aligned}
\left[ \begin{array}{c} n_{o_2,RG,k+1} \\ T_{RG,k+1} \end{array} \right] &= A \left[ \begin{array}{c} n_{o_2,RG,k} \\ T_{RG,k} \end{array} \right] + \left[ \begin{array}{c} b_{12} \\ b_{22} \end{array} \right] \theta_{INJ,k} + \left[ \begin{array}{cc} b_{11} & b_{13} \\ b_{21} & b_{23} \end{array} \right] \left[ \begin{array}{c} Q_{fuel,k} \\ P_{boost,k} \end{array} \right] \\
P_{PEAK,k} &= \left[ \begin{array}{cc} c_{21} & c_{22} \end{array} \right] \left[ \begin{array}{c} n_{o_2,RG,k} \\ T_{RG,k} \end{array} \right] + d_{22} \theta_{INJ,k} + \left[ \begin{array}{cc} d_{21} & d_{23} \end{array} \right] \left[ \begin{array}{c} Q_{fuel,k} \\ P_{boost,k} \end{array} \right]
\end{aligned}
\tag{3}
$$

where, $b_{ij}, c_{ij}, d_{ij}$ are elements of each system matrix $B, C, D$.

Consequently, the engine combustion system with $\theta_{INJ,k}$ (fuel injection timing) as the control input and $P_{PEAK,k}$ (peak pressure) as the output can be expressed by the following form with disturbances.

$$
\begin{aligned}
\boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + \boldsymbol{b}u(k) + \boldsymbol{d}_1(k) \\
y(k) &= \boldsymbol{c}^T \boldsymbol{x}(k) + du(k) + d_2(k)
\end{aligned}
\tag{4}
$$

where

$$\boldsymbol{x}(k) = \begin{bmatrix} n_{o_2,RG,k} \\ T_{RG,k} \end{bmatrix}, \quad y(k) = P_{PEAK,k}, \quad u(k) = \theta_{INJ,k},$$

$$\boldsymbol{b} = \begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix}, \quad \boldsymbol{c}^T = \begin{bmatrix} c_{21} & c_{22} \end{bmatrix}, \quad d = d_{22},$$

$$\boldsymbol{d}_1(k) = \begin{bmatrix} b_{11} & b_{13} \\ b_{21} & b_{23} \end{bmatrix} \begin{bmatrix} Q_{fuel,k} \\ P_{boost,k} \end{bmatrix}, \quad d_2(k) = \begin{bmatrix} d_{21} & d_{23} \end{bmatrix} \begin{bmatrix} Q_{fuel,k} \\ P_{boost,k} \end{bmatrix}$$

Furthermore, we impose the following assumptions.

**Assumption 1** *The system given in (4) is ASPR.*

**Assumption 2** *$\boldsymbol{d}_1(k)$ can be represented as*

$$\boldsymbol{d}_1(k) = \boldsymbol{b}\frac{d_2(k)}{d} \tag{5}$$

**Assumption 3** *Considering a reference signal $y_r(k)$, which the output $y(k) := P_{PEAK,k}$ required to track, there exists an ideal input $u^*(k)$ (ideal fuel injection timing: $\theta^*_{INJ,k}$) such that the perfect output tracking is attained. That is, there exists an ideal state $\boldsymbol{x}^*(k)$ and ideal input $u^*(k)$ such that*

$$\begin{aligned} \boldsymbol{x}^*(k+1) &= A\boldsymbol{x}^*(k) + \boldsymbol{b}u^*(k) \\ y^*(k) &= y_r(k) = \boldsymbol{c}^T\boldsymbol{x}^*(k) + du^*(k) \end{aligned} \tag{6}$$

*is satisfied.*

**Remark 1** *The sufficient condition for the system be ASPR has been provided as follows [6]:*

*(1) The system has the relative degree of 0.*

*(2) The system is minimum-phase.*

*(3) The high frequency gain of the system is positive.*

*The considered engine system satisfies the above mentioned ASPR conditions.*

The objective here is to design an output feedback controller based on the ASPR properties of the system for engine combustion systems under given assumptions.

## 4. Combustion control system design

As mentioned in Remark 1, since the considered system is ASPR, if the system's output $y(k)$ is available at the present time instance, one can design a feedback controller with a feedforward input as follows.

$$u(k) = -\theta^* e(k) + u^*(k) + v_d^*(k) \tag{7}$$

where $e(k) := y(k) - y_r(k)$ is the output tracking error, and $\theta^*$ is the ideal feedback gain which renders the SPR closed-loop system. $v_d^*(k)$ is the ideal forward input for disturbance rejection which is given by

$$v_d^*(k) = -\frac{1}{d}d_2(k) = -\begin{bmatrix} \frac{d_{21}}{d} & \frac{d_{23}}{d} \end{bmatrix} \begin{bmatrix} Q_{fuel,k} \\ P_{boost,k} \end{bmatrix} \tag{8}$$

Then it is easy to confirm that one can attain the output tracking such that $\lim_{k\to\infty} e(k) = 0$. Unfortunately, however, since the system has its own direct feedthrough term of the input, we can not directly use the output $y(k)$ for the controller design due to causality problem.

Therefore we consider introducing a pre-compensator and a parallel feedforward compensator (PFC) for remaking the system ASPR again in order to design a stable output feedback based output tracking control system without causality problem.
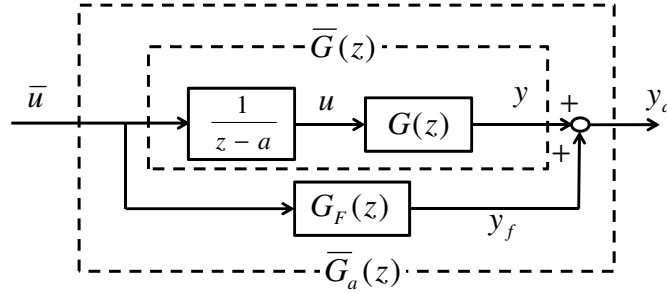
**Figure 2.** Augmented system with a PFC

*4.1. Introduction of pre-compensator and PFC*

Let's denote the transfer function of the considered system by $G(s)$. This is a proper transfer function with the relative degree of 0.

For this system, we consider introducing an integral type pre-compensator $\frac{1}{z-1}$ and then designing a PFC which renders the resulting augmented system ASPR for the expanded system with the pre-compensator.

To this end, firstly introduce a virtual stable pre-compensator $\frac{1}{z-a}$, $|a| < 1$ (See Fig. 2). The resulting expanded system can be represented by

$$\bar{G}(z) = \frac{1}{z-a}G(z), \ |a| < 1 \tag{9}$$

For this expanded system $\bar{G}(z)$, we design a PFC: $G_F(s)$ based on the model based design [10] scheme as follows.

$$G_F(z) = G_{ASPR}(z) - \bar{G}(z) \tag{10}$$

where $G_{ASPR}(z)$ is a designed ASPR model given by controller designer.

As mentioned in Remark 1, the considered engine system is ASPR itself. So, we consider designing the ASPR model as follows by utilizing the engine model $G(z)$:

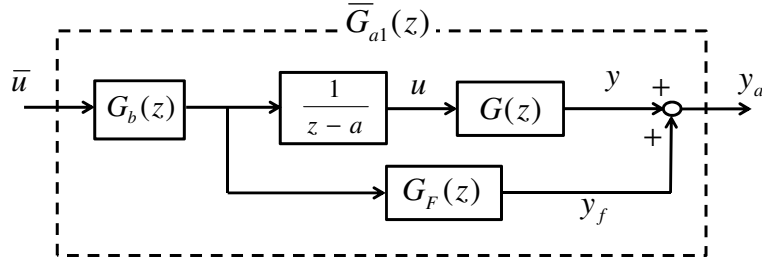$$G_{ASPR}(z) = \frac{1}{1-a}G(z) \tag{11}$$

In this case, the PFC is obtained by

$$\begin{aligned} G_F(z) &= \frac{1}{1-a}G(z) - \bar{G}(z) \\ &= \frac{1}{1-a}G(z) - \frac{1}{z-a}G(z) \\ &= \frac{z-1}{(1-a)(z-a)}G(z) \end{aligned} \tag{12}$$
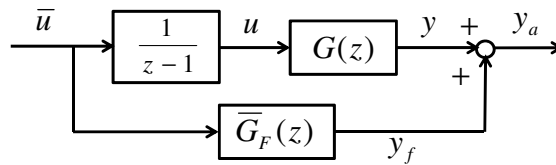
The resulting augmented system shown in Fig. 2:

$$\bar{G}_a(z) = \bar{G}(z) + G_F(z) = G_{ASPR}(z) = \frac{1}{1-a}G(z) \tag{13}$$

is ASPR by definition.

**Figure 3.** Introduction of second pre-compensator



**Figure 4.** Equivalent augmented system

For the obtained ASPR augmented system $\bar{G}_a(z)$, we further introduce a second virtual pre-compensator:

$$G_b(z) = \frac{z-a}{z-1} \tag{14}$$

The obtained expanded system can be expressed by (See Fig. 3)

$$
\begin{aligned}
\bar{G}_{a1}(z) &= \bar{G}_a(z)G_b(z) \\
&= \left(\bar{G}(z) + G_F(z)\right)\frac{z-a}{z-1} \\
&= \frac{1}{z-1}G(z) + \frac{z-a}{z-1}G_F(z)
\end{aligned}
\tag{15}
$$

The equivalent augmented system can be expressed as shown in Fig. 4, where

$$
\begin{aligned}
\bar{G}_F(z) &= \frac{z-a}{z-1}G_F(z) \\
&= \frac{z-a}{z-1}\cdot\frac{z-1}{(1-a)(z-a)}G(z) \\
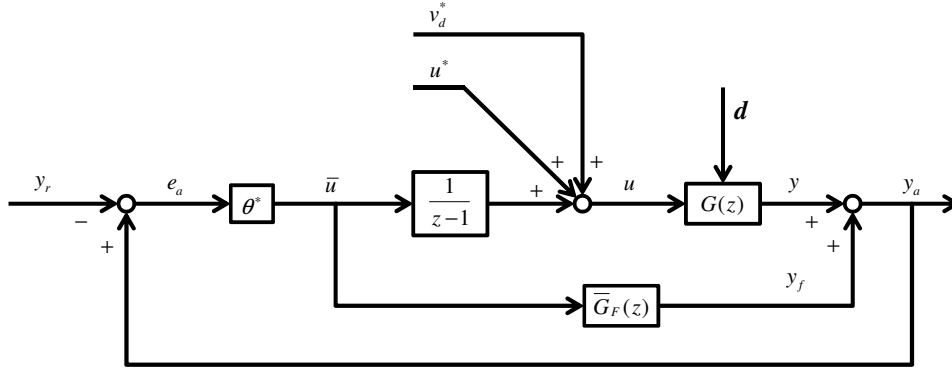&= \frac{1}{1-a}G(z)
\end{aligned}
\tag{16}
$$

Thus, by designing PFC as

$$\bar{G}_F(z) = \frac{1}{1-a}G(z) \tag{17}$$

for the expanded system with the pre-compensator, the resulting augmented system:

$$G_a(z) = \frac{1}{z-1}G(z) + \bar{G}_F(z) \tag{18}$$

is ASPR. Finally, for the expanded system $\frac{1}{z-1}G(z)$ with the pre-compensator $\frac{1}{z-1}$, the PFC can be designed by $\bar{G}_F(z) = \frac{1}{1-a}G(z)$ in order to render the resulting augmented system ASPR.

**Figure 5.** Output feedback control system

*4.2. Control system design*

For the designed augmented system, we consider designing ASPR based stable output feedback control with simple feedforward control inputs.

Since the designed augmented system is ASPR, there exists a static output feedback with a gain $\theta^* \geq \theta_{\min}$ such that that the resulting closed-loop is SPR and thus it is stable. Using this feedback gain $\theta^*$, we design tracking control system as follows (See Fig. 5).

$$u(k) = \frac{1}{z-1}[\bar{u}(k)] + u^*(k) + v_d^*(k) \tag{19}$$

$$\bar{u}(k) = -\theta^* e_a(k) \tag{20}$$

$$e_a(k) = y_a(k) - y_r(k) = y(k) + y_f(k) - y_r(k) \tag{21}$$

$$y_f(k) = \bar{G}_F(z)[\bar{u}(k)] \tag{22}$$

where $u^*(k)$ and $v_d^*(k)$ are ideal feedforward input given in (6) and (8) for output following and disturbance rejection, respectively. The notation $W(z)[q(k)]$ denotes the output of the system with a transfer function $W(s)$ and input $q(k)$. Thus $y_f(k)$ is the PFC output. $y_a(k)$ is the augmented system's output and then $e_a(k)$ is the tracking error of the augmented system.

It is easily confirmed that the error system can be stabilized with a sufficient large feedback gain $\theta^*$ and thus we have $\lim_{k\to\infty} e_a(k) = 0$ and $\lim_{k\to\infty} y_f(k) = 0$ for a stable PFC. Consequently, the tracking error of the original system $e(k) = y(k) - y_r(k)$ also converges to zero.

**Remark 2** *It should be noted that since the augmented system has direct input feedthrough term, one can directory implement the control input given in (20). However, fortunately, since the feedback control signal can be represented by*

$$\bar{u}(k) = -\theta^* \{ y(k) + \boldsymbol{c}_f^T \boldsymbol{x}_f(k) + d_f \bar{u}(k) - y_r(k) \} \tag{23}$$

*from (20) and (21), where $\boldsymbol{x}_f(k)$ is a state vector of the PFC $\bar{G}_f(z)$ with the input $\bar{u}(k)$ so that a realization of $\bar{G}_f(z)$ can be represented by*

$$\begin{aligned} \boldsymbol{x}_f(k+1) &= A_f \boldsymbol{x}_f(k) + \boldsymbol{b}_f \bar{u}(k) \\ y_f(k) &= \boldsymbol{c}_f^T \boldsymbol{x}_f(k) + d_f \bar{u}(k) \ , \end{aligned} \tag{24}$$

*we have the following equivalent feedback control signal which can be constructed by available signals.*

$$\bar{u}(k) = -\tilde{\theta}^* \bar{e}_a(k) \tag{25}$$

$$\tilde{\theta}^* = \frac{\theta^*}{1 + d_f \theta^*} \tag{26}$$

$$\bar{e}_a(k) = y(k) + \boldsymbol{c}_f^T \boldsymbol{x}_f(k) - y_r(k) \tag{27}$$

**Remark 3** *In practical sense, it is difficult to obtain ideal feedforward input $u^*(k)$ and $v_d^*(k)$. In this paper, we simply approximate these values from simple step response test and adjust the errors by the feedback control based on systems ASPR-ness.*

## 5. Validation through numerical simulations

We confirmed the effectiveness of the proposed method through numerical simulations on the nonlinear TD Model given in (1). We supposed in the simulation that the following linear approximated model of the considered combustion model is known.

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + \boldsymbol{b}u(k) + \boldsymbol{d}_1(k) \\ y(k) &= \boldsymbol{c}^T \boldsymbol{x}(k) + du(k) + d_2(k) \end{aligned} \tag{28}$$

with

$$A = \begin{bmatrix} 2.990 \times 10^{-2} & -2.028 \times 10^3 \\ 3.286 \times 10^{-9} & 3.910 \times 10^{-2} \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} 1.595 \\ -5.400 \times 10^{-7} \end{bmatrix}, \; \boldsymbol{c} = \begin{bmatrix} 8.687 \times 10^{-4} \\ 1.812 \times 10^2 \end{bmatrix}$$

$$d = d_{22} = -1.423 \times 10^{-1}, \; d_{21} = 9.225 \times 10^{-2}, \; d_{23} = 4.317 \times 10^{-2}$$

In order to design a stable and implementable output feedback control system for the combustion system, we design a PFC given as in Fig. 4 by

$$\bar{G}_{F_1} = \frac{1}{1-a} G(z)$$

where $G(z)$ is the transfer function of the model (28).

For the obtained augmented system, the control input is designed as in (19) – (22) and (25) with

$$u^*(k) = G(z)^{-1}[y_r(k)], \quad v_d^*(k) = - \begin{bmatrix} \frac{d_{21}}{d} & \frac{d_{23}}{d} \end{bmatrix} \begin{bmatrix} Q_{fuel,k} \\ P_{boost,k} \end{bmatrix}$$
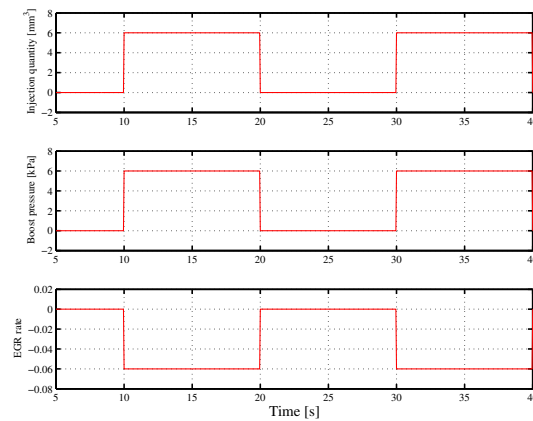
and

$$a = 0.5, \; \theta^* = 1.0 \times 10^5$$

In this simulation, we supposed that Fuel injection quantity ($Q_{fuel,k}$), boost pressure ($P_{boost,k}$) and external EGR ratio ($\psi_k$), which are considered as disturbances, are changing as in Fig. 6 during operation.

Fig. 7, Fig. 8 and Fig. 9 show simulation results. Fig. 7 is a result only with the output feedback. The output quickly converges to reference signal and thus good result is obtained. However, in the transient state, overshoot behavior is appeared. Fig. 8 is a result only with the feedforward inputs. Since the practical combustion system is a nonlinear system, the feedforward input must have a miss match using the approximated linear model information. Fig. 9 is a result with the proposed method constructed with feedback and feedforward controls. Without overshoot and offset error, the result shows pretty good control performance even if the controller was designed using a simple approximated linear model, and it also shows robust performance for unconsidered disturbances such as EGR rate change.

**Figure 6.** Disturbances: Fuel injection quantity ($Q_{fuel,k}$), boost pressure ($P_{boost,k}$) and external EGR ratio ($\psi_k$)
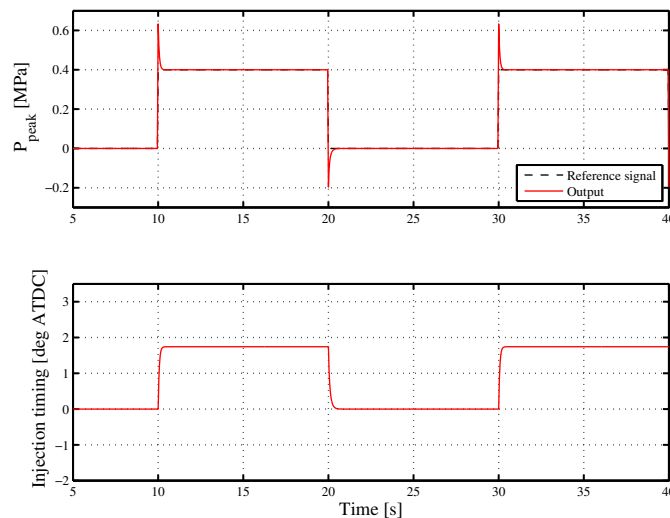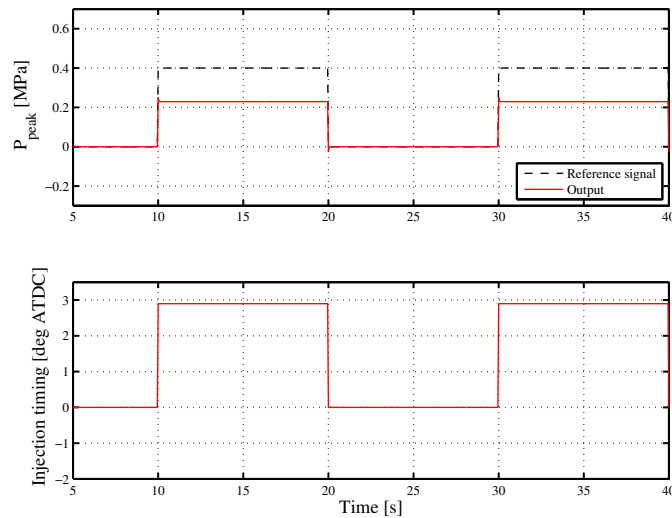


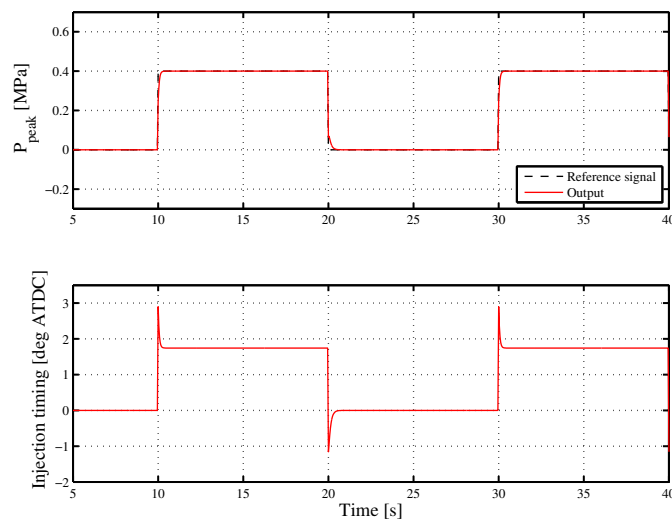**Figure 7.** Simulation result by a feedback input only

## 6. Conclusions

In this paper, a direct combustion control of engine was considered and an output feedback based control system design scheme for engine combustion control was proposed. The proposed method is an output feedback control based on system's ASPRness. For a TD Model based SISO system, in which the fuel injection timing is considered as a control input and the peak pressure is considered as an output of the system, a stable and simple output feedback control system design scheme for engine combustion control was presented and a simple feedforward control design strategy was also proposed for eliminating the affect from the disturbances. The effectiveness of the proposed method was confirmed through numerical simulations based on the TD Model.

## Acknowledgements

**Figure 8.** Simulation result by a feedforward input only



**Figure 9.** Simulation result by proposed method with feedback and feedforward inputs

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