Fatigue Damage Spectrum calculation in a Mission Synthesis procedure for Sine-on-Random excitations

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Abstract. In many real-life environments, certain mechanical and electronic components may be subjected to Sine-on-Random vibrations, i.e. excitations composed of random vibrations superimposed on deterministic (sinusoidal) contributions, in particular sine tones due to some rotating parts of the system (e.g. helicopters, engine-mounted components, …). These components must be designed to withstand the fatigue damage induced by the "composed" vibration environment, and qualification tests are advisable for the most critical ones. In the case of an accelerated qualification test, a proper test tailoring which starts from the real environment (measured vibration signals) and which preserves not only the accumulated fatigue damage but also the "nature" of the excitation (i.e. sinusoidal components plus random process) is important to obtain reliable results. In this paper, the classic time domain approach is taken as a reference for the comparison of different methods for the Fatigue Damage Spectrum (FDS) calculation in case of Sine-on-Random vibration environments. Then, a methodology to compute a Sine-on-Random specification based on a mission FDS is proposed.

1. Introduction

In most real-life environments, mechanical or electronic components are subjected to vibrations. Some of these components may have to pass qualification tests to verify that they can withstand the fatigue damage they will encounter during their operational life. In order to conduct a reliable test, the environmental excitations can be taken as a reference to synthesize the test profile: this procedure is referred to as "test tailoring". Due to cost and feasibility reasons, accelerated qualification tests are usually performed. In this case, the duration of the original excitation which acts on the component for its entire life-cycle, typically hundreds or thousands of hours, has to be reduced.

In particular, the "Mission Synthesis" procedure permits to quantify the induced damage of the environmental vibration and synthesize a new profile with a reduced duration, but the same amount of induced damage [1]. In order to focus on the damage potential associated with a vibratory excitation, a generic component is represented by a series of linear Single Degree of Freedom (SDOF) systems, with a fixed damping ratio and the natural frequency that varies in the range of the component frequencies of interest. It is assumed that if two dynamic excitations produce the same damage on the SDOF linear system taken as reference then they produce the same damage also on the real component under test. Under the assumption of three main hypotheses (1. stress proportional to the relative displacement between mass and base of the SDOF system; 2. Wöhler's curve and Basquin's law

$N \sigma^b = C$ where N is the number of cycles to failure under stress of amplitude σ, whereas b and C are characteristic constants of the material; 3. Miner's rule for the linear damage accumulation) this simplification permits to reduce the problem of the damage quantification in finding the response of a linear SDOF system. To this aim a frequency-domain function, i.e. the so-called Fatigue Damage Spectrum (FDS), is defined to quantify the fatigue damage. When a new profile with the same amount of damage and a reduced duration is required for accelerated laboratory tests, it can be synthesized starting from environmental excitation by maintaining the same FDS.

In case the original excitation has random characteristics with a Gaussian distribution of its values, the procedure is well-known [1, 2] and a Power Spectral Density (PSD) can be obtained as a test profile which closely represents the original excitation (as an alternative, a purely deterministic synthesis in the form of a sine sweep is also possible). However, in a number of cases, the vibration does not follow a Gaussian distribution. In particular when a rotating part is present in the system (e.g. helicopters, engine-mounted components, …), typically the excitations assume Sine-on-Random characteristics. Deterministic components in the form of sinusoids, due to the rotors in the system, are superimposed on a random excitation. In this case, the value distribution is not Gaussian, due to the presence of the sine tones, and a synthesized PSD (possibly with a reduced duration) has not the adequate characteristics to properly represent the original excitation in laboratory (accelerated) tests. Indeed a proper test tailoring should not only preserve the accumulated fatigue damage, but also the "nature" of the excitation (in this case the sinusoidal components superimposed on the random process) in order to obtain reliable results. Thus, in case of Sine-on-Random environments, a Sine-on-Random specification is supposed to better represent the original excitation compared to a purely random profile.

In this work, the Mission Synthesis procedure is applied in the case of Sine-on-Random vibrations. The classic time-domain approach is taken as a reference for the comparison of different methods for the FDS calculation in presence of Sine-on-Random vibrations [1, 3]. Then, a methodology to compute a Sine-on-Random specification based on a mission FDS is presented. This is a novelty, since in the literature different methods to obtain a PSD specification from a reference Sine-on-Random profile are available [4, 5], but a way to synthesize a Sine-on-Random specification from a reference FDS is missing.

The paper is organized as follows: Section 2 compares the different frequency-domain approaches for the FDS calculation in case of Sine-on-Random vibrations, Section 3 presents a new method for the synthesis of a Sine-on-Random profile starting from a reference FDS and its advantages are discussed in Section 4. Section 5 reports some concluding remarks.

2. Fatigue Damage Spectra (FDS) calculation

In this paragraph, different frequency-domain approaches for the evaluation of the fatigue damage in the case of Sine-on-Random excitations are compared, investigating the pros and cons of each one. There is a need for a different method than the time-domain approach (which can process any signal) due to the long time necessary for the calculations in the case of very long signals. Nevertheless, due to its reliability, the time-domain method is taken as reference to evaluate the damage estimation of the frequency-domain methods.

2.1. Sufficiently-spaced sinusoids

If the frequencies of the sine tones are "sufficiently spaced", a simplified method is available. In fact, it is possible to treat each sinusoid independently [1].

In a first instance, the simple case of a single sinusoid superimposed on a random excitation is considered. The signal can be written as:

$$I(t) = B \cos(2\pi f_s t + \varphi) + r(t) \tag{1}$$

where B, f_s and φ are the amplitude, frequency and phase of the sinusoid and where $r(t)$ is the random signal.

If the values of the excitation have a Gaussian distribution, under the narrowband response assumption, the peaks of the response follow a Rayleigh distribution [6]. Such a stochastic approach permits to immediately predict the response relative displacement peaks and thus (under the three assumptions mentioned in Section 1) to estimate the fatigue damage avoiding the time-consuming calculations of the time-domain method. The idea is to apply a similar statistical procedure also in the case of Sine-on-Random excitation but, due to the presence of the sinusoid, the input signal does not follow a Gaussian distribution, so that the corresponding peaks distribution must first be found.

The component is represented with a series of SDOF systems, each one characterized by its natural frequency f_n and quality factor Q ($Q = 1/(2\varsigma)$, where ς is the damping factor). The first step of the procedure is the calculation of the relative displacement response of the system. If the random signal of the excitation is represented by a PSD of amplitude $G_{\ddot{r}}(f_n)$ and the sinusoid by the amplitude B in the form of an acceleration, the corresponding responses can be characterized by means of the following expressions [1, 2]:

$$z_{a_{rms}} = \sqrt{\frac{Q \cdot G_{\ddot{r}}(f_n)}{4 \cdot (2\pi f_n)^3}} \tag{2}$$

$$z_s = \frac{B}{(2\pi f_n)^2 \sqrt{\left[1 - \left(\frac{f_s}{f_n}\right)^2\right]^2 + \left(\frac{f_s}{Q\, f_n}\right)^2}} \tag{3}$$

where $z_{a_{rms}}$ collects the root mean square (rms) values of the response of the SDOF systems to the random excitation and z_s represents the maximum values of the response to the sinusoidal component. It is worth stressing out that both $z_{a_{rms}}$ and z_s are functions of f_n. Consequently, $\dot{z}_{a_{rms}}$, $\ddot{z}_{a_{rms}}$ are the rms value functions of the relative velocity and acceleration random responses and $z_{s_{rms}}$, $\dot{z}_{s_{rms}}$, $\ddot{z}_{s_{rms}}$ are the rms value functions of the relative displacement, velocity and acceleration responses to the sinusoidal excitation.

The average number of positive peaks of the response per unit time $n_{p\,SoR}^+$ is defined as [1]:

$$n_{p\,SoR}^+ = \frac{1}{2\pi} \sqrt{\frac{\ddot{z}_{a_{rms}}^2 + \ddot{z}_{s_{rms}}^2}{\dot{z}_{a_{rms}}^2 + \dot{z}_{s_{rms}}^2}} \tag{4}$$

Defining $S(I)$ as the envelope of I (Eq. 1), the probability density function of this envelope was deduced by Rice [7]:

$$P(S) = \frac{S}{r_{rms}} J_0\left(\frac{S \cdot B}{r_{rms}}\right) e^{-\frac{S^2 + B^2}{2\, r_{rms}}} \tag{5}$$

$$\text{with: } J_0(x) = \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^{2n} \frac{1}{(n!)^2} \tag{6}$$

where r_{rms} is the rms value of the random signal and where J_0 is the Bessel function of the first kind of order zero.

The probability density of the envelope has the same form of the probability density of maxima [6, 11]. Due to the linearity of the SDOF system, the response to a Sine-on-Random excitation still has the Sine-on-Random characteristics, thus the expression can be used to estimate the peaks distribution of the response:

$$P\left(z_p\right) = P(S) = \frac{z_p}{z_{a_{rms}}} J_0\left(\frac{z_p \cdot z_s}{z_{a_{rms}}}\right) e^{-\frac{z_p{}^2 + z_s{}^2}{2\,z_{a_{rms}}}} \tag{7}$$

where z_p is the peak amplitude of the response.

Knowing the peaks distribution, it is now possible to exploit a statistical approach. In fact, it allows for a straightforward estimation of the Rainflow cycle histogram:

$$N\left(z_p\right) = P\left(z_p\right) n_p^+{}_{SoR}\, T \tag{8}$$

where T is the duration and $N\left(z_p\right)$ is the number of estimated cycles at the displacement amplitude z_p of the response.

Considering, according to the Mission Synthesis procedure, the Basquin's law for the fatigue damage calculation and the Miner's rule for the linear accumulation of the damage, the following expression for the fatigue damage D can be obtained:

$$D = \frac{K^b\, n_p^+{}_{SoR}\, T}{C} \int_0^\infty z_p^b\, P\left(z_p\right) dz_p \tag{9}$$

where b is the Basquin's coefficient (i.e. the parameter determining the Wöhler's curve slope), K and C are constants of the material. It is worth recalling that when $D = 1$ the component undergoes a fatigue failure. After the substitution of the peaks distribution, the fatigue damage can be expressed as:

$$D = \frac{K^b\, n_p^+{}_{SoR}\, T}{C} \int_0^\infty z_p^b\, \frac{z_p}{z_{a_{rms}}} J_0\left(\frac{z_p \cdot z_s}{z_{a_{rms}}}\right) e^{-\frac{z_p{}^2 + z_s{}^2}{2\,z_{a_{rms}}}} dz_p \tag{10}$$

In the particular case of a single sinusoid, the integral above can be arranged in the following form, giving the expression for the FDS calculation [1]:

$$FDS = D = \frac{K^b}{C}\, n_p^+{}_{SoR}\, T\left(\sqrt{2}\, z_{a_{rms}}\right)^b \Gamma\left(1 + \frac{b}{2}\right)\, {}_1F_1\left(-\frac{b}{2}, 1, -a_0^2\right) \tag{11}$$

$$with: a_0 = \frac{z_{s_{rms}}}{z_{a_{rms}}} \tag{12}$$

$$_1F_1(\propto, \delta, \pm \mathrm{x}) = \sum_{j=0}^\infty \frac{(\propto)_j\,(\pm x)^j}{(\delta)_j\, j!} \tag{13}$$

where $_1F_1$ is referred to as the hypergeometric function [8] and a_0 is the ratio between the sinusoidal and random responses.

In the case of multiple (N) sine tones, if their frequencies are sufficiently spaced, where "sufficiently spaced" depends on the amplitude of the sinusoids and the damping of the SDOF system [1], only the nearest sinusoid to the natural frequency of the SDOF system has relevance on the damage, while the other ones are negligible. In this case, the sinusoids can indeed be considered independently, so that after the calculations of the separate FDS functions of each sine tone plus the random signal, the complete spectrum is given by their envelope:

$$FDS_{total} = envelope\left(FDS_{Sine_1-on-Random}, FDS_{Sine_2-on-Random}, \dots, FDS_{Sine_N-on-Random}\right) \tag{14}$$

As an illustration, the method is applied to a Sine-on-Random signal composed by a flat PSD with amplitude $1\ (m/s^2)^2/Hz$ in the bandwidth 0–400 Hz and two sine tones with amplitude $20\ m/s^2$ and frequencies 20 Hz and 60 Hz. The comparison with the time-domain approach shows the effectiveness

of the envelope of the different FDS calculated with (Eq. 11) in case of "spaced" sinusoids (Fig. 1a). In case the frequencies of the sinusoids are close, the damage contribution of the close sine tones is coupled, i.e. it is influenced by both. As a consequence the method, which considers only one sinusoid at a time, would give an underestimation of the fatigue damage, as shown in Fig. 1b where the formulation is applied in the case that the two sinusoids are at 20 Hz and 30 Hz.
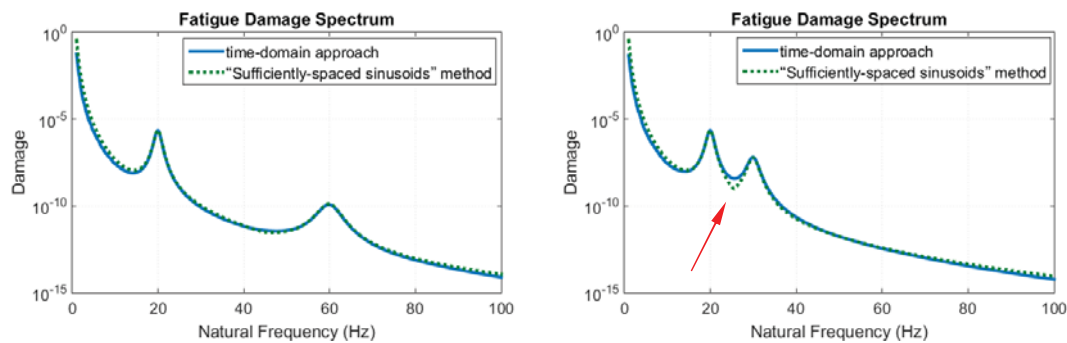

Figure 1: FDS calculation – Time-domain approach (blue) vs. "Sufficiently-spaced sinusoids" method (green), (a): sine tones at 20 and 60 Hz; (b) sine tones at 20 and 30 Hz

2.2. Closely-spaced sinusoids

In case of closely-spaced sinusoids, a more effective statistical approach, that takes into account all the sinusoids at the same time, was proposed [3]. The signal can be written as:

$$I(t) \; = \; r(t) + \sum_{i=1}^{N} B_i \, cos(2\pi f_i t + \varphi_i) \tag{15}$$

where B_i, f_i, φ_i are the amplitudes, frequencies ad phases of the N sinusoids and $r(t)$ is the random signal. For this kind of signal, the probability density function of the envelope $S(I)$ is, according to Rice [7]:

$$P(S) = S \int_{0}^{\infty} x \, J_0(S \, x) \, e^{-\frac{r_{rms}^{2} * x^2}{2}} \prod_{i=1}^{N} J_0(B_i \, x) \; dx \tag{16}$$

where r_{rms} is the rms value of $r(t)$ and J_0 is the Bessel function of the first kind of order zero. Thus, considering the correct estimation of the average number of positive peaks of the response per unit time $n_{p\,SoR}^{+}$ that takes into account all the sinusoidal responses $z_{s\,rms_i}$:

$$n_{p\,SoR}^{+} = \frac{1}{2\pi} \sqrt{\frac{\ddot{z}_{a\,rms}^{2} + \sum_{i=1}^{N} \ddot{z}_{s\,rms_i}^{2}}{\dot{z}_{a\,rms}^{2} + \sum_{i=1}^{N} \dot{z}_{s\,rms_i}^{2}}} \, , \tag{17}$$

it is possible to follow the same statistical procedure for the damage calculation, using the new probability density function as expression of the peak distribution of the response. This leads to the following FDS formulation:

$$FDS = \; D = \frac{K^b}{C} \, n_{p\,SoR}^{+} \, T \int_{0}^{\infty} z_p^{b+1} \int_{0}^{\infty} x \, e^{-\frac{z_{a\,rms}^{2} x^2}{2}} J_0(x \, z_p) \prod_{i=1}^{N} J_0(x \, z_{s_i}) \, dx \; dz_p \tag{18}$$

The comparison of this method with the previous one and the time domain approach applied to the example of Fig. 1b (with sine tones at 20 Hz and 30 Hz) shows that the expression in (Eq. 18) solves

the problem of the damage underestimation of the previous method in case of "closely-spaced" sinusoids (Fig. 2). The main drawback of this formulation is that a closed-form solution for the FDS expression is not available so that a numerical integration is required.
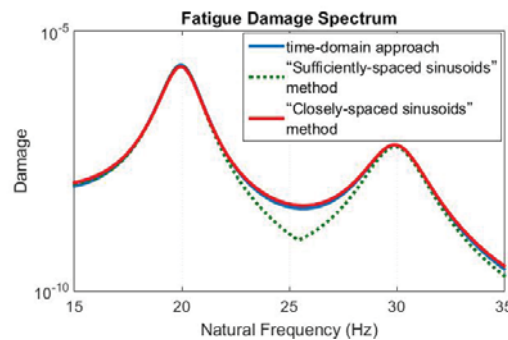

Figure 2: FDS calculation in case of closely-spaced sinusoids – Time-domain approach (blue), "Sufficiently-spaced sinusoids" method (green), "Closely-spaced sinusoids" method (red)

3. Sine-on-Random Profile synthesis

If a critical mechanical or electronic component is subjected to one of these Sine-on-Random excitations during its operational life, a qualification test may be necessary to verify its endurance to the induced fatigue damage. In order to conduct a qualification test, a new specification profile has to be synthesized. In particular, this profile has to be based on the real environment to be realistic, but at the same time its duration has to be limited to make the laboratory test feasible.

Assuming that the fatigue damage has to be preserved, the FDS of a certain excitation is calculated (with one of the described methods) and then targeted as the mission FDS to synthesize a new profile with the same damage and a reduced duration. However, the main objective of this work is to also preserve the "characteristics" of the environmental excitation: the synthesized profile should be a Sine-on-Random profile (which still matches the mission FDS).

In this section, a new method for the synthesis of a Sine-on-Random profile is compared with the standard PSD profile synthesis. A closed-form expression for the damage estimation in case of a single sine tone superimposed on a random vibration was shown in Section 2.1. Its application even in presence of multiple sinusoids is possible in case of sufficiently-spaced frequencies. The inversion of this closed-form formula can be used to synthesize a Sine-on-Random profile starting from a mission FDS.

Unfortunately, if the sine tones are not sufficiently-spaced this approach cannot be adopted due to the error in the damage estimation between closed sinusoids (Fig. 1b). On the other hand the method for the FDS computation presented in Section 2.2 (that solves the problem of damage underestimation) requires a numerical integration and is not reversible, so that it cannot be used for synthesis purposes.

3.1. Synthesis procedure

Starting from the expression for the FDS calculation of a (single sinusoid) Sine-on-Random excitation described in the previous paragraph, it is possible to obtain a procedure for a Sine-on-Random profile synthesis. A parameter which represents the ratio between the amplitude of the sinusoids and the random signal in the original environmental data is needed to this aim.

The average number of peaks per second $n_{p\,SoR}^+$ (Eq. 4) can be rewritten in function of the a_0 parameter (Eq. 12), that is the ratio between the sine and the random amplitudes of the response:

$$n_{p\,SoR}^+ = \frac{1}{2\pi}\frac{\ddot{z}_{rms}}{\dot{z}_{rms}} = \frac{1}{2\pi}\sqrt{\frac{\ddot{z}_{a\,rms}^2 + \ddot{z}_{s\,rms}^2}{\dot{z}_{a\,rms}^2 + \dot{z}_{s\,rms}^2}} = \sqrt{\frac{f_n^4 + f_s^4\,a_0^2}{f_n^2 + f_s^2\,a_0^2}} \tag{19}$$

Hence if a_0 is known or obtainable, it is possible to invert the procedure and obtain a synthesized Sine-on-Random profile with a specified duration T'.

In particular, the first step is to obtain the responses given by the new profile from the inversion of (Eq. 11):

$$z_{a_{rms}}' = \left[\frac{D}{\frac{K^b}{C} \, n^+_{p\,SoR} \, T' \left(\sqrt{2}\,\right)^b \Gamma\left(1 + \frac{b}{2}\right) \, {}_1F_1\left(-\frac{b}{2}, 1, -a_0^2\right)} \right]^{\frac{1}{b}} \quad (20)$$

$$z_{s_{rms}}' = a_0 \cdot z_{a_{rms}}' \quad (21)$$

where $z_{a_{rms}}'$ is the relative displacement response induced by the new random signal (i.e. the synthesized PSD) and $z_{s_{rms}}'$ is the relative displacement response induced by the new synthesized sinusoid. The duration T' can be set at the desired value for the synthesized profile.

Then, the expressions (Eq. 2), (Eq. 3) can be inverted and the amplitudes of the sinusoid B' and of the PSD $G_{\ddot{r}}'$ of the new profile can be obtained:

$$G_{\ddot{r}}'(f_n) = \frac{4 \cdot (2\pi f_n)^3}{Q} \cdot \left(z_{a_{rms}}'\right)^2 \quad (22)$$

$$B' = (2\pi f_n)^2 \cdot \sqrt{\left[1 - \left(\frac{f_s}{f_n}\right)^2\right]^2 + \left(\frac{f_s}{Q\,f_n}\right)^2} \cdot z_s' \quad (23)$$

so that the synthesized Sine-on-Random profile (with a single sinusoid) is obtained.

In case of several sinusoids superimposed on random vibration, if the sine frequencies are sufficiently spaced, as already mentioned, a similar extension is applicable: each sine tone is treated independently. The procedure can be summarized as:

- Considering a single sine tone at a time superimposed on the random signal, the method reported above is applied in order to obtain a synthesized Sine-on-Random profile with a single sinusoid.
- The procedure is repeated for each sinusoid.
- In order to perform the "inverse envelope" of (Eq. 14), only the minimum value among the different calculations of the random component is taken at each frequency.
- The complete Sine-on-Random specification is composed by all the synthesized sinusoids and the "minimum" random signal.

Since the measured environmental data are usually in the form of timeseries, some manipulations are necessary in order to obtain the ratio between the amplitude of the sinusoids and the random signal, i.e. the a_0 parameter (Eq. 12), before the actual synthesis can be performed. In particular, the following procedure is followed:

- By knowing the fundamental frequency of the rotating part, it is possible to extract the harmonic components from the overall signal, e.g. using the "harmonic filter" presented in [9].
- The Fourier transform is applied to the extracted harmonic components, in order to find the amplitudes and the phases of the fundamental sinusoid and its harmonics.
- The residual part (i.e. the overall signal minus the extracted harmonic components), is considered to be the random signal component and thus the corresponding PSD is computed.
- The sinusoids and the random excitations are applied to the SDOF system, in order to obtain the characteristic values of the relative displacement responses (Eq. 2), (Eq. 3).

Therefore, the parameter a_0 is obtainable and the Sine-on-Random synthesis can be performed. The extracted phases of the sinusoids are kept in the new profile, to preserve as much as possible the

characteristics of the original excitation. The complete procedure (comprising both the harmonic extraction and the Sine-on-Random synthesis) is schematized in Fig. 3.
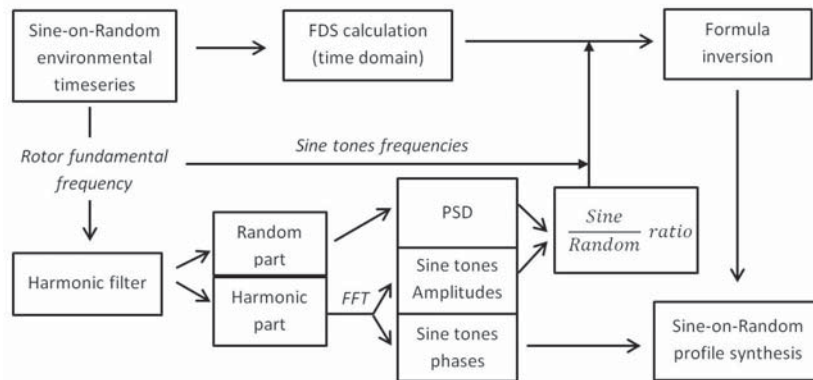

Figure 3: Schematic diagram for the synthesis procedure of a Sine-on-Random profile

3.2. Application example: helicopter data

Starting from real environmental data with Sine-on-Random properties, the Sine-on-Random synthesis is compared with the classical PSD synthesis. Data were acquired during an experimental campaign on a helicopter whose rotor has the following characteristics: fundamental frequency at 392 rpm (~ 6.53 Hz), blade passage at 32.65 Hz (five blades).

Starting from the measured timeseries, the time-domain approach is applied for the FDS calculation. Then, a PSD is synthesized with the standard Mission Synthesis procedure. For the synthesis of a Sine-on-Random profile, instead, the procedure described in Section 3.1 is applied. In this case, the fundamental sinusoid and its first 14 harmonics are considered.

In order to evaluate the two different methods, in a first instance, the duration of the synthesized profiles is taken equal to the original measured vibration. Then, a comparison between the original FDS and the FDSs of the synthesized profiles is carried out. In particular, a timeseries is derived from the synthesized profiles and the time domain approach is used, for the purpose of a comparison which is as reliable as possible.
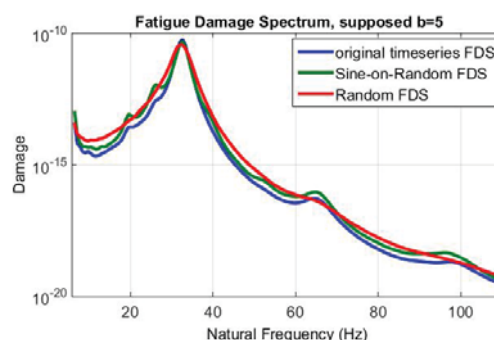

Figure 4: FDS comparison (b=5) – Original profile (blue), Sine-on-Random synthesis (green), Random synthesis (red)

From the comparison illustrated in Fig. 4, it can be noticed that the FDS associated with the synthesized Sine-on-Random profile is closer to the shape of the FDS of the original excitation with respect to the synthesized PSD. Though, some discrepancies can be noted. The damage overestimation of the synthesized Sine-on-Random profile is due to the fact that the approximated formula of Section

2.1 was used. In fact it was shown that, if the sine tones are closely spaced, the envelope of the different FDS calculated with (Eq. 11) gives an underestimation of the damage. In the synthesis procedure, if the starting mission FDS is properly calculated (without damage underestimation, e.g. by using the approach in Section 2.2), the procedure will overestimate the random part in the bandwidth between two closely-spaced sinusoids. In fact, considering only one sine tone at a time, the residual damage due to the neglected sinusoids (taken into account by the mission FDS but not by the synthesis formula) will be added to the random excitation, leading to a more severe synthesized profile. If the severity overestimation is seen as a further safety factor of the procedure, the error is acceptable. However, in order to minimize the discrepancy a recursive procedure to effectively and efficiently reduce the overestimation will be investigated in future work.

In the described Mission Synthesis procedure, the value of the b coefficient typical of electronic components $(b = 5)$ was used [10]. If it is assumed that the real value of the b coefficient was different, the actual FDSs of the original signal and the synthesized profiles can be computed and compared. The examples with $b = 7$ (Fig. 5), $b = 12$ (Fig. 6) and $b = 3$ (Fig. 7) show that synthesized the Sine-on-Random profile is more representative of the original environment. In fact, while the recalculated FDS of the Sine-on-Random profile is still close to the original FDS, the FDS of the synthesized PSD shows a remarkable distance from the reference one. The example with $b = 3$ is particularly significant and highlights the possible severity underestimation of the synthesized purely random profile that would lead to an improper qualification test.
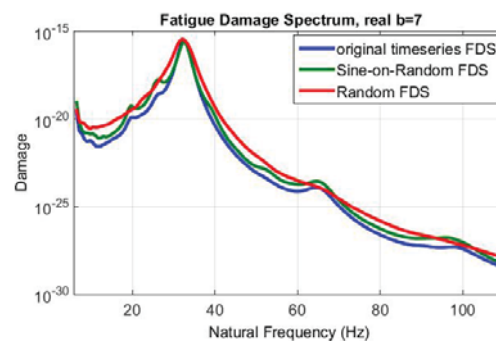

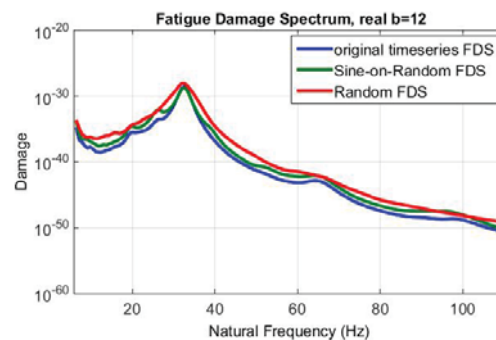Figure 5: FDS comparison (b=7) – Original profile (blue), Sine-on-Random synthesis (green), Random synthesis (red)

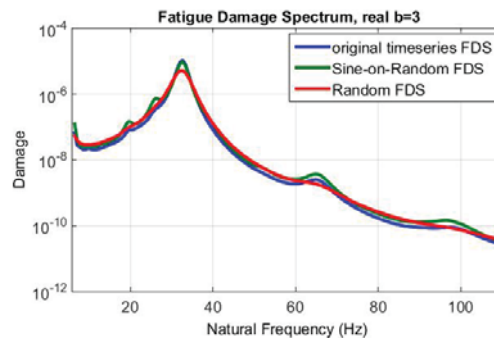Figure 6: FDS comparison (b=12) – Original profile (blue), Sine-on-Random synthesis (green), Random synthesis (red)

Figure 7: FDS comparison (b=3) – Original profile (blue), Sine-on-Random synthesis (green), Random synthesis (red)

4. Discussion

The advantage of the synthesis of a Sine-on-Random profile with respect to the traditional PSD synthesis is that it does not simply attempt to match the reference FDS but it also better describes the peaks distribution of the original signal. Indeed the maxima of a Sine-on-Random excitation do not follow a Rayleigh distribution as in the case of Gaussian random vibrations.

Given that there is an exponential relation between the fatigue damage and the relative displacement peaks, with the Basquin's coefficient b as exponent, the dependence on the accurate knowledge of the b coefficient during the mission synthesis procedure is reduced if the peaks distribution is well-described by the synthesized excitation. Since this coefficient is usually taken from the literature, possibly in the form of a range of uncertain values, it is frequently a source of possible errors.

Consequently, in case of Sine-on-Random real-life environments, it is expected that a synthesized Sine-on-Random profile will be less subjected to errors due to a possibly inaccurate choice of the b coefficient, thus providing a strong advantage over the standard PSD synthesis.

Thus, as shown in Section 3.2, in case the synthesized specification has the same duration as the original signal, it is confirmed that the synthesized Sine-on-Random profile is less sensitive to a possibly uncertain knowledge of the Basquin's coefficient b, compared to a PSD synthesis. The reason relies on the better representation of the peaks distribution of the Sine-on-Random profile. In fact, while the random profile aims at matching the damage with a Rayleigh's peaks distribution, the Sine-on-Random specification better represents the peaks distribution of the original vibration, in addition to the FDS correspondence.

A better damage matching from the Sine-on-Random specification is expected also in case of accelerated lifetime profiles, but further investigations are needed to demonstrate this hypothesis.

5. Conclusions

An overview of two frequency-domain methods for the FDS calculation in case of Sine-on-Random vibrations was presented, with a focus on the pros and cons of each procedure. In particular, a closed-form solution is available in case of sufficiently-spaced sinusoids, but it tends to underestimate the damage in case of closely-spaced sinusoids. On the other hand, a method for an accurate damage estimation even in case of closely-spaced sinusoids is also available: the main drawback however is that a closed-form solution cannot be achieved so that a numerical integration is required.

The first method can be used to obtain a new procedure for the synthesis of a Sine-on-Random specification starting from a reference FDS (and an application example of a helicopter vibration was shown). This is the main novelty of the work and is considered significant by the authors, since it is nowadays becoming more acknowledged that in presence of non-Gaussian excitations (such as Sine-on-Random) the purpose of a proper test tailoring should be to not only match the damage (i.e. the FDS), but also to preserve the nature of the excitation.

In case the sine tones are not sufficiently spaced, a more severe profile is synthesized due to the usage of approximated formulations. Since the severity overestimation can be seen as a further safety factor of the procedure, the error is acceptable. However, a way to reduce the overestimation (e.g. a recursive procedure) is under investigation to improve the proposed approach.

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