On the influence of nonlinearities on vibrational energy transduction under band-limited noise excitations

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Abstract. Vibrational energy harvesters are often excited by band-limited noise excitations. In this paper, the influence of the stiffness nonlinearity on the transduction of the energy harvester and the relative performance of linear, monostable hardening-type and bistable energy harvesters are compared and investigated. The performance is experimentally compared under band-limited noise excitations of different levels, bandwidths, and centre frequencies at first. The rms output power delivered to the same load resistance is measured and compared under the same excitation levels, which indicts the constant electrical damping level. It is shown that the effect of nonlinearities is strongly dependent on the excitation parameters. Under a moderate excitation level it is shown that the monostable hardening-type oscillator performs worse than its linear counterpart under band-limited excitation. However, the results also illustrate that for the most part of the frequency and bandwidth range considered, the bistable harvester can outperforms the linear variant but for around the peak output area. Moreover, the comparison is also numerically conducted with the consideration of the optimised electrical damping level and the displacement constraint of the device. General conclusions are drawn based on the experimental observations.

1. Introduction

Different configurations of energy transducers have been used to transform kinetic vibration energy into electricity, among which the traditional linear devices operate based on

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the principle of resonance which offers significantly decreased performance under off-resonance conditions and very limited bandwidth [1, 2]. In order to overcome the shortcoming of linear oscillators, the explorations of nonlinear systems have been widely discussed by some researchers, in particular the hardening monostable oscillator with a cubic nonlinearity [3-6], and bistable devices with double-well potential [7-10]. Those nonlinear systems showed an increase in the bandwidth of the device and the capability to outperform linear resonance under harmonic excitations. However, the excitation force is not always periodic in reality and in fact most ambient vibrations sources can actually be somewhat stochastic in nature.

The influence of the nonlinearity on the performance of energy harvesters under noise excitations has been investigated in some of the literature [11-16]. However, it was demonstrated that under Gaussian white noise excitation both the nonlinearity of the hardening-type monostable oscillator and the bistabilities in the potential did not provide any enhancement of energy harvesting over that of the traditional linear generator with a single well [17, 18]. Different conclusions can be found when the value of the constant of the harvesting circuit is taken into consideration [19, 20]. While many environmental excitations exhibit characteristics of white noise excitations, many others have most of their energy trapped within a narrow bandwidth possessing the characteristics of a band-limited coloured excitation. Notably, Daqaq [17] analysed the responses of linear and hardening-type monostable systems subjected to band-limited noise by computing the probability density function (PDF) and made certain comparisons. It was shown that the nonlinearity can adversely influence the mean output power under band-limited noise excitations, but it can be noted that experimental validation was not involved in this particular study. Sebald et al. [21, 22] arrived at a similar conclusion for a hardening-type monostable harvester. After that, by applying a static compressive axial load at one end of a beam, Masana, et al [23] experimentally investigated the performance of a monostable hardening-type energy harvester in the pre-buckling condition, and a bistable energy harvester in the post-buckling condition, when the harvesters were subjected to random base excitations of different levels, bandwidths, and centre frequencies. The bistable harvester has been shown to outperform the monostable design.

On the other hand, Meimukhin [24] made a comparison of a bistable energy harvester and its linear counterpart and showed that nonlinear bistable oscillators generally perform better than the linear one under band-limited noise excitation, in certain regions. This was only under the condition that a random excitation with a cut-off frequency was analysed, and without considering the independent influence of the various centre frequencies. However, Joo, et al [25] numerically demonstrated that under coloured noise excitations the linear energy harvester outperformed the monostable nonlinear harvester, while the bistable energy harvester showed the lowest performance, and this rather conflicts with the conclusions discussed above. Moreover, the optimised electrical damping and the practical displacement constraint of the device are not considered in the previous investigations. The work presents in this paper provides a full numerical and experimental performance comparison between the monostable hardening-type configuration, the bistable configuration, and the linear counterpart, under a band-limited base force. The influences of various excitations levels, bandwidths and centre frequencies are all taken into account.
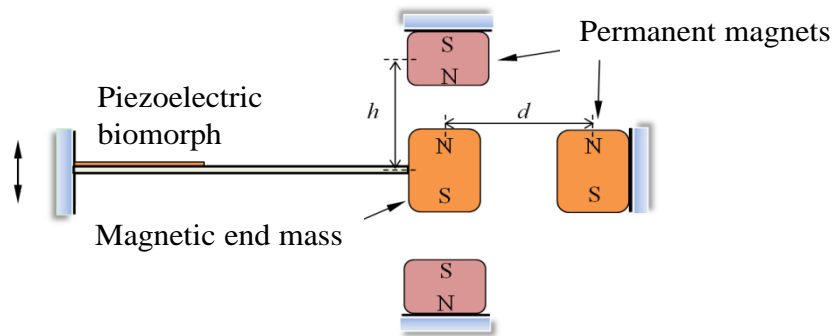
2. Energy harvester configurations

Fig. 1 Schematic diagram of the energy harvesters in different configurations

Fig. 1 presents a schematic diagram from which two types of energy harvester can be synthesised by means of intentionally introduced nonlinearities. Bistability can be created by positioning two permanent magnets with opposite polarities, respectively on the piezoelectric cantilever tip and on another fixed support at a distance d in the horizontal direction. The corresponding expression for the interaction force can be obtained by the dipole model as [6]

$$F(x) = \frac{\mu_0 v^2}{4\pi}\Theta(x) \tag{1}$$

where the function $\Theta(x)$ is defined by

$$\Theta(x) = \frac{3\left(M_{cy}\left(dM_{fx} + xM_{fy}\right) + M_{cx}\left(dM_{fy} + 3xM_{fx}\right)\right)}{\left(d^2 + x^2\right)^{5/2}} - \frac{15x\left(xM_{fx} + dM_{fy}\right)\left(M_{cx} + M_{cy}\right)}{\left(d^2 + x^2\right)^{7/2}} \tag{2}$$

To achieve the monostable nonlinear configuration, another two permanent magnets are fitted symmetrically at an equilibrium distance h at either side of the magnetic end mass, and the magnets are arranged in a repulsive configuration to produce the hardening-type nonlinearity. The restoring forces of the magnetic end mass caused by the top and bottom magnets can be expressed by

$$F(x) = \frac{\mu_0 v^2}{4\pi}\left(\Theta(h+x) - \Theta(h-x)\right) \tag{3}$$

3. Experimental comparative performance analysis under constant damping

The experiments were conducted using a seismic shaker (IMV corporation, m060) controlled by a dSPACE 1103 controller. The main objective was to make a comparative performance study in the linear, monostable and bistable configurations under coloured noise excitations. These devices allow for the chosen harvester system to be subjected to random base acceleration at specified excitation levels, frequency bandwidths and centre frequencies. The band-limited noise signals are created by applying different band-pass filters to the original Gaussian white noise signal for driving the shaker. The shaker table acceleration is measured by a vibrometer then the root-mean-square value is calculated by the dSPACE controller, and

this is kept approximately equal for noise excitations with different bandwidths and centre frequencies by tuning the amplifier gain (noting that a constant power spectral density of the excitation produced by the shaker cannot be guaranteed under different bandwidths and centre frequencies using a fixed gain because of the dynamic characteristics of the shaker). The delivered rms power on a load resistance of $1\ \text{M}\Omega$ connected to the piezoelectric bimorph attached on the cantilever can be obtained and used for comparison.
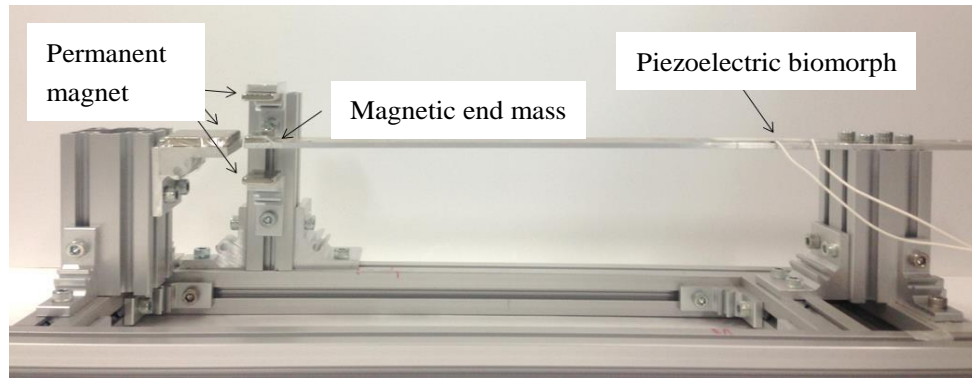

Fig. 2 Photo of the energy harvesters in different configurations

A photograph of the experimental device is shown in Fig. 2. The permanent magnets are attached to sliders on rails that allow the distance to be adjusted. In order to conduct a convenient comparison, the fundamental frequencies in different configurations are set to be equal to 15 Hz by adjusting the mass on the cantilever tip and changing the distances h and d.

Tests consist of response comparisons under various centre frequencies and excitation levels at a constant bandwidth, and the influence of the bandwidth and centre frequency on the performance is comprehensively assessed by keeping the same excitation level.

3.1. Performance comparison under different centre frequencies and excitation levels

The responses are investigated with the change of centre frequency in different configurations in this section. Fig. 3 shows the comparison of the output voltage variance in the open-circuit configuration with band-limited noise excitation (using a bandwidth of 4 Hz) of several levels. The sampling window of the output voltage is kept wide enough for accuracy.

The results in Fig. 3(a) show that at a small excitation level the peak of the output voltage occurs when the centre frequency nears the fundamental frequency for both of the monostable hardening-type and bistable configurations, and this is similar to the linear configuration. There is also not a great deal of difference for the amplitude of the peak output voltage between the results, and this indicates that these energy harvesters are operating near their equilibrium positions and do not obviously exert strong nonlinear effects.
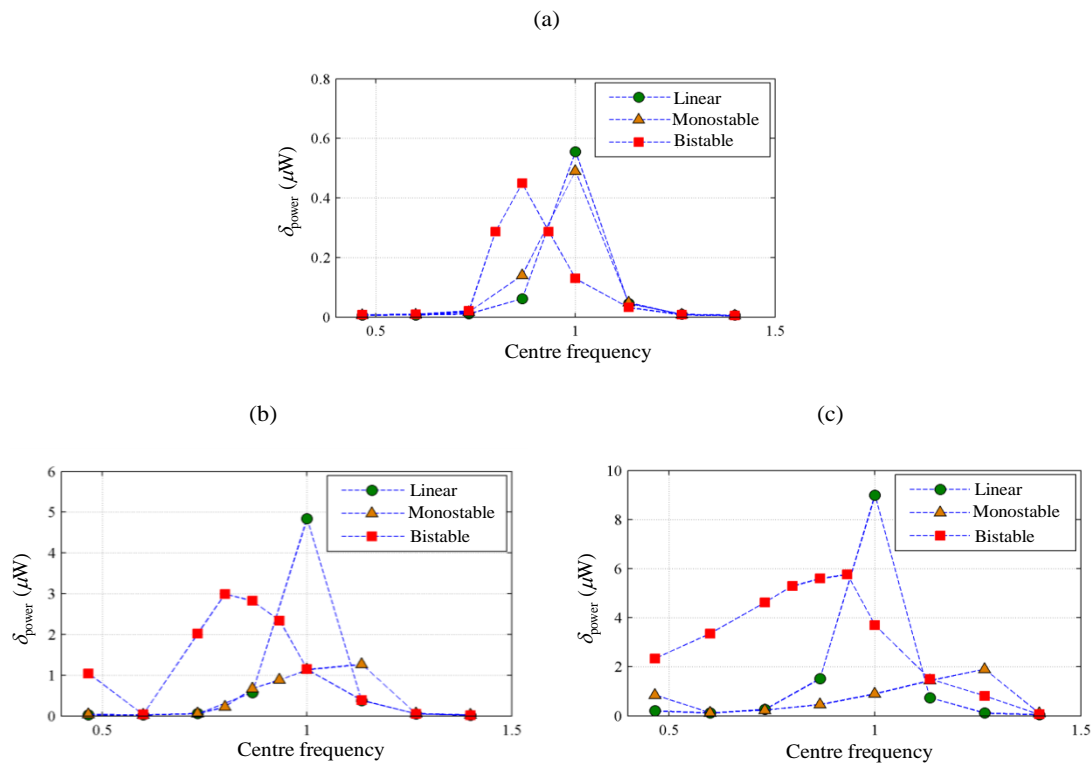
(a)

(b)   (c)

Fig. 3 Variation of the rms power with the excitation's centre frequency under the bandwidth

of 4 Hz: (a) $\sigma^2_{acc} = 0.0286\,\mathrm{m^2 s^{-4}}$, (b) $\sigma^2_{acc} = 2.49\,\mathrm{m^2 s^{-4}}$, and (c) $\sigma^2_{acc} = 7.38\,\mathrm{m^2 s^{-4}}$.

   With the increase of the acceleration level the peak output voltage of the monostable energy harvester shifts towards higher frequencies because of the hardening effect of the stiffness, as shown in Fig. 3(b). However, it can be noted that the amplitude of the peak voltage is lower when compared with the linear configuration. It is known that for a monostable hardening-type oscillator, both low- and high-energy responses can coexist for the same parameter combinations at relatively high harmonic excitation levels. This indictates that when the energy harvester is randomly excited, the high energy orbit cannot be maintained. The performance decreased due to a noticeable hopping between the different energy orbits with the variation in excitation level. On the other hand the peak voltage of the bistable configuration shifted towards to the lower frequency away from the fundamental frequency. This phenomenon is caused by asymmetric stiffness softening effect within one potential well.

   The peak output voltage of the monostable energy harvester shifts further to the high frequency side as the excitation level is increased, as presented in Fig. 3(c). However, the tendency of the bistable configuration is to show a reversion and to start to move to the higher frequency side. The increase of the excitation makes it relatively easy to overcome this barrier and hop between the two potential wells. In this condition the hardening-effect starts to become the dominant factor at higher centre frequencies. For a bistable energy harvester the presence of the hopping oscillation between the double potential wells yields a substantially larger power output. However the large amplitude hopping oscillation is considerably

dependent upon the excitation level.

3.2. Comprehensive performance comparison

Fig. 4 shows the experimental output voltage variance as a function of the centre frequency and bandwidth of the noise excitation for the different configurations and under an excitation level of 2.49 m/s^2. For the monostable case as shown in Fig. 4(b), the peak response can be observed at a higher centre frequency than the fundamental frequency. For different centre frequencies the optimum bandwidth can be found.

For the linear energy harvester a foreseeable peak output can be noted at the fundamental frequency in Fig. 4(a). The influence of the bandwidth shows a similar effect to the monostable nonlinear case. However an overall higher output can be observed for the linear energy harvester when compared with the monostable nonlinear case, and this is different from the general impression due to the assumed advantages inherent in nonlinearity in this context.

Noting that it is different to the linear and monostable nolinear configurations, the bistable system result of Fig. 4(c) shows that the peak voltage variance is obtained at a lower centre frequency than the fundamental frequency. The voltage variance obtained is much higher than that of the other configurations in most areas. It is confirmed that the bistable configuration is less susceptible to variations in the centre frequency and bandwidth of the noise excitation, especially for the low centre frequency areas. However, it can also be noted that the peak value of the linear configuration is still greater than that of the bistable energy harvester.

(a)

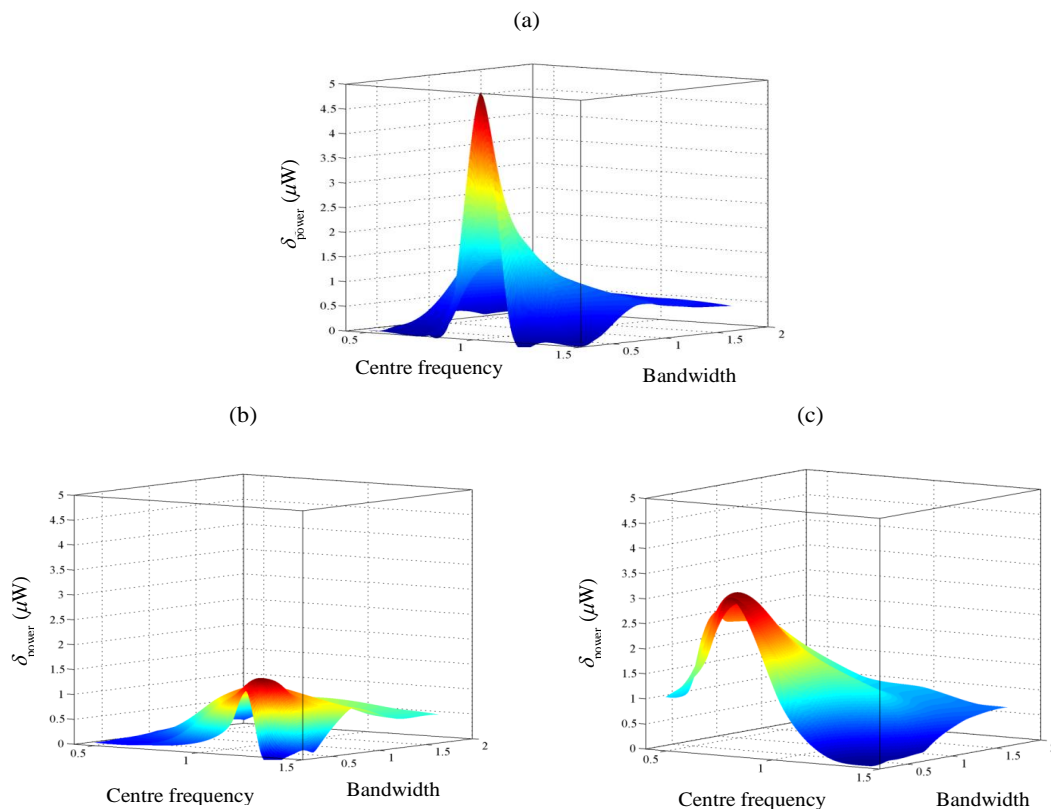(b)   (c)

Fig. 4 Variation of the rms power with the excitation's centre frequency and the bandwidth: (a) linear, (b) hardening-type monostable, and (c) bistable. ($\sigma_{acc}^2 = 2.49 \, \mathrm{m^2 s^{-4}}$)

Moreover, for all configurations and centre frequencies, as the bandwidth of the excitation increases to a large enough value the influence of the nonlinearity and centre frequencies decreases and the similar amplitude can be observed for different configurations when the bandwidth exceeds certain critical values.

4. Numerical comparative performance analysis under optimised damping

Thus far, the above discussion is the experimental comparative investigation of different configurations under the constant electrical damping level, however, the optimised damping level is not taken into consideration. For further fair comparison, numerical simulations with the optimised electrical damping for each combination of the centre frequency and bandwidth are carried out. Considering the achievable higher electrical damping compared with the piezoelectric-type, the electromagnetic type transducer is adopted. The governing equation of the system is written as

$$m\ddot{x} + c\dot{x} \pm kx + \alpha x^3 = N(t) \tag{4}$$

where c is the total damping, and it is the sum of the mechanical damping c_m and the electrical damping c_e.

Table 1. Parameters used in the investigations of stiffness on the frequency response.

Parameter	Value	Unit
m	0.028	kg
c	0.036	Ns/m
k (monostable/linear)	248	N/m
k (bistable)	124	N/m
α (monostable)	2.5×10^8	N/m^3
α (bistable)	3.8×10^7	N/m^3

Table 1 shows the parameters for numerical simulations, which also the representative of the experimental setup. For each combination of the bandwidth and centre frequency, the electrical damping is varied to find the optimised value and get the maximum output power.

From Fig. 5, when the damping level is optimised, the similar responses to the experimental results can be observed, whatever the overall tendency or the quantitative analysis. Generally, the hardening-type monostable configuration does not show improvement compared with the linear case.
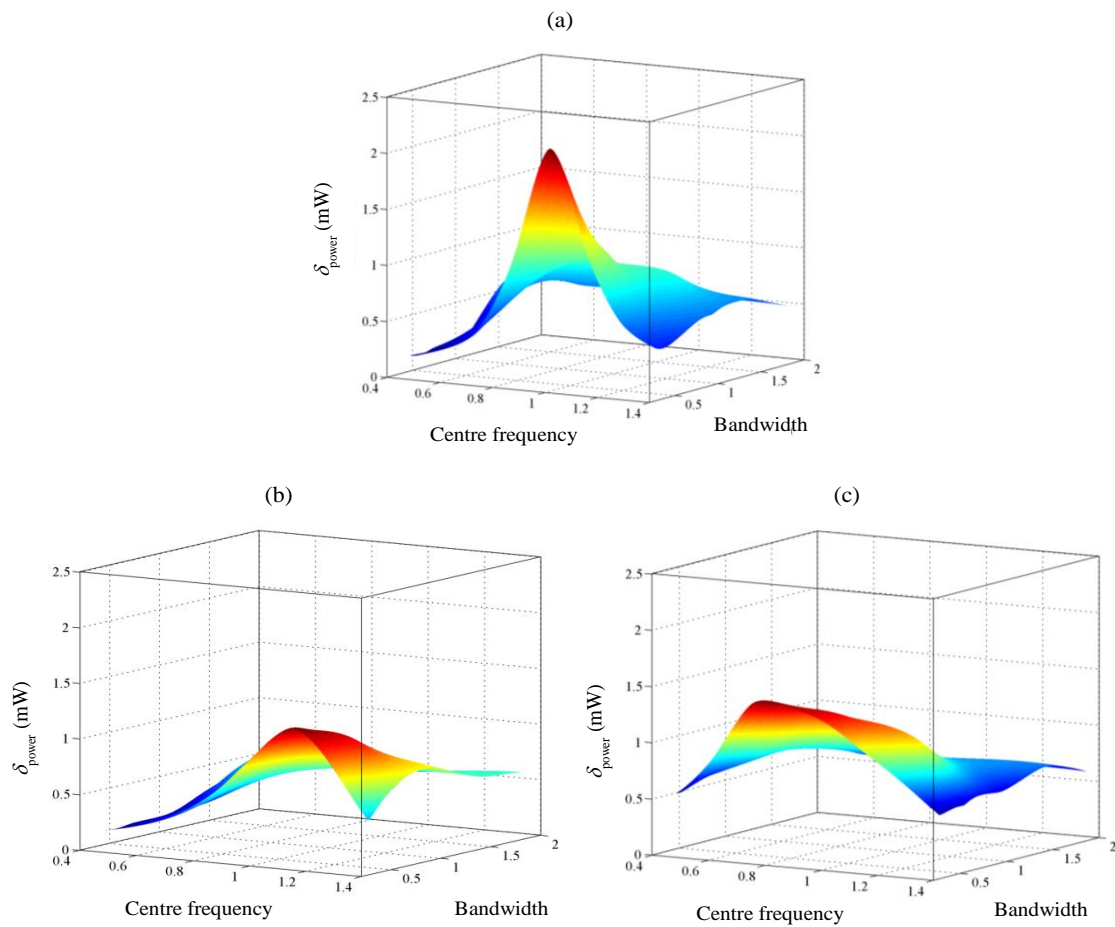
Fig. 5 Variation of the rms power with the excitation's centre frequency and the bandwidth: (a)

linear, (b) hardening-type monostable, and (c) bistable. ($\sigma_{acc}^2 = 2.49\,\mathrm{m^2 s^{-4}}$)

5. Numerical comparative performance analysis under constrained displacement

Considering the practical energy harvester has limit of the stroke, the condition when the displacement of the mass is constrained to a certain value is investigated for further comparison of different configurations in this section.

It should be mentioned that a special characteristic of the bistable configuration is that it has two equilibrium points, which indicts that there is the displacement exists even if there is no excitation is applied to the system. For the performance comparison in the previous sections, the variance of the excitation acceleration is 2.49 $\mathrm{m^2 s^{-4}}$, where the displacement of the linear energy harvester cannot exceed the displacement of the equilibrium point of the bistable energy harvester for most cases. Thus, a higher excitation level of 15 $\mathrm{m^2 s^{-4}}$ is used for performance comparison under constrained displacement. It is assumed that the maximum displacement amplitude limit is set to be 4 mm. Figs. 6 and 7 show the rms output power under different centre frequencies and bandwidths for the bistable and hardening-type energy harvesters, respectively, where the displacement response is smaller than the constraint.

Moreover, Figure 8 shows the performance comparison of the linear energy harvester between the conditions that with displacement constraint (grid curve surface) and without the constraint (smooth curve surface). It can be noted that there is an obvious decrease of the

output power around the peak response area.

To summarise, it is also predictable that if the displacement constraint is very close to the equilibrium points of the bistable energy harvester, the performance of the bistable energy harvester will suffer an obvious decrease because it can only achieve the intra-well motion instead of the large amplitude inter-well motion because of the displacement constraint.

However, when the excitation level is large enough to make the displacement of the linear energy harvester exceed that of the bistable configuration, the linear energy harvester will present the decreased performance as shown in Figure 5.47, while there is little influence on the bistable energy harvester.
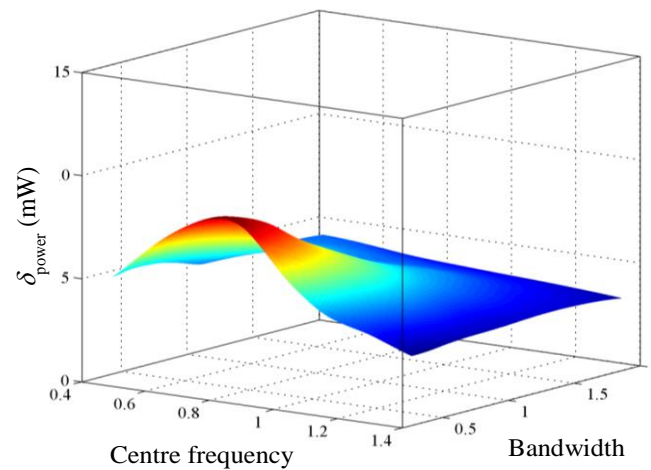

Fig. 6 Variation of the rms output power with the excitation's bandwidth and centre frequency.

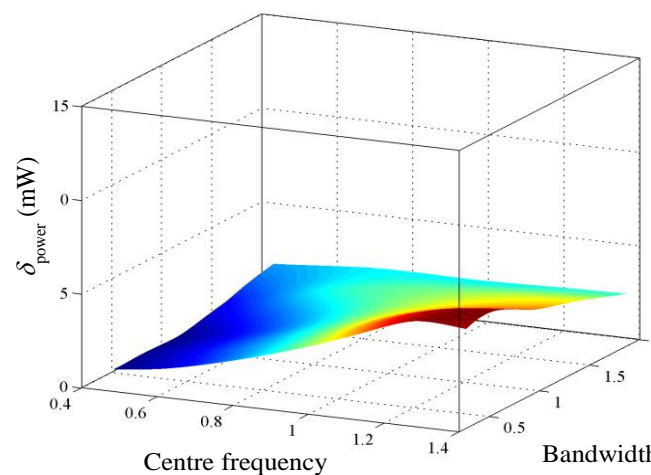($\sigma_{\mathrm{acc}}^2 = 15\,\mathrm{m}^2\mathrm{s}^{-4}$ bistable configuration)

Fig. 7 Variation of the rms output power with the excitation's bandwidth and centre frequency.

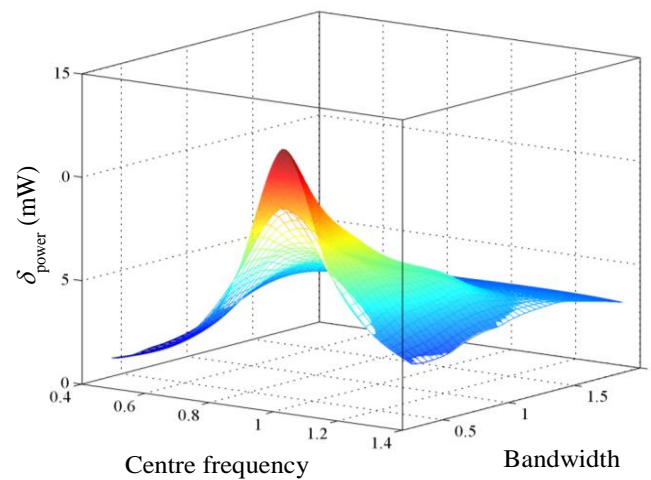($\sigma_{\mathrm{acc}}^2 = 15\,\mathrm{m}^2\mathrm{s}^{-4}$ hardening-type monostable configuration)

Fig. 8 Comparison of the optimised output power. (grid surface: with displacement limit;

smooth surface: without damping limit $\sigma_{acc}^2 = 15\,\mathrm{m}^2\mathrm{s}^{-4}$ linear configuration)

Conclusions

   The performance of an energy harvester under band-limited noise excitations was analyzed. The output voltage variance of the linear, hardening-type monostable and bistable configurations under different centre frequencies, bandwidths, and excitation levels was compared to generalise the relative performance. The remarks obtained are as follows.

(1) When the input excitation level is small the peak response can be observed around the fundamental frequencies, and it shows similar amplitude irrespective of the centre frequency in all configurations. This indicates that the nonlinearities of the monostable and bistable energy harvesters have little effect on the response.

(2) As the input acceleration is increased to a relatively high level the peak response shifts toward a higher centre frequency for the monostable nonlinear energy harvester, and the same tendency is seen as the excitation level is further increased to a higher level still because of the hardening nonlinearity effect. However the fact that the peak response of the bistable configuration moves to a lower centre frequency indicates the effect of the softening effect within the single potential as the excitation level is increased. Then the presented reversion effect demonstrates that the hardening phenomenon takes over when the large amplitude hopping oscillation between the potential wells becomes apparent.

(3) In contrast to the commonly held assumption that a hardening-type monostable nonlinear energy harvester shows a wider bandwidth of large amplitude voltages as compared to the linear variant, this study experimentally and numerically validate that the hardening-type nonlinearity does not provide enhancement, it should be avoided for the band-limited noised excited energy harvesting.

(4) For a given band-limited noise excitation with certain bandwidth and centre frequency, the linear configuration can be selected, and the fundamental frequency of the device should be around the centre frequency of the excitation frequency bandwidth. That's because it has been validated that the optimised condition for the hardening-type monostable and bistable energy harvesters is when the parameter is appropriately selected so that they are operated like a linear energy harvester.

(5) If the excitation presents the time-varying property of the frequency bandwidth or centre frequency, in other words, considering that sometimes the designed fundamental frequency of the system is excluded from excitation frequency range or becomes far away from the centre excitation frequency, the bistable configuration is the best candidate. That is because for most of the frequency and bandwidth range considered the bistable harvester outperforms the linear system but for area of the peak output power.

(6) The bistable energy harvester requires the overall lower electrical damping compared with the linear and monostable configurations. If the maximum available damping is constrained, the linear and the hardening-type energy harvester will suffer energy loss at both the low and high centre frequency area when the excitation bandwidth is narrow.

(7) Because of the existence of the equilibrium point of the bistable energy harvester, there is a minimum stroke requirement for the bistable configuration. However, when the excitation level is large enough to make the displacement amplitude become bigger than that of the bistable energy harvester, the constrained displacement will cause the obvious performance decrease around the peak response area for the linear energy harvester.

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