The Parameter Identification Method of Blade Asynchronous Vibration under Sweep Speed Excitation

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Abstract. The non-contact blade tip timing (BTT) measurement, with the advantage of measuring all the disk blades, has obtained widespread applications in industries. The measured data is under-sampled. Generally asynchronous vibrations will appear with the rotating speed sweeping under the excitation of the flow field, which can create a great increase of the blade vibration amplitude that is potentially harmful to the operation reliability of the turbine. In this paper, the investigation on the parameter identification method of blade asynchronous vibration under variable speed excitations was deeply explored. A novel method of parameter identification has been developed, based on the two sensors data interpolation in order to increase the frequency that can be identified. The spectral analysis with all phases Fast Fourier Transform (apFFT) was performed to extract the accuracy amplitude and phase of the blade. The order tracking calculation was carried out based on the minimum angle error. A mathematical model with four blades was developed to simulate the BTT testing data which can produce the simulation of speed sweep asynchronous vibration signal of the four blades. The correctness and accuracy of the algorithm was verified with the parameters identification of blade asynchronous vibration using the simulation model.

1. Introduction

As an important part of a mechanical impeller, the blade's reliable operation is the key to guarantee the safety of a rotating machine. Nowadays, the non-contact measurement of blade tip timing(BTT) has obtained widespread applications in industries[1][2]. The BTT technology measures the blade tip vibration with sensors mounted on the rig case to detect the arriving time of individual blades. Vibration amplitudes are extracted by subtracting arriving times to theoretical ones. The sampling frequency of BTT, which is just a few hundred Hertz, is once per revolution, far less than vibration frequencies of blades which can reach up to several thousand Hertz. The BTT technology can obtain the under-sampled sequence data of the blade vibration, with a blade tip timing sensor mounted on the rig case of the rotating machine. Therefore, it is necessary to develop the parameter identification techniques of the blade vibration based on the BTT measurement.

The blade vibration can be considered as two different forms: synchronous vibration and asynchronous vibration. In the case of synchronous vibration, the blade frequency of vibration is an integer multiple of the rotational frequency of the assembly. The commonly-used BTT methods for

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synchronous vibration include both the single parameter method developed by Zablotskiy and Korostelev[3][4], and the two parameters plot by Zielinski, Ziller[5]and Heath[6]. NSMS[7] put forward the Circumferential Fourier fit analysis Method and Gallego-Garrido exploited the auto-regression method[8][9].

For the asynchronous vibration excited by rotating stall, surging and flow, known as blade flutter, the frequency is the non-integer time the rational speed. Generally, asynchronous vibration will appear when the speed changes under the excitation of the flow field, which can create great increase of the blade vibration amplitude and harm the reliable operation of the turbine. Asynchronous vibration is identified by individual spectra for each blade and a single all-blade spectrum[10] which means that all the blades vibrate at the same frequencies and with the same amplitudes. The amplitude and frequency of asynchronous vibration of blade can be determined. The multi-sensor uniform and the '5+2' distribution[11] are involved in the asynchronous vibration.

There are alternative methods for the blade parameter identification. The tip timing signals are under-sampled and the spectrums are aliased. A minimum variance filter to updating the autocorrelation matrix iteratively for spectral estimation of blades vibration[12]. Mistuned blade vibration with high level amplitude and with close modes will cause estimation error of frequency of the blade. The spectral estimation was the better performance on identification for the tip timing data[13].

In this paper, a novel parameter identification method of blade asynchronous vibration under sweep excitation has been developed based on the two sensor data interpolation to increase detection frequency. The spectral analysis with all phases Fast Fourier Transform (apFFT)[14][15] was performed to extract the amplitude and phase with high accuracy. Then the tracking order was derived based on the minimum angle error.

2. Parameter identification method of blade asynchronous vibration under sweep excitation

The principle of BTT is shown in figure 1. In one cycle, the time of an arrival of a blade to the BTT sensor is t_{nij0} without vibration where n is the index of the cycle and j is the index of the blade. When the blade is vibrating, the time of the arrival of the blade to the BTT sensor is t_{nij}. T is the time period of one cycle. The equation of a single blade angular displacement with single degree of freedom vibration can be written as

$$\beta_i = \frac{2\pi(t_{ni} - t_{ni0})}{T} \tag{1}$$

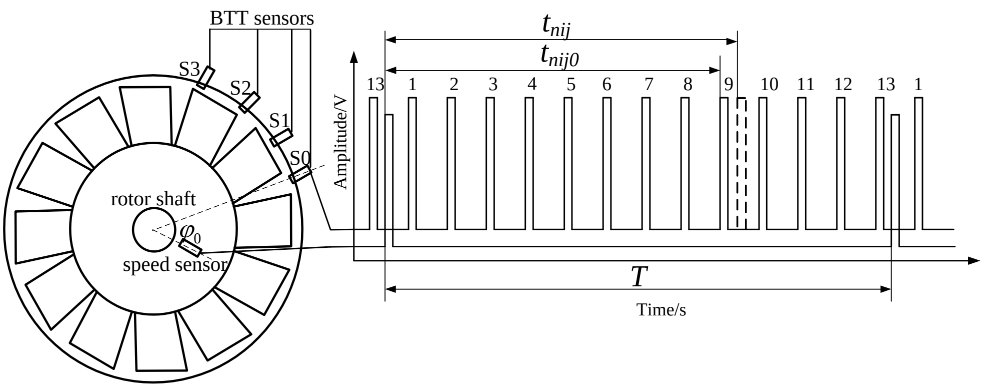

Figure 1. The principle of blade tip timing.

Its vibration inflection can be expressed in the following equation as

$$y_i = A\cos(\omega t_{ni} + \varphi_0) \tag{2}$$

where A is the vibration amplitude, φ_0 is the initial angle position, i, from 0 to $M-1$, is the index of the sensor and M represents the number of sensors.

The frequency of the blade asynchronous vibration is non-integer times as much as the rotational speed. Assuming that

$$\omega = (m + \Delta m)\Omega \tag{3}$$

where m is a positive integer, Δm is a decimal number from 0 to 1, and Ω is the rotational frequency.

Then, t_{ni} is given by

$$t_{ni} = (\theta_i + 2\pi n)/\Omega \tag{4}$$

where θ_i is the mounted angle position of the sensor, and n is the revolution index. So equation (2) becomes

$$y_i(n) = A\cos((m + \Delta m)\theta_i + 2\pi n\Delta m + \varphi_0) \tag{5}$$

where $\Delta m = \Delta\omega / \Omega$, and $\Delta\omega$ is the detection frequency of the signal blade spectrum. We can define $\varphi_i = (m + \Delta m)\theta_i + \varphi_0$, so Equation (5) can be written as

$$y_i(n) = A\cos(\Delta\omega \frac{2\pi n}{\Omega} + \varphi_i) \tag{6}$$

$\Delta\varphi_i$, the measured phase of the other sensors relative to the first sensor is given as

$$\Delta\varphi_i = (m + \Delta m)(\theta_i - \theta_0) \tag{7}$$

where θ_0 is the angle position of the first sensor. Due to the periodicity of the angle, the range of $\Delta\varphi_i$ is [0, 360). So equation (7) becomes

$$\Delta\varphi_i = \left(m \pm \frac{\Delta\omega}{\Omega}\right)(\theta_i - \theta_0) \tag{8}$$

θ_i and Ω in equation (8) can be obtained from the test system. The detection frequency $\Delta\omega$ is given by the spectrum analysis of the blade vibration signals and then Δm can be determined. The signal sampling frequency f_s is calculated by $N_b \cdot M \cdot \Omega$, where N_b is the blade number. When the simulation data is the single blade vibration signals sampled by the single sensor, f_s equals Ω and the actual spectrum of the vibration signals is symmetrical. Based on the detection frequency $\Delta\omega$ from 0 to $\Omega/2$, it is necessary to conduct the interpolation of the vibration signals of two sensors to increase the sampling frequency. Thus each Δm from 0 to 1 can be identified and the range of the detection frequency $\Delta\omega$ becomes from 0 to Ω. The vibration phase is given by the apFFT spectrum and then m can be solved by order tracking[16] with the relationship of the vibration phase and the angle position of the sensor in equation (7). Finally, the blade vibration frequency can be obtained by $\omega = (m \pm \Delta m)\Omega_n$. The above process of parameter identification can be divided into three steps.

(1) Interpolation of vibration signals.

The interpolation procedure means that the blade vibration data measured by the latter sensor is inserted into the data measured by the former sensor to form a new vector. For example, *TIP*1 and *TIP*2 are the blade data vectors acquired by the first sensor and the second sensor, as $[x_{11}, x_{12}, x_{13}, \cdots x_{1n}]^T$ and $[x_{21}, x_{22}, x_{23}, \cdots x_{2n}]^T$. After the interpolation, the new first data vector $TIP1^*$ becomes $[x_{11}, x_{21}, x_{12}, x_{22}, x_{13}, x_{23}, \cdots x_{1n}, x_{2n}]^T$. In the same way, the data acquired by the third sensor is inserted into the second data to form the new second data vector $TIP2^*$. At last, if measured by five sensors, after the interpolation, the number of the data vectors becomes four. The sampling frequency of the single blade by the single sensor f_s increases into 2Ω, which makes sure that the range of $\Delta\omega$ is from 0 to Ω. Then, each Δm of asynchronous vibration from 0 to 1 can be identified by this algorithm.

(2) The detection frequency and phase of the blade vibration signals by apFFT.

Generally, the blade vibration phase calculated by conventional FFT is not accurate except that when the sampling frequency is integer times as much as the FFT frequency. And also the spectrum aliases often appear in conventional FFT which has averse effects on the accuracy of the vibration phase. All phases Fast Fourier Transform(apFFT) described in the literature can improve the accuracy of the vibration phase estimation.

Equation (6) can be written as the single frequency complex exponential form

$$y_i(n) = Ae^{j(\Delta\omega\frac{2\pi n}{\Omega}+\varphi_i)} \tag{9}$$

The blade vibration response including N points $y_i(n)$ is obtained by FFT as

$$Y_i(k) = A\sum_{n=0}^{N-1} e^{j(\Delta\omega\frac{2\pi n}{\Omega}+\varphi_i)} e^{-j\frac{2\pi kn}{N}} = A\frac{\sin[\pi(\frac{N\Delta\omega}{\Omega}-k)]}{\sin[\pi(\frac{N\Delta\omega}{\Omega}-k)/N]} e^{\left[\varphi_i+\frac{N-1}{N}(\frac{N\Delta\omega}{\Omega}-k)\pi\right]} \tag{10}$$

where k= 0 , 1, 2 , …, N–1.

Fourier transform of a cycle shift measured data was averaged to derive the apFFT. A sample point in the vibration displacement sequence $y(0)$ only contains N-dimensional vectors

$$\begin{cases} y^0 = [y(0), y(1), \cdots, y(N-1)]^T \\ y^1 = [y(-1), y(0), \cdots, y(N-2)]^T \\ \quad \cdots\cdots \\ y^{N-1} = [y(-N+1), y(-N+2), \cdots, y(0)]^T \end{cases} \tag{11}$$

Then each vector is ordered again by cyclic shift with $y(0)$ in the first place. Another N-dimensional vectors can be obtained

$$\begin{cases} y'^0 = [y(0), y(1), \cdots, y(N-1)]^T \\ y'^1 = [y(0), y(1), \cdots, y(-1)]^T \\ \quad \cdots\cdots \\ y'^{N-1} = [y(0), y(-N+1), \cdots, y(-1)]^T \end{cases} \tag{12}$$

According to the shift property of DFT, the relationship of $Y'^l(k)$, Discrete Fourier Transform of $y'^l(n)$ in equation (12) and $Y^l(k)$, Discrete Fourier Transform of $y^l(n)$ in equation (11) can be written as

$$Y'^l(k) = Y^l(k)e^{j\frac{2\pi}{N}lk} \tag{13}$$

where l, k = 0,1,2,…,N–1.

The apFFT result can be derived by the following equation

$$Y_i^{ap}(k) = \frac{1}{N}\sum_{i=0}^{N-1} Y'_{il}(k) = \frac{Ae^{j\varphi_i}}{N}\frac{\sin^2[\pi(\frac{N\Delta\omega}{\Omega}-k)]}{\sin^2[\pi(\frac{N\Delta\omega}{\Omega}-k)/N]} \tag{14}$$

where i is the index of the sensor.

In equations (10) and (14), the phase in conventional FFT is affected by the frequency deviation value $\frac{N\Delta\omega}{\Omega}-k$. However, the phase in apFFT is the value of the measured point $y(0)$ no matter how the frequency deviation value is. The detection frequency and phase can be derived from apFFT. The value m can be derived from order tracking with the relationship of the vibration phase and the mounted angle of the sensor in equation (8).

(3) The vibration order by order tracking.

With M sensors and the measured phase $\Delta\varphi_i^*$ from 0 to 2π, the equation can be written as

$$\Delta\Phi^* = [\Delta\varphi_0^*, \Delta\varphi_1^*, \cdots, \Delta\varphi_{M-1}^*] \tag{15}$$

The maximum in the apFFT spectrum is the detection frequency of the blade vibration. Seen in the phase spectrum, the phase of the detection frequency is the measured phase by the sensor. It should be noted that the phase measured by each sensor is under the same detection frequency. $\Delta\Phi^*$ is the difference between the phase measured by each sensor and the one by the first sensor.

The traced phase $\Delta\varphi_{ki}$ can be obtained from 0 to 2π with different frequency multiplications and written as

$$\Delta\Phi_k = [\Delta\varphi_{k0}, \Delta\varphi_{k1}, \cdots, \Delta\varphi_{kM-1}] \tag{16}$$

The residual of the measured phase and the initial phase is expressed as

$$e_k = \Delta\Phi_k - \Delta\Phi^* \tag{17}$$

where $e_k = [e_{k0}, e_{k1}, \cdots, e_{kM-1}]$.

Then S_k, the RMS of e_k characterizes the degree of deviation from the traced phase to the measured phase

$$S_k = \left(\frac{\sum_{i=1}^{n} e_k^2}{n} \right)^{1/2} \tag{18}$$

In equation (8) the measured phase of the sensor doesn't exactly equal to the initial phase of the sensor due to the measurement errors. However the equation can be considered correct approximately when m is the real value. The asynchronous vibration has two kinds of fractional parts, '+' and '–', which demands to calculate S_k in different situations and determine Δm '+' or '–'. The order of the vibration can be obtained with m when S_k is the minimum and by $\omega = (m \pm \Delta m)\Omega_n$, the vibration frequency can be solved. The flow chart of the parameter identification algorithm of sweep asynchronous vibration is given by figure 2.

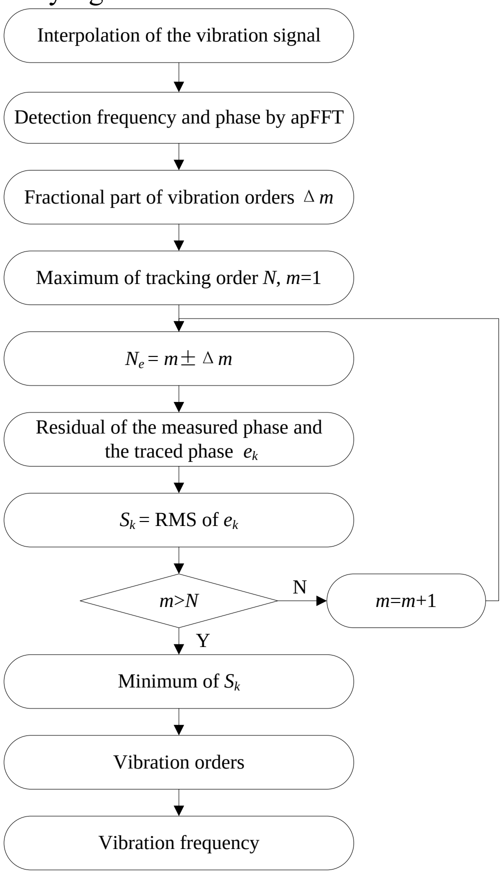

Figure 2. The flow chart of the parameter identification algorithm of asynchronous vibration under sweep.

3. Simulation verification

In order to verify the algorithm, the four-blade assembly mathematical model, similar with the one developed by G. Dimitridis[17][18], is used and simulation data of asynchronous responses in Simulink were available.

The basic physical system is illustrated in figure 3. Each blade of the assembly is idealised by a single mass-spring-damper. The stiffness of each blade is represented by a spring-damper attached to the disk and inter-blade coupling is simulated by spring-damper systems connected to adjacent blades.

The equations of motion for this system are given as

$$\mathbf{M}\ddot{\boldsymbol{y}}+\mathbf{C}\dot{\boldsymbol{y}}+\mathbf{K}\boldsymbol{y}=\boldsymbol{F}(t) \tag{16}$$

where $\boldsymbol{y}$ is the vibration inflection of the single blade, $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, and $\mathbf{K}$ is the stiffness matrix. They are given by

$$\mathbf{M}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{m_2}{m_1} & 0 & 0 \\ 0 & 0 & \frac{m_3}{m_1} & 0 \\ 0 & 0 & 0 & \frac{m_4}{m_1} \end{bmatrix} \tag{17}$$

$$\mathbf{C}=2\xi\omega_n\begin{bmatrix} \frac{c_1+c_{12}+c_{41}}{c_1} & -\frac{c_{12}}{c_1} & 0 & -\frac{c_{14}}{c_1} \\ -\frac{c_{12}}{c_1} & \frac{c_2+c_{12}+c_{23}}{c_1} & -\frac{c_{23}}{c_1} & 0 \\ 0 & -\frac{c_{23}}{c_1} & \frac{c_3+c_{23}+c_{34}}{c_1} & -\frac{c_{34}}{c_1} \\ -\frac{c_{14}}{c_1} & 0 & -\frac{c_{34}}{c_1} & \frac{c_4+c_{34}+c_{14}}{c_1} \end{bmatrix} \tag{18}$$

$$\mathbf{K}=\omega_n^{\ 2}\begin{bmatrix} \frac{k_1+k_{12}+k_{41}}{k_1} & -\frac{k_{12}}{k_1} & 0 & -\frac{k_{14}}{k_1} \\ -\frac{k_{12}}{k_1} & \frac{k_2+k_{12}+k_{23}}{k_1} & -\frac{k_{23}}{k_1} & 0 \\ 0 & -\frac{k_{23}}{k_1} & \frac{k_3+k_{23}+k_{34}}{k_1} & -\frac{k_{34}}{k_1} \\ -\frac{k_{14}}{k_1} & 0 & -\frac{k_{34}}{k_1} & \frac{k_4+k_{34}+k_{14}}{k_1} \end{bmatrix} \tag{19}$$

Note that ξ is the damping ratio given by

$$\xi=c_1/(2m_1\omega_n) \tag{20}$$

and ω_n is the natural frequency of the first blade alone given by

$$\omega_n^{\ 2}=k_1/m_1 \tag{21}$$

In the case where $m_1=m_2=m_3=m_4$, $c_1=c_2=c_3=c_4$, $k_1=k_2=k_3=k_4$, the simulation model of the blade vibration system is established using Matlab Simulink tool. For the single degree of freedom system, equation (16) can be rewritten as

$$m\ddot{y}+c\dot{y}+ky=F(t) \tag{22}$$

where $F(t)$ is the exciting force with input time t. The expression of $F(t)$ is given by

$$F(t)=F_A\cos(2\pi N_e f_v t+\varphi) \tag{23}$$

where F_A is the exciting force amplitude, N_e is the resonance order, f_v is the rotation frequency, φ is the initial phase. The model of the single blade vibration system in Matlab Simulink is shown in figure 4.

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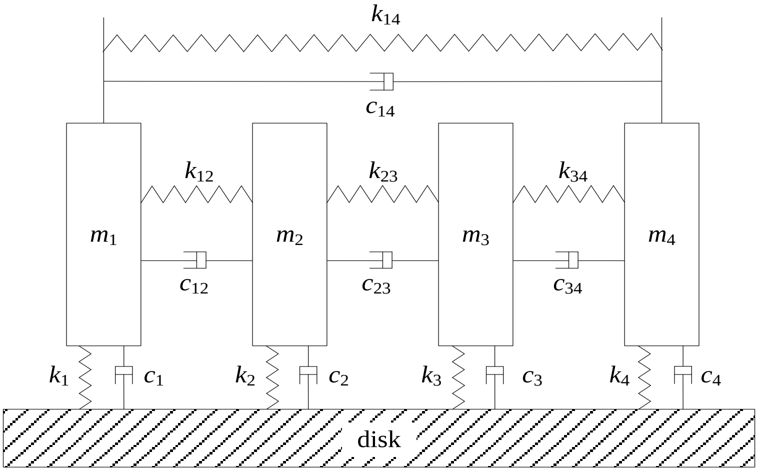

Figure 3. Model of four-blade vibration system.

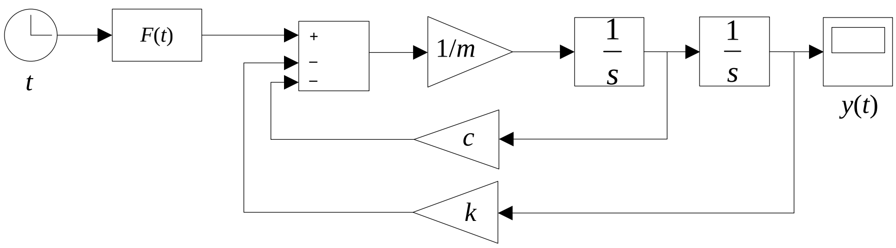

Figure 4. Model of the single blade vibration system.

The sweep speed range is selected from 1287Hz to 123Hz linearly according to the formula $f_n / f_{v1} < N_e < f_n / f_{v2}$, where f_{v1} is the initial frequency and f_{v2} is the stop frequency. The data contain asynchronous vibration orders 1.1, 2.2, 3.3, 4.4, 5.5, 6.6 and 7.7 which can produce the asynchronous vibration data of the blade with sensor angle position at 10°, 15°, 20° and 25°. The simulated parameter of each blade is around $f_{n1} = f_{n2} = f_{n3} = f_{n4} = 118032Hz$, $\xi_1 = \xi_2 = \xi_3 = \xi_4 = 0.06\%$. The simulation time is about 100s, with 30% white noise , the coupling stiffness and damping of the blades are assuming zero. After the settings, the deceleration asynchronous vibration signal with 10° the interval mounted angle of the sensors, is given by figure 5. The process of order tracking on 1.1 frequency multiplications is given by figures 6 and figure 7.

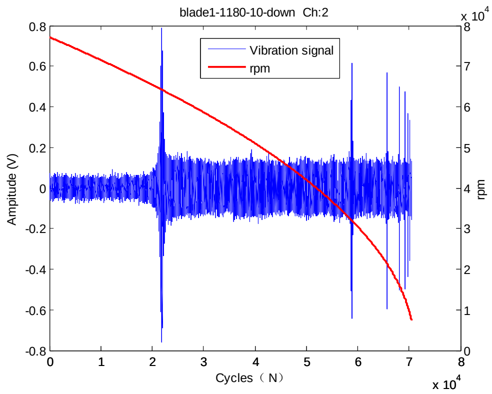

Figure 5. Asynchronous vibration simulation signal and speed change.

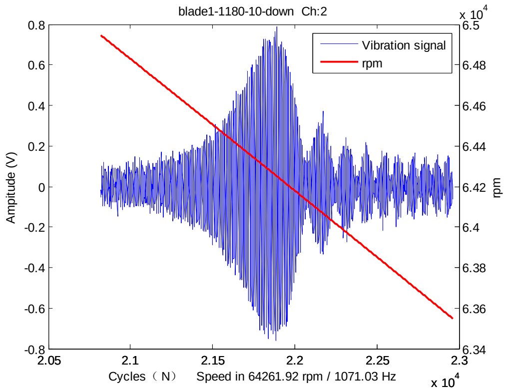

Figure 6. Data segment of 1.1 frequency multiplications signals and speed change.

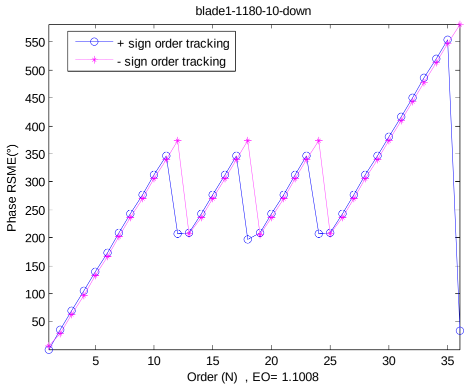

Figure 7. Order tracking on 1.1 frequency multiplications of Blade 1.

The parameter identification of asynchronous multi-octave deceleration simulation can be obtained by order tracking. When interval setting angles of the sensors are 10°, 15°, 20° and 25° respectively, the identification results are listed in table 1 to table 4.

The relative error of the identification, ΔE, is given by

$$\Delta E = \frac{|I - S|}{S} \times 100\% \tag{24}$$

where I is the identified value and S is the simulated value.

In the tables, both relative frequency errors and the order errors are less than 0.2% by tracking order, so the results are in the range of allowable errors and are considered correct. Tracking order can identify blade asynchronous vibration in any order if improved slightly. The detection frequency was derived with the spectrum analysis based on interpolation of the vibration signals. The simulated signals are better uniform under sampling data. Above simulated signals are un-uniform. So the decimal order of asynchronous has an error when it is greater than 0.5. It will be improved in the future.

Table 1. Parameter identification of Blade 1 with 10 °of the sensors in deceleration by tracking order.

	Orders						
Simulated order	1.1	2.2	3.3	4.4	5.5	6.6	7.7
Identified order	1.1	2.2	3.3	4.4	5.5	6.61	7.71
ΔE (%)	0	0	0	0	0	0.15	0.13
Simulated frequency(Hz)	1178	1177	1176.6	1175.4	1175.6	1173.5	1173.7
Identified frequency(Hz)	1179	1178	1178	1177	1175	1175	1175
ΔE (%)	0.08	0.08	0.12	0.14	0.05	0.13	0.11

Table 2. Parameter identification of Blade 1 with 15 °of the sensors in deceleration by tracking order.

	Orders						
Simulated order	1.1	2.2	3.3	4.4	5.5	6.6	7.7
Identified order	1.1	2.2	3.3	4.4	5.5	6.61	7.71
ΔE (%)	0	0	0	0	0	0.15	0.13
Simulated frequency(Hz)	1178.1	1177	1176.2	1175.5	1174.5	1175	1174.1
Identified frequency(Hz)	1179	1178	1177	1177	1176	1176	1176
ΔE (%)	0.08	0.08	0.07	0.13	0.13	0.09	0.16

Table 3. Parameter identification of Blade 1 with 20 °of the sensors in deceleration by tracking order.

	Orders						
Simulated order	1.1	2.2	3.3	4.4	5.5	6.6	7.7
Identified order	1.1	2.2	3.3	4.4	5.5	6.61	7.71
ΔE (%)	0	0	0	0	0	0.15	0.13
Simulated frequency(Hz)	1179	1177.3	1176.5	1175.3	1175.2	1174.4	1174.3
Identified frequency(Hz)	1179	1178	1177	1176	1174	1176	1176
ΔE (%)	0	0.06	0.04	0.06	0.1	0.14	0.14

Table 4. Parameter identification of Blade 1 with 25 °of the sensors in deceleration by tracking order.

	Orders						
Simulated order	1.1	2.2	3.3	4.4	5.5	6.6	7.7
Identified order	1.1	2.2	3.3	4.4	5.51	6.61	7.71
ΔE (%)	0	0	0	0	0.18	0.15	0.13
Simulated frequency(Hz)	1178.2	1177.1	1176.2	1175.6	1174.7	1174.4	1174.6
Identified frequency(Hz)	1179	1178	1177	1177	1177	1176	1175
ΔE (%)	0.07	0.08	0.07	0.12	0.2	0.14	0.03

4. Conclusion

Vibration is one of the main causes of mechanical failure in the rotating machinery. Blade tip timing technique is an approach to parameter identification using under-sampled signals. Interpolation and tracking order method can improve the sampling rate and identify parameters of blade asynchronous vibration. The detection frequency and phase can be obtained by all phases Fast Fourier Transform. Blade vibration can be obtained by order tracking with the relationship of the vibration phase and the setting angle of the sensor. Finally, the blade vibration frequency can be derived. The four blades vibration simulation model with asynchronous multi-octave deceleration is used to verify the algorithm. The results demonstrated that the algorithm can identify parameters of blade asynchronous vibration correctly and effectively.

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