

Suppressing self-excited vibrations of mechanical systems by impulsive force excitation

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Abstract. In this contribution, self-excited mechanical systems subjected to force excitation of impulsive type are investigated. It is shown that applying force impulses which are equally spaced in time, but whose impulsive strength depends in a certain manner on the state-variables of the mechanical system, results in a periodic energy exchange between lower and higher modes of vibration. Moreover, in the theoretical case of Dirac delta impulses, it is possible that no energy crosses the system boundary while energy is transferred across modes, i.e. neither external energy is fed to the mechanical system, nor energy is extracted from the mechanical system. Shifting energy to higher modes of vibration, whose natural damping is larger compared to lower ones, results in a faster dissipation of energy. An analytical stability investigation is presented using the assumption of impulsive forcing of Dirac delta type, which allows deciding easily about the stability by evaluating the eigenvalues of the coefficient matrix of a corresponding set of difference equations. It is shown that the developed impulsive forcing concept is capable to suppress self-excited vibrations of mechanical systems. Some numerical results of a simple mechanical system with two degrees of freedom underline the presented approach.

1. Introduction

Since a long time, self-excited vibrations of mechanical systems are, due to their dangerous nature, of interest to the engineering community. Hence, a lot of measures have been developed to avoid them. Their characteristic of feeding external energy to a mechanical system naturally leads to the question if this energy can be redirected in a way to finally achieve a stable system. One possibility is the introduction of nonlinear energy sinks (NES), see [1], for example. NES were first introduced for reducing vibrations of mechanical systems with initial excitation, see [2] for a variety of detailed investigations. It was shown that lightweight NES which are passively coupled to a primary mechanical structure allow to efficiently transfer vibration energy from the primary system to the NES in an unidirectional manner. Within the NES, the vibration energy is dissipated, and hence, a reduction of vibrations of the primary structure is achieved.

An other mechanism using nonlinear energy sinks is investigated in [3]. Therein, vibro-impact NES are introduced to transfer energy across modes of vibration. On a primary structure, two additional moveable masses are installed, which may exhibit impacts with the primary structure. It is demonstrated, that these impacts can lead to a modal redistribution of energy in the sense, that energy is transferred from lower to higher modes, resulting in a faster decrease of vibrations. The modal transfer of energy in systems with a periodically and continuously varying stiffness parameter was investigated in [4] and [5], for example. At certain frequencies of the varying



stiffness parameter, which were reported in [6], a periodic and bidirectional modal transfer of energy can be observed. By contrast to NES-systems, which are of passive nature, external energy is necessary in this case. In [7], the effect of switching a stiffness between two predefined levels is studied. It is shown that a transfer of energy from low modes of one stiffness state, to high modes of the other stiffness state is possible.

Unidirectional modal energy transfers applying stiffness impulses were investigated in [8], and the capabilities of suppressing self-excited vibrations were demonstrated in [9]. A correct timing of the application of the impulses is necessary in both cases to ensure energy transfers in only one direction, i.e. from low to high modes, and in general, the applied impulses are not equidistant in time.

In this contribution, a method using impulsive forcing is proposed to rearrange the modal energy distribution of self-excited dynamical systems in a way which allows to stabilize the otherwise unstable system. Therefore, impulsive forces, which are equidistant in time, but whose strength is state-dependent, are introduced. It is shown, that a transfer of discrete amounts of energy across modes of vibrations is possible, where the direction of transfer depends on the state of the system at the instant of time the impulse is applied. Moreover, it is demonstrated that certain states of the mechanical system exist where, in the theoretical case of Dirac delta force impulses, neither energy is fed to the mechanical system, nor energy is extracted from it. The repeated application of such impulses according to the proposed approach results in a description of the system dynamics at discrete, equidistant instants of time by a set of difference equations. Therewith, the stability of the trivial solution can be investigated by calculating the eigenvalues of the corresponding coefficient matrix. Numerical stability investigations of a mechanical system with two degrees of freedom show that the proposed method allows to suppress self-excited vibrations.

2. Analytical investigations

Assume that the equations of motion of a n -dimensional mechanical system are given in the following form

$$\mathbf{M}\mathbf{x}'' + \mathbf{C}\mathbf{x}' + \mathbf{K}\mathbf{x} = \sum_{k=1}^K \varepsilon_k \delta(\tau - \tau_k) \mathbf{f}, \quad (1)$$

where $\mathbf{x}$ represents the displacement vector, $(\)'$ denotes differentiation with respect to time τ , and $\mathbf{M}$ and $\mathbf{K}$ are the constant and symmetric mass and stiffness matrices. It is further assumed, that the matrix $\mathbf{C}$ can be decomposed in a damping and a self-excitation part according to

$$\mathbf{C} = \mathbf{C}_d + \mathbf{C}_{se}, \quad (2)$$

hence, self-excitation mechanisms which can be modelled by negative damping coefficients are investigated in this contribution. Moreover, the mechanical system is subjected to a sequence of K force impulses of Dirac delta type applied at instants of time τ_k , see right side of Eq. (1), where $\delta(\tau - \tau_k)$ represents the Dirac delta function. Each Dirac delta impulse can be seen as the simplest approximation of e.g. a half-sine shaped impulse. The constant vector

$$\mathbf{f} = [f_1, f_2, \dots, f_n]^T \quad (3)$$

specifies which masses are subjected to force impulses. Introducing the scaling factor ε_k allows the impulsive strength to be different for each impulse.

2.1. Energy transfer

The effect of a single Dirac delta force impulse to the displacements and velocities of the mechanical system is investigated following [10]. Therefore, Eq. (1) is rewritten for a single impulse applied at an instant of time τ_k .

$$\mathbf{M}\mathbf{x}'' + \mathbf{C}\mathbf{x}' + \mathbf{K}\mathbf{x} = \varepsilon_k \delta(\tau - \tau_k) \mathbf{f} \quad (4)$$

The impulse on the right hand side of the above equation must be balanced with an impulse on the left hand side. If, for example, $\mathbf{x}'$ would be of impulsive type, $\mathbf{x}''$ would contain the derivative of the Dirac delta function, which would have to be balanced on the right hand side. Since no such function appears on the right hand side, $\mathbf{x}'$ is not of impulsive type. Similar considerations hold for $\mathbf{x}$. Hence, the only possibility is that $\mathbf{x}''$ is an impulsive function, and therefore, $\mathbf{x}$ and $\mathbf{x}'$ remain bounded at τ_k . Integrating Eq. (4) over an infinitesimally small interval from τ_{k-} to τ_{k+} , where the $-/+$ sign denotes the instants of time just before/after application of the impulse, leads to

$$\mathbf{M} \underbrace{\int_{\tau_{k-}}^{\tau_{k+}} \mathbf{x}'' d\tau}_{\mathbf{x}'(\tau_{k+}) - \mathbf{x}'(\tau_{k-})} + \mathbf{C} \underbrace{\int_{\tau_{k-}}^{\tau_{k+}} \mathbf{x}' d\tau}_{=0} + \mathbf{K} \underbrace{\int_{\tau_{k-}}^{\tau_{k+}} \mathbf{x} d\tau}_{=0} = \mathbf{f} \underbrace{\int_{\tau_{k-}}^{\tau_{k+}} \delta(\tau - \tau_k) d\tau}_{=1} \varepsilon_k, \quad (5)$$

and finally

$$\mathbf{x}'(\tau_{k+}) = \mathbf{M}^{-1} \mathbf{f} \varepsilon_k + \mathbf{x}'(\tau_{k-}), \quad (6)$$

and

$$\mathbf{x}(\tau_{k+}) = \mathbf{x}(\tau_{k-}). \quad (7)$$

A force impulse of Dirac delta type has no effect on the displacements $\mathbf{x}$, see Eq. (7), i.e. the potential energy of the mechanical system remains constant across an impulse, whereas the velocities $\mathbf{x}'$, in general, are subject of variation (6). The corresponding variation of the kinetic energy ΔT of the overall system is given by

$$\Delta T_k = T(\tau_{k+}) - T(\tau_{k-}), \quad (8)$$

where $T(\tau_{k+})$ and $T(\tau_{k-})$ denote the kinetic energy just after/before application of the impulse. Using Eq. (6), ΔT_k can be written in the following form

$$\Delta T_k = \frac{1}{2} (\mathbf{f}^T \mathbf{M}^{-1} \mathbf{f} \varepsilon_k^2 + 2 \mathbf{f}^T \mathbf{x}'(\tau_{k-}) \varepsilon_k). \quad (9)$$

Obviously, ΔT_k is a quadratic function of the strength scaling factor ε_k , and depends on the velocities $\mathbf{x}'(\tau_{k-})$ just before the impulse. Assuming that the coefficients of ε_k and ε_k^2 in Eq. (9) are unequal zero, the equation

$$\Delta T_k = 0 \quad (10)$$

is fulfilled, of course, in the trivial case

$$\varepsilon_k = \varepsilon_{k,z1} = 0, \quad (11)$$

and in the case

$$\varepsilon_k = \varepsilon_{k,z2} = -\frac{2}{\mathbf{f}^T \mathbf{M}^{-1} \mathbf{f}} \mathbf{f}^T \mathbf{x}'(\tau_{k-}). \quad (12)$$

Interestingly, in the latter case, the impulse does not have any effect on the overall energy content of the mechanical system, although the impulsive strength $\varepsilon_k = \varepsilon_{k,z2} \neq 0$. For this reason, this

case is investigated in the following in more detail. Assume that an impulse with $\varepsilon_k = \varepsilon_{k,z2}$ is applied to only one vibrating mass of the n -dimensional mechanical system, i.e. for the vector $\mathbf{f}$ in Eq. (6), $f_i \neq 0$ for $i = p$, and $f_i = 0$ for $i \neq p$, $i, p \in [1, n]$ holds. It can easily be seen, that in this case, the velocity of the respective mass M_p changes its sign, but preserves its absolute value to fulfill Eq. (10), i.e. the kinetic energy of the mass M_p remains unchanged. Nevertheless, considering modal coordinates, the kinetic energy of each mode will, in general, be subject of variation. Therefore, a conventional modal transformation according to

$$\mathbf{x} = \mathbf{\Phi} \mathbf{y} \quad (13)$$

is introduced, where $\mathbf{\Phi}$ comprises the eigenvectors of the corresponding undamped system of Eq. (1) without external forcing. Equations (6) and (7) can now be written in the form

$$\mathbf{y}'(\tau_{k+}) = \mathbf{\Phi}^{-1} \mathbf{M}^{-1} \mathbf{f} \varepsilon_k + \mathbf{y}'(\tau_{k-}), \quad (14)$$

and

$$\mathbf{y}(\tau_{k+}) = \mathbf{y}(\tau_{k-}). \quad (15)$$

In general, the vector $\mathbf{\Phi}^{-1} \mathbf{M}^{-1} \mathbf{f}$ in Eq. (14) is fully occupied even if only one mass is subjected to force impulses, contrary to the vector $\mathbf{M}^{-1} \mathbf{f}$ in Eq. (6) which possesses elements unequal zero only at the positions corresponding to the respective masses. Hence, applying impulses only to one mass, allows to affect all modes of vibration y_i , $i = 1, \dots, n$ of the mechanical system. Splitting ΔT_k in Eq. (10) in the modal components leads to

$$\Delta T_k = \sum_{i=1}^n \Delta T_k^{(i)} = 0, \quad (16)$$

which is fulfilled for $\varepsilon_k = \varepsilon_{k,z2}$ as shown earlier. Therein, the superscript (i) denotes the i -th mode of vibration. The variations $\Delta T_k^{(i)}$ in Eq. (16) can be further grouped in modes which receive energy, and in modes from whom energy is extracted. Denoting the grouped modes as set A and set B , Eq. (16) reads

$$\Delta T_k = \Delta T_{A,k} + \Delta T_{B,k} = 0. \quad (17)$$

This means that energy is transferred from mode-set A to mode-set B or vice versa. It has to be pointed out, that Eq. (10) as well as (17) mean that no energy crosses the system boundary while energy is transferred, as the work of the impulsive forces is equal to the variation of the kinetic energy of the overall system, which is zero. Consequently, neither external energy is needed to initiate the transfer, nor is the transfer accompanied by extracting energy from the mechanical system. It is evident, that this only holds in the theoretical case of force-impulses of Dirac delta type.

At that point, the question about the benefit of transferring kinetic energy across modes of vibration arises. One possible advantage comes clear if the fact that higher modes possess larger damping ratios than lower ones is taken into consideration. Hence, shifting energy from a lower, less damped mode, to a higher mode with larger damping results in a faster dissipation of the transferred energy than in the lower mode. In the case where no self-excitation is present, a faster decay of free vibrations of the mechanical system is expected. In the self-excited case, a repeated energy transfer across modes, initiated by a sequence of impulses is maybe capable to suppress the self-excited vibrations, i.e. naturally leads to the question about the stability of the mechanical system.

2.2. Stability

To investigate the stability of the trivial solution of the mechanical system, the equations of motion (1) are written in first order form according to

$$\mathbf{q}' = \mathbf{A}\mathbf{q} + \sum_{k=1}^K \varepsilon_k \delta(\tau - \tau_k) \mathbf{b} \quad (18)$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{f} \end{bmatrix}, \quad (19)$$

where $\mathbf{I}$ represents the identity matrix. Between adjacent impulses, the mechanical system Eq. (18) has the well-known solution

$$\mathbf{q}(\tau) = \mathbf{e}^{\mathbf{A}(\tau - \tau_k)} \mathbf{q}(\tau_{k+}) \quad \text{for} \quad \tau_k < \tau \leq \tau_{k+1}, \quad (20)$$

see [11], for example. The equidistance of the impulses allows to regard the timeline as a sequence of timespans $\Delta\tau$, where the mechanical system is of autonomous type, followed by an impulse. With the abbreviation $\mathbf{D} = \mathbf{e}^{\mathbf{A}\Delta\tau}$, and Eqs. (6) and (7), the relation between the state-vector $\mathbf{q}(\tau_{k+})$ at the beginning τ_{k+} of an autonomous timespan, to the state-vector $\mathbf{q}(\tau_{k+1,+})$ just after the following impulse is of the form

$$\mathbf{q}(\tau_{k+1,+}) = \mathbf{q}(\tau_{k+1,-}) + \varepsilon_{k+1} \mathbf{b} = \mathbf{D}\mathbf{q}(\tau_{k+}) + \varepsilon_{k+1} \mathbf{b}. \quad (21)$$

Choosing ε_k to be state-dependent according to Eq. (12) leads to

$$\mathbf{q}(\tau_{k+1,+}) = \mathbf{R}\mathbf{q}(\tau_{k+}), \quad (22)$$

which is a set of linear difference equations with constant coefficients. The coefficient matrix reads

$$\mathbf{R} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} - 2\mathbf{G} \end{bmatrix} \mathbf{D} \quad (23)$$

with

$$\mathbf{G} = \frac{1}{\mathbf{f}^T \mathbf{M}^{-1} \mathbf{f}} \mathbf{M}^{-1} \mathbf{f} \mathbf{f}^T. \quad (24)$$

Equation (22) relates the state-vector at an instant of time τ_{k+} to the state at $\tau_{k+1,+}$, where $\tau_{k+1,+} - \tau_{k+} = \Delta\tau$ holds, i.e. $\mathbf{R}$ describes the *growth* of the state-vector $\mathbf{q}$. Following [11], $\mathbf{R}$ is denoted as the *growth matrix* of the system. Now, we can introduce a function $\chi(\tau)$, similarly to [11], comprising the values of the state-vector $\mathbf{q}(\tau)$ at the instants of time $0, \tau_{1+}, \tau_{2+}, \dots, \tau_{k+}, \dots$, where with Eq. (22),

$$\chi(\tau_{k+1,+}) = \mathbf{R}\chi(\tau_{k+}) \quad (25)$$

holds. The question about the stability of the trivial solution of the mechanical system Eq. (1) with ε_k according to Eq. (12) can be investigated by observing the evolution of the state-vectors in χ , which is described by the eigenvalues of the matrix $\mathbf{R}$. Hence, the mechanical system Eqs. (1) and (12), is asymptotically stable if, and only if, for the absolute value of all eigenvalues λ_i , $i \in [1, n]$ of $\mathbf{R}$

$$|\lambda_i| < 1 \quad (26)$$

holds, and unstable if the absolute value of at least one eigenvalue of $\mathbf{R}$ is larger than one. Deciding about the stability can either be done by numerically calculating the eigenvalues of $\mathbf{R}$, or analytically by using the Schur-Cohn criterium, see [12], for example.

3. Numerical example

Figure (1) shows a sketch of the investigated mechanical model. The lateral flow with constant

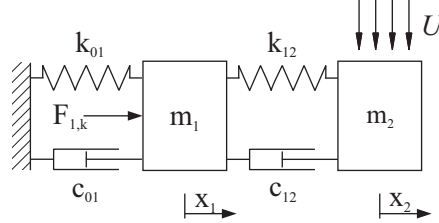


Figure 1. Investigated self-excited mechanical system with impulsive forcing of mass m_1 .

velocity U results in an axial force F_s on mass m_2 in the direction of x_2 . Using a linearized Rayleigh model, F_s can be written in the form

$$F_s = (a - bU^2)\dot{x}_2. \quad (27)$$

Denoting $c_s = a - bU^2$, self-excitation takes place if $c_s < 0$ holds. The mass m_1 is subjected to a sequence of Dirac delta force impulses

$$F_{1,k} = \varepsilon_k \delta(\tau - \tau_k) f_1 \quad (28)$$

where $k = 1, 2, \dots$, and f_1 was set to $f_1 = 1$. Applying a time transformation $\tau = \Omega_R t$, where $\Omega_R = \sqrt{k_R/m_R}$ represents a reference angular velocity, and introducing the non-dimensional system parameters according to

$$M_i = \frac{m_i}{m_R}, \quad K_{i-1,i} = \frac{k_{i-1,i}}{k_R}, \quad C_{i-1,i} = \alpha K_{i-1,i}, \quad C_s = \frac{c_s \Omega_R}{k_R}, \quad (29)$$

$i = 1, 2$, the equations of motion are of the form of Eq. (1). The following non-dimensional system parameters

$$M_1 = M_2 = 1, \quad K_{01} = K_{12} = 1, \quad \alpha = 0.01, \quad C_s = -0.02 \quad (30)$$

are used for the numerical investigations. Calculating the eigenvalues of the corresponding system matrix $\mathbf{A}$ leads to

$$\Lambda_{1,2} = -0.0103 \pm 1.6179i, \quad \text{and} \quad \Lambda_{3,4} = 0.0053 \pm 0.6180i, \quad (31)$$

just to show that C_s is tuned in a way that the mechanical system without impulsive excitation is unstable. The eigenvalues λ_i of the matrix $\mathbf{R}$, according to Eq. (23), decide about the stability of the system with impulsive force excitation, as described in section 2.2. Obviously, they depend on the constant time $\Delta\tau$ between impulses, i.e. $\lambda_i = \lambda_i(\Delta\tau)$, as the matrix $\mathbf{D}$ in Eq. (23) is a function of $\Delta\tau$. Hence, the effect of $\Delta\tau$ on the stability of the trivial solution is investigated in the following.

The maximum absolute value of the eigenvalues λ_i for different values of $\Delta\tau$ is depicted in Fig. 2. For most values of $\Delta\tau$, the impulsive force excitation has no stabilizing effect, as $\max|\lambda_i| > 1$ holds. Interestingly, two small intervals with the center at about $\Delta\tau_{p1}$ and $\Delta\tau_{p2}$ are observed where $\max|\lambda_i| < 1$, i.e. the originally unstable system becomes asymptotically stable by impulsive forcing. To give insight into the physical mechanism behind, it turned out,

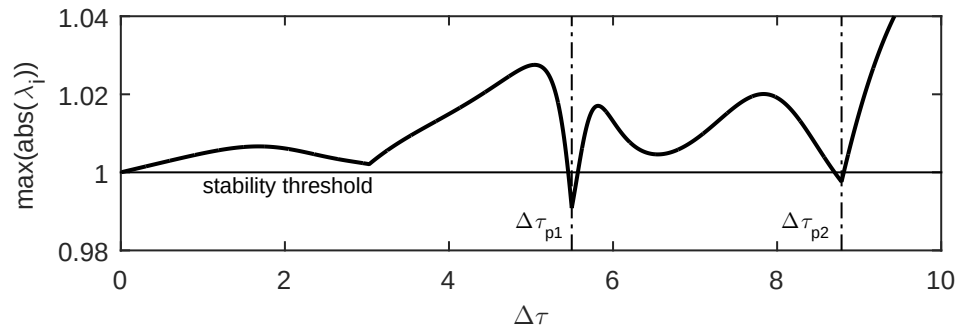


Figure 2. Maximum absolute value of the eigenvalues λ_i of the matrix $\mathbf{R}$ for different values of the constant time $\Delta\tau$ between impulses.

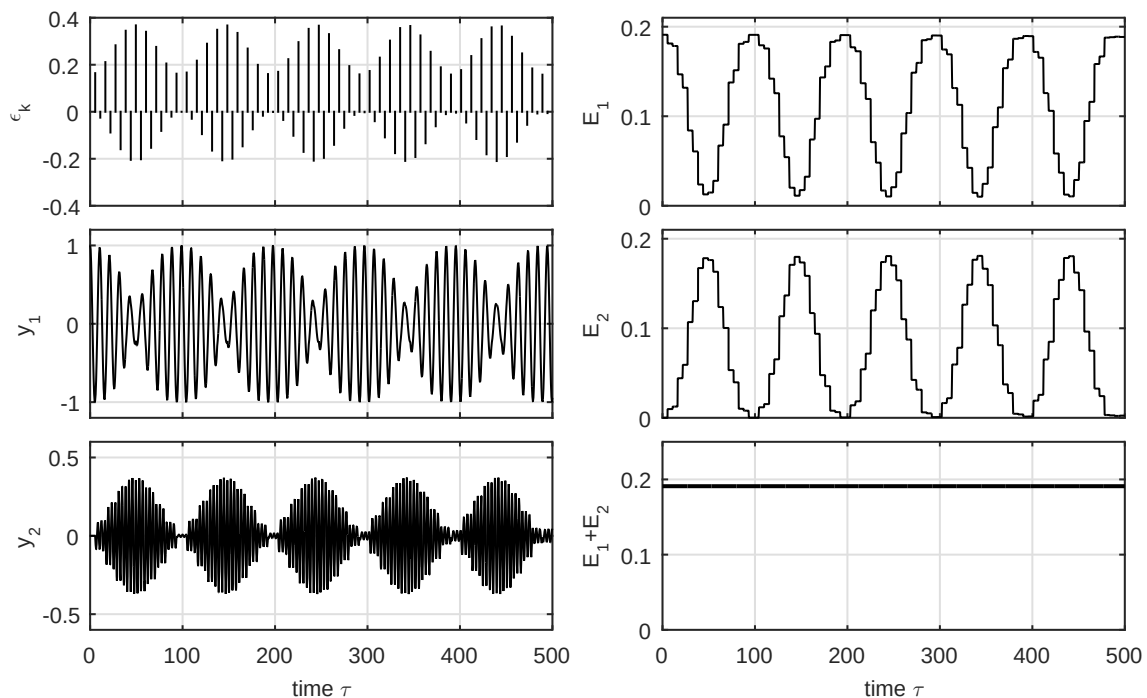


Figure 3. Timeseries of the strength scaling factor ε_k , the modal coordinates y_1 and y_2 , and the modal energies E_1 and E_2 for $\Delta\tau = \Delta\tau_{p1}$ of the mechanical system without natural damping and turned off self-excitation. Initial condition: first mode deflection.

that it is useful to look at the behaviour of the system when neglecting natural damping as well as switching off the self-excitation mechanism, i.e. in the equations of motion $\alpha = 0$ and $C_s = 0$. Figure 3 shows the corresponding timeseries of the strength scaling factor ε_k , the modal coordinates y_1 and y_2 , and the modal energies E_1 and E_2 for $\Delta\tau = \Delta\tau_{p1}$. As initial condition, a first mode deflection according to $y_1(\tau = 0) = 1$ and $y_2(\tau = 0) = 0$, ($y'_i(\tau = 0) = 0$, $i = 1, 2$) is used, i.e. initially, only the first mode contains energy. At $\tau = \Delta\tau_{p1}$, the first impulse is applied with a strength according to Eq. (12). Now, the second mode contains a part of the energy of the first mode. Until the second impulse is applied, the energy content of both modes remains constant, as there is no natural damping present, and the self-excitation mechanism is turned

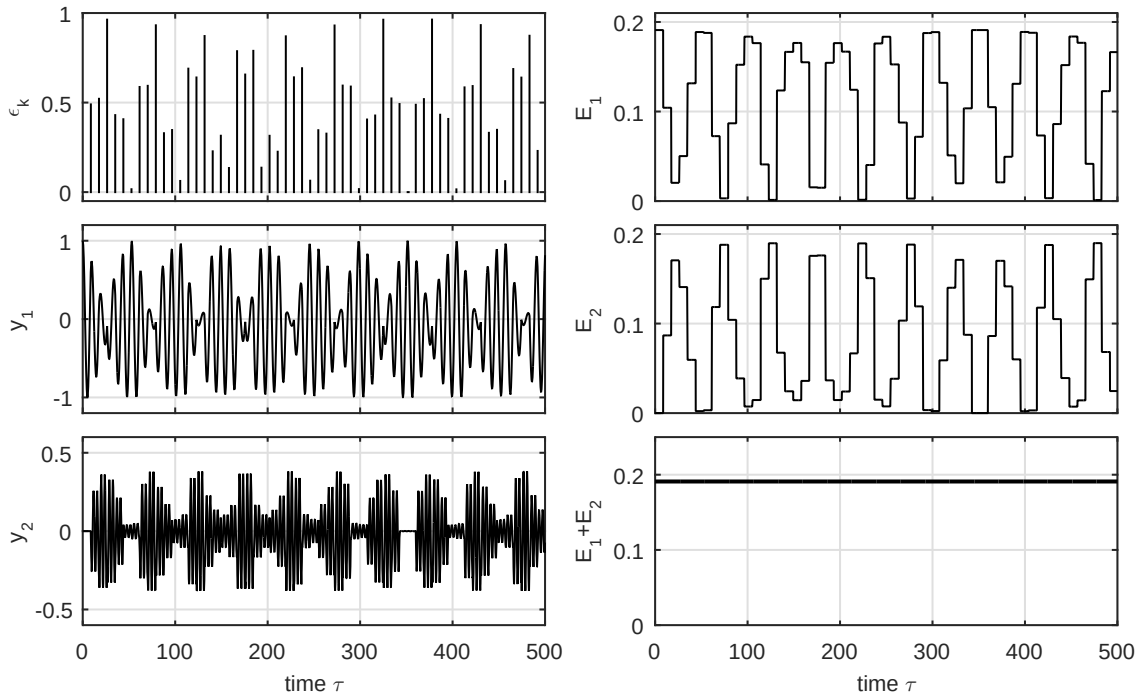


Figure 4. Timeseries of the strength scaling factor ϵ_k , the modal coordinates y_1 and y_2 , and the modal energies E_1 and E_2 for $\Delta\tau = \Delta\tau_{p2}$ of the mechanical system without natural damping and turned off self-excitation. Initial condition: first mode deflection.

off. In the following, a quasi-periodic sequence of impulses with strength ϵ_k is observed, see Fig. 3 (left column, top), which is not symmetric with respect to the time-axis. This sequence of impulses results in a periodic decrease and increase of first mode vibrations $y_1(\tau)$, see Fig. 3 (left column, center) and an opposed modulation of second mode vibrations $y_2(\tau)$, see Fig. 3 (left column, bottom). This means, that vibration energy is transferred between modes one and two in a periodic manner.

The modal exchange of energy is shown clearly if the total modal energy contents E_i , $i = 1, 2$, comprising potential and kinetic energy are introduced. Figure 3 (right) shows the corresponding results. At the initial time $\tau = 0$, $E_1(0) \neq 0$ and $E_2(0) = 0$. With ongoing time, E_1 decreases, as more and more energy is shifted to the second mode, resulting in an increase of E_2 . At about $\tau \approx 50$ time-units, E_1 has approached a minimum value (approximately 6.7% of the initial energy), and E_2 a maximum one. Thereafter, E_1 and E_2 increase and decrease periodically, but in the opposite direction. It has to be pointed out that the impulsive forces, per definition according to Eqs. (10) and (12), neither add energy to, nor extract energy from the mechanical system. For this reason, the total energy content $E_1 + E_2$ of the mechanical system remains constant throughout all the time, see Fig. 3 (right column, bottom).

If the timespan $\Delta\tau$ between the impulses is set to $\Delta\tau = \Delta\tau_{p2}$, the results depicted in Fig. 4 are obtained. First of all, one recognizes that ϵ_k takes only positive values, contrary to Fig. 3. Moreover, a faster exchange of energy between the modes of vibration is observed. In principle, the same behavior as in Fig. 3 occurs, namely a periodic modal exchange of energy where the total energy content of the mechanical system remains constant. For this reason, only the case $\Delta\tau = \Delta\tau_{p1}$ is investigated in the following.

To further demonstrate how transferring energy across modes is capable to stabilize unstable, self-excited systems, natural damping is introduced, i.e. $\alpha = 0.01$ according to Eqs. (30).

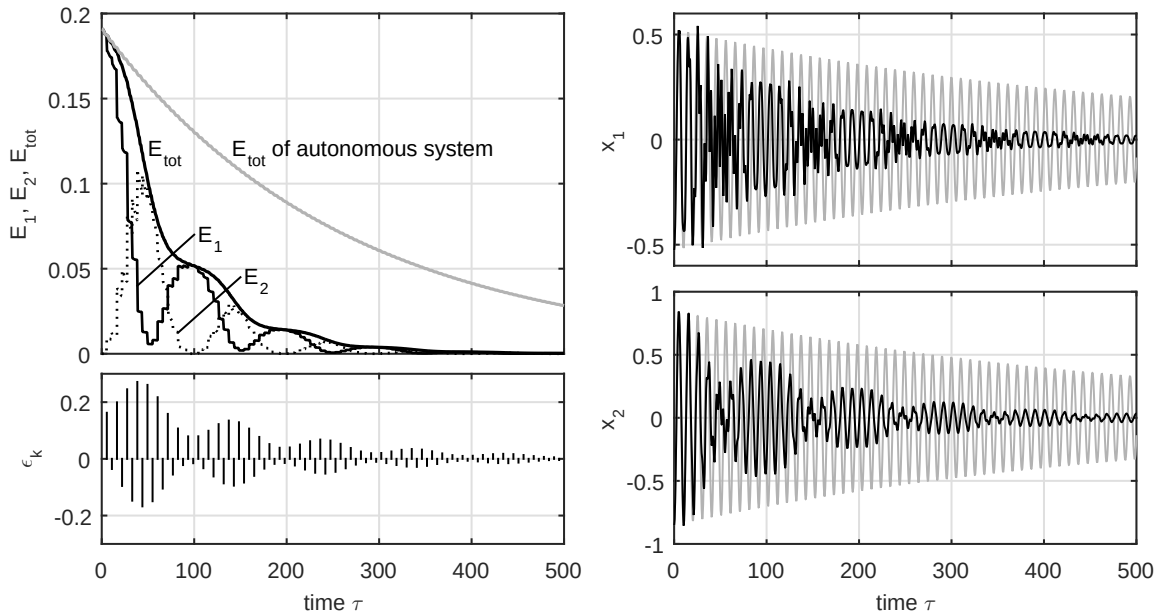


Figure 5. Timeseries of the mechanical system with natural damping but in the absence of self-excitation. Modal energy contents E_1 (solid) and E_2 (dotted), total energy content $E_{tot} = E_1 + E_2$, impulsive strength scaling factor ε_k (left), and physical displacements x_1 and x_2 (right) for $\Delta\tau = \Delta\tau_{p1}$. Grey-colored lines denote results of the corresponding system without impulsive excitation. Initial condition: first mode deflection.

The self-excitation mechanism is still turned off, and a first mode deflection is used as initial condition. Figure 5 (left) shows the modal energy contents E_1 and E_2 , and the total energy content $E_{tot} = E_1 + E_2$. As before, a modal exchange of energy between first and second mode is observed, resulting now in a much faster decrease of E_{tot} compared to the autonomous system (see grey colored line). In the presence of natural damping, shifting energy to the higher mode allows to utilize the enhanced damping properties of the higher mode compared to the lower one in a better way. A discrete amount of energy, which is transferred to a higher mode is dissipated in a shorter period of time in the higher mode than in the lower one. Consequently, the total energy content of the mechanical system decreases faster than in the case where no impulsive excitation is active (see grey-colored line). Figure 5 (right) shows the corresponding physical displacements x_1 and x_2 . According to the enhanced dissipation of energy, x_1 and x_2 decrease much faster compared to the system without impulsive excitation (grey colored lines). Hence, shifting energy to higher modes using the proposed approach can also be seen as a method of active vibration suppression of mechanical systems.

Henceforth, the self-excitation mechanism is turned on. According to the eigenvalues Eq. (31), the trivial solution of the autonomous system is unstable. As shown in Fig. 2, impulsive excitation with $\Delta\tau = \Delta\tau_{p1}$ allows to stabilize the unstable system. Figure 6 shows the corresponding results in the time domain, where as initial condition an initial velocity of mass #2 according to $x'_1(\tau = 0) = 0$ and $x'_2(\tau = 0) = 1$, ($x_i(\tau = 0) = 0$, $i = 1, 2$) is used. As previously, the impulsive force excitation causes a repeated transfer of energy between first and second mode in both directions, but with higher frequency. Due to the enhanced utilization of the damping properties of the mechanical system, the total energy content E_{tot} decreases to zero. In the case without impulsive excitation, E_{tot} increases beyond all limits (see grey-colored line). The same applies for the physical displacements x_1 and x_2 , see Fig. 6 (right).

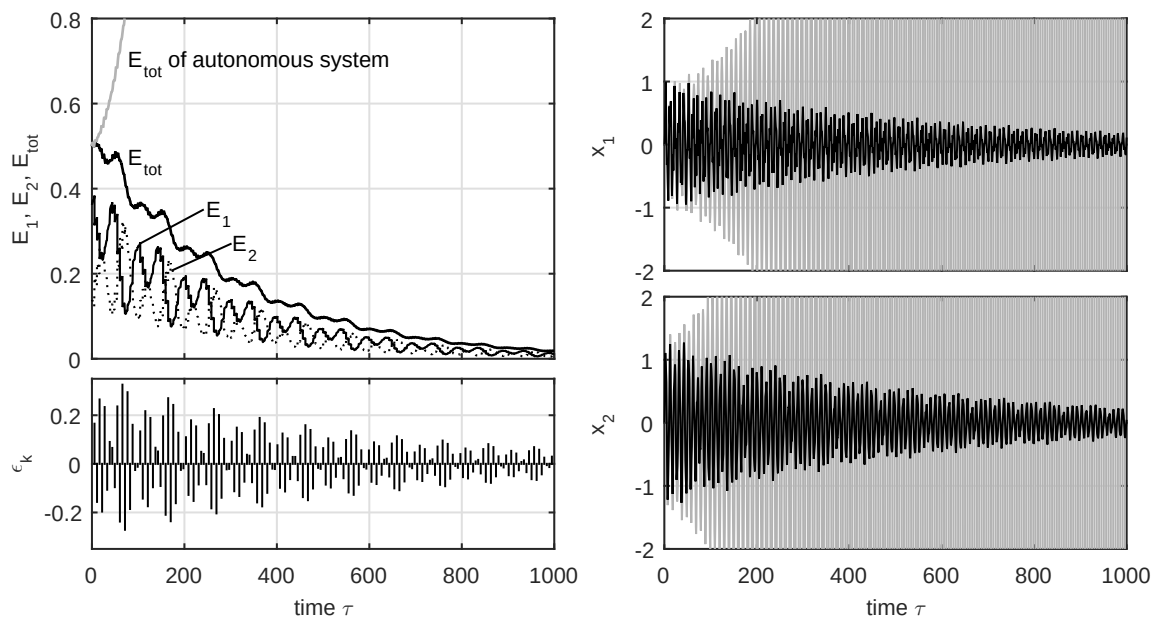


Figure 6. Timeseries of the self-excited mechanical system. Modal energy contents E_1 (solid) and E_2 (dotted), total energy content $E_{tot} = E_1 + E_2$, impulsive strength scaling factor ϵ_k (left), and physical displacements x_1 and x_2 (right) for $\Delta\tau = \Delta\tau_{p1}$. Grey-colored lines denote results of the corresponding system without impulsive excitation. Initial condition: initial velocity of mass #2.

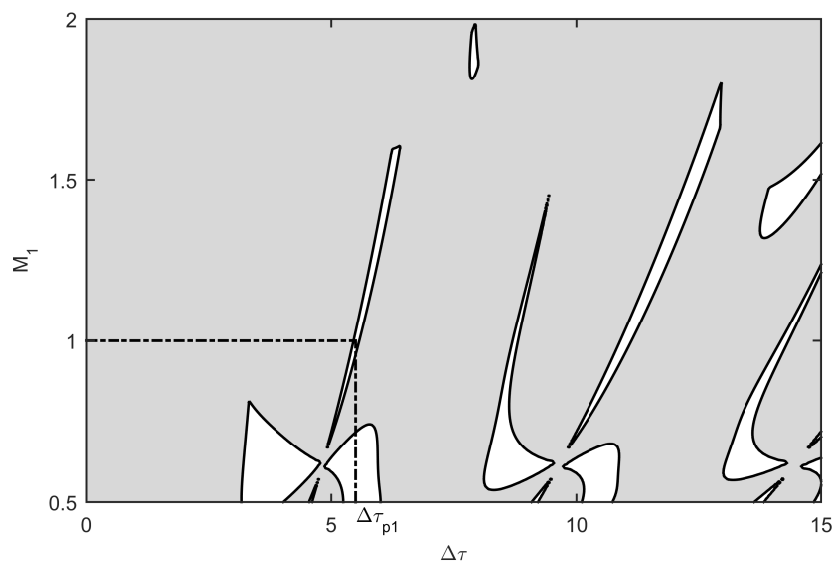


Figure 7. Stability chart for variation of the system parameter M_1 and the constant timespan $\Delta\tau$ between impulses. Grey=unstable, white=asymptotically stable trivial solution.

Up to now, the stability properties for a constant set of system parameters was studied. Figure 7 displays the results of a stability investigation for different values of the mass M_1 and the

timespan $\Delta\tau$ between impulses. For all values of M_1 within the interval $[0.5, 2]$, the system without impulsive force excitation is unstable. Gray-colored areas in Fig. 7 denote instability, whereas white-colored ones represent an asymptotically stable trivial solution. The stability threshold $\max|\lambda_i| = 1$ is marked by the solid black lines. The previously investigated set of parameters $M_1 = 1$ and $\Delta\tau = \Delta\tau_{p1}$ is indicated by the dashed-dotted lines, and is part of a narrow asymptotically stable area which becomes wider with increasing M_1 and shifts to higher values of $\Delta\tau$. Two large areas of stability are observed in the vicinity of $\Delta\tau \approx 4.9$ and $M_1 \approx 0.6$. Moreover, additional areas of stability appear near $\Delta\tau \approx 9.5$ and $\Delta\tau \approx 14.4$, which look similar to the foregoing. Figure 7 also demonstrates that the proposed impulsive forcing allows to suppress self-excited vibrations for almost all values of M_1 within the interval $[0.5, 2]$ by choosing a proper value of $\Delta\tau$.

4. Conclusions

A new method of suppressing self-excited vibrations of mechanical systems by introducing impulsive forcing was presented in this contribution. It was shown that a state-dependent impulsive strength exists, which allows to shift discrete amounts of energy from one mode of vibration to the other. In this special case, no energy is added to the mechanical system by the impulsive forces. The stability properties of the mechanical system can be investigated easily by calculating eigenvalues of a set of difference equations. It was demonstrated, that this method is capable to suppress self-excited vibrations of mechanical systems.

Acknowledgments

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