

# Active vibration control of a plate using vibration gradients

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**Abstract.** Minimization of the squared transverse velocity at a measurement point does not guarantee the global vibration reduction for the whole structure, and the control result is dependent on the measurement point. Flexibility of the sensor placement is usually limited in practice. If the measurement point is near the nodal line of the mode, this mode cannot be decreased effectively and even increased by the control force. This study investigates the control method with the error criterion being the sum of the squared vibration velocity and the squared vibration gradients (spatial gradients) at a measurement point. Since the spatial distributions of the vibration velocity and its gradients are different, the aforementioned problem caused by the nodal line are mitigated. The numerical examples indicate that the performance of the control including the vibration gradients is less dependent on the measurement point, and this method achieves a better global vibration reduction, than the conventional method, i.e., minimization of the squared vibration velocity.

## 1. Introduction

Minimization of the vibration velocity at a single measurement point does not guarantee a global vibration reduction of the entire structure. Although the global vibration reduction could be achieved by introducing a number of measurement points over the whole structure, sensors may not be placed as planned in practice due to some design limitation. Hence, it is needed to reduce the global vibration by controlling only the local vibration.

The aforementioned technical problem is the same in active noise control. Minimization of the sound pressure at a single measurement point does not guarantee a global noise reduction in the whole enclosure. To overcome this problem, controlling the total energy density, i.e., the sum of the acoustic potential energy density and acoustic kinetic energy density, was proposed [1]. The acoustic potential energy density, which is proportional to the squared sound pressure, and the acoustic kinetic energy density, which is proportional to the squared particle velocity, have different spatial distributions to each other, since the particle velocity is a spatial gradient of the sound pressure. Hence, setting the sum of the both energy densities as control criterion mitigates the observability problem of acoustic modes, which leads to a degradation in the global noise reduction. Multiple microphones are placed around a point of interest, and the sound pressure and its gradients are obtained. Though multiple microphones are used, the measurement point is essentially one, which means that the sensor system is local. It was found in earlier works that active noise control with the criterion being the sum of the squared sound pressure and the squared sound pressure gradients can produce better global control effect than conventional active noise control the criterion of which is only the squared sound pressure [1]. This idea was also expanded to active structural acoustic control, and it was shown that the global acoustic power



radiated from a vibrating panel can be reduced by controlling the weighed sum of the squared vibration velocity and squared vibration gradients [2].

The purpose of this paper is to investigate active vibration control with the criterion being the sum of the squared vibration velocity and the squared vibration gradients for rectangular panels. Basic vibration control with a single measurement point and cluster control with a pair of four measurement points [3] are investigated, respectively. The objective of basic vibration control is to suppress all the structural modes, and the objective of cluster control is to independently suppress the structural modes with high radiation efficiency. Numerical examples are presented and the control performance, i.e., the global vibration reduction, is compared between the conventional control with the criterion being only the vibration velocity and the control with the criterion being the vibration velocity and its gradients.

In section 2, the criteria which consists of not only the vibration velocity but also the vibration gradients are formulated for the basic vibration control and cluster control, respectively. In section 3, the global control performance of these two vibration control methods are verified via numerical examples. In section 4, findings of this study are summarized.

## 2. Theory

In this section, the control criterion composed of vibration velocity and its gradients is formulated. The formulations for basic vibration control and cluster control are presented in section 2.1 and 2.2, respectively.

### 2.1. Active vibration control with vibration gradients

Consider a rectangular panel with an arbitrary boundary condition. According to the modal expansion theory, the velocity of the panel is written as

$$v(x, y) = \boldsymbol{\varphi}^T(x, y)\mathbf{a}, \quad (1)$$

where  $(x, y)$  denotes an arbitrary location on the panel,  $\boldsymbol{\varphi}(x, y)$  is the real vector the terms of which are the structural modal functions, the superscript T denotes transpose, and  $\mathbf{a}$  is the complex vector the terms of which are complex modal amplitude. It is assumed that the panel is excited by forces with angular frequency  $\omega$  and the time dependence  $\exp(i\omega t)$  is omitted for brevity. Spatial gradients of the velocity are then given by

$$\frac{\partial v(x, y)}{\partial x} = \boldsymbol{\varphi}_x^T(x, y)\mathbf{a}, \quad (2)$$

$$\frac{\partial v(x, y)}{\partial y} = \boldsymbol{\varphi}_y^T(x, y)\mathbf{a}, \quad (3)$$

$$\frac{\partial^2 v(x, y)}{\partial x \partial y} = \boldsymbol{\varphi}_{xy}^T(x, y)\mathbf{a}, \quad (4)$$

where the terms of  $\boldsymbol{\varphi}_x(x, y)$ ,  $\boldsymbol{\varphi}_y(x, y)$ , and  $\boldsymbol{\varphi}_{xy}(x, y)$  are gradients of the structural modal functions in the  $x$ ,  $y$ , and  $x$  and  $y$  directions, respectively. Nodes of the structural modal function and its  $x$ ,  $y$ , and  $x$  and  $y$  directional gradients are usually different. Hence, the deterioration in global control performance due to the observability problem can be mitigated by setting the control criterion,  $J(x, y)$ , as

$$J(x, y) = |v(x, y)|^2 + \left| \frac{\partial v(x, y)}{\partial x} \right|^2 + \left| \frac{\partial v(x, y)}{\partial y} \right|^2 + \left| \frac{\partial^2 v(x, y)}{\partial x \partial y} \right|^2. \quad (5)$$

### 2.2. Cluster control with vibration gradients

The panel is assumed to be symmetric in the  $x$  and  $y$  directions. The structural modal functions are then classified into the following four groups: even function in the  $x$  and  $y$  directions (even/even cluster); odd function in the  $x$  direction and even function in the  $y$  direction (odd/even cluster); even function in

the  $x$  direction and odd function in the  $y$  direction (even/odd cluster); and odd function in the  $x$  and  $y$  directions (odd/odd cluster). It is known that the even/even cluster is the most contributive to the radiated acoustic power at low frequencies, and controlling the even/even cluster independently is a good strategy for noise reduction [3]. Equations (1) through (4) can be rewritten as

$$v(x, y) = \begin{bmatrix} \Phi_{e/e}(x, y) \\ \Phi_{o/e}(x, y) \\ \Phi_{e/o}(x, y) \\ \Phi_{o/o}(x, y) \end{bmatrix}^T \begin{bmatrix} \mathbf{a}_{e/e} \\ \mathbf{a}_{o/e} \\ \mathbf{a}_{e/o} \\ \mathbf{a}_{o/o} \end{bmatrix}, \quad (6)$$

$$\frac{\partial v(x, y)}{\partial x} = \begin{bmatrix} \Phi_{e/e,x}(x, y) \\ \Phi_{o/e,x}(x, y) \\ \Phi_{e/o,x}(x, y) \\ \Phi_{o/o,x}(x, y) \end{bmatrix}^T \begin{bmatrix} \mathbf{a}_{e/e} \\ \mathbf{a}_{o/e} \\ \mathbf{a}_{e/o} \\ \mathbf{a}_{o/o} \end{bmatrix}, \quad (7)$$

$$\frac{\partial v(x, y)}{\partial y} = \begin{bmatrix} \Phi_{e/e,y}(x, y) \\ \Phi_{o/e,y}(x, y) \\ \Phi_{e/o,y}(x, y) \\ \Phi_{o/o,y}(x, y) \end{bmatrix}^T \begin{bmatrix} \mathbf{a}_{e/e} \\ \mathbf{a}_{o/e} \\ \mathbf{a}_{e/o} \\ \mathbf{a}_{o/o} \end{bmatrix}, \quad (8)$$

$$\frac{\partial^2 v(x, y)}{\partial x \partial y} = \begin{bmatrix} \Phi_{e/e,xy}(x, y) \\ \Phi_{o/e,xy}(x, y) \\ \Phi_{e/o,xy}(x, y) \\ \Phi_{o/o,xy}(x, y) \end{bmatrix}^T \begin{bmatrix} \mathbf{a}_{e/e} \\ \mathbf{a}_{o/e} \\ \mathbf{a}_{e/o} \\ \mathbf{a}_{o/o} \end{bmatrix}. \quad (9)$$

It is assumed that four measurement points are placed symmetrically as  $(x_1, y_1)$ ,  $(x_2, y_2) = (L_x - x_1, y_1)$ ,  $(x_3, y_3) = (x_1, L_y - y_1)$ , and  $(x_4, y_4) = (L_x - x_1, L_y - y_1)$ , where  $L_x$  and  $L_y$  are the lengths of the panel in the  $x$  and  $y$  directions. Considering that the derivative of an even function is odd and *vice versa*, the following equations are satisfied

$$\Phi_{e/e}(x_1, y_1) = \Phi_{e/e}(x_2, y_2) = \Phi_{e/e}(x_3, y_3) = \Phi_{e/e}(x_4, y_4), \quad (10a)$$

$$\Phi_{o/e}(x_1, y_1) = -\Phi_{o/e}(x_2, y_2) = \Phi_{o/e}(x_3, y_3) = -\Phi_{o/e}(x_4, y_4), \quad (10b)$$

$$\Phi_{e/o}(x_1, y_1) = \Phi_{e/o}(x_2, y_2) = -\Phi_{e/o}(x_3, y_3) = -\Phi_{e/o}(x_4, y_4), \quad (10c)$$

$$\Phi_{o/o}(x_1, y_1) = -\Phi_{o/o}(x_2, y_2) = -\Phi_{o/o}(x_3, y_3) = \Phi_{o/o}(x_4, y_4), \quad (10d)$$

$$\Phi_{e/e,x}(x_1, y_1) = -\Phi_{e/e,x}(x_2, y_2) = \Phi_{e/e,x}(x_3, y_3) = -\Phi_{e/e,x}(x_4, y_4), \quad (11a)$$

$$\Phi_{o/e,x}(x_1, y_1) = \Phi_{o/e,x}(x_2, y_2) = \Phi_{o/e,x}(x_3, y_3) = \Phi_{o/e,x}(x_4, y_4), \quad (11b)$$

$$\Phi_{e/o,x}(x_1, y_1) = -\Phi_{e/o,x}(x_2, y_2) = -\Phi_{e/o,x}(x_3, y_3) = \Phi_{e/o,x}(x_4, y_4), \quad (11c)$$

$$\Phi_{o/o,x}(x_1, y_1) = \Phi_{o/o,x}(x_2, y_2) = -\Phi_{o/o,x}(x_3, y_3) = -\Phi_{o/o,x}(x_4, y_4), \quad (11d)$$

$$\Phi_{e/e,y}(x_1, y_1) = \Phi_{e/e,y}(x_2, y_2) = -\Phi_{e/e,y}(x_3, y_3) = -\Phi_{e/e,y}(x_4, y_4), \quad (12a)$$

$$\Phi_{o/e,y}(x_1, y_1) = -\Phi_{o/e,y}(x_2, y_2) = -\Phi_{o/e,y}(x_3, y_3) = \Phi_{o/e,y}(x_4, y_4), \quad (12b)$$

$$\Phi_{e/o,y}(x_1, y_1) = \Phi_{e/o,y}(x_2, y_2) = \Phi_{e/o,y}(x_3, y_3) = \Phi_{e/o,y}(x_4, y_4), \quad (12c)$$

$$\Phi_{o/o,y}(x_1, y_1) = -\Phi_{o/o,y}(x_2, y_2) = \Phi_{o/o,y}(x_3, y_3) = -\Phi_{o/o,y}(x_4, y_4), \quad (12d)$$

$$\Phi_{e/e,xy}(x_1, y_1) = -\Phi_{e/e,xy}(x_2, y_2) = -\Phi_{e/e,xy}(x_3, y_3) = \Phi_{e/e,xy}(x_4, y_4), \quad (13a)$$

$$\Phi_{o/e,xy}(x_1, y_1) = \Phi_{o/e,xy}(x_2, y_2) = -\Phi_{o/e,xy}(x_3, y_3) = -\Phi_{o/e,xy}(x_4, y_4), \quad (13b)$$

$$\Phi_{e/o,xy}(x_1, y_1) = -\Phi_{e/o,xy}(x_2, y_2) = \Phi_{e/o,xy}(x_3, y_3) = -\Phi_{e/o,xy}(x_4, y_4), \quad (13c)$$

$$\Phi_{o/o,xy}(x_1, y_1) = \Phi_{o/o,xy}(x_2, y_2) = \Phi_{o/o,xy}(x_3, y_3) = \Phi_{o/o,xy}(x_4, y_4). \quad (13d)$$

Therefore, the even/even cluster can be extracted as

$$v(x_1, y_1) + v(x_2, y_2) + v(x_3, y_3) + v(x_4, y_4) = 4\Phi_{e/e}^T(x_1, y_1)\mathbf{a}_{e/e}, \quad (14a)$$

$$\frac{\partial v(x_1, y_1)}{\partial x} - \frac{\partial v(x_2, y_2)}{\partial x} + \frac{\partial v(x_3, y_3)}{\partial x} - \frac{\partial v(x_4, y_4)}{\partial x} = 4\Phi_{e/e,x}^T(x_1, y_1)\mathbf{a}_{e/e}, \quad (14b)$$

$$\frac{\partial v(x_1, y_1)}{\partial y} + \frac{\partial v(x_2, y_2)}{\partial y} - \frac{\partial v(x_3, y_3)}{\partial y} - \frac{\partial v(x_4, y_4)}{\partial y} = 4\Phi_{e/e,y}^T(x_1, y_1)\mathbf{a}_{e/e}, \quad (14c)$$

$$\frac{\partial^2 v(x_1, y_1)}{\partial x \partial y} - \frac{\partial^2 v(x_2, y_2)}{\partial x \partial y} - \frac{\partial^2 v(x_3, y_3)}{\partial x \partial y} + \frac{\partial^2 v(x_4, y_4)}{\partial x \partial y} = 4\Phi_{e/e,xy}^T(x_1, y_1)\mathbf{a}_{e/e}. \quad (14d)$$

The signal processing described in equation (14a) is conventional cluster filtering [3], and those described in equations (14b) through (14d) are the expansion to the vibration gradients. It should be noted that each of the other clusters can also be extracted in a similar fashion, i.e., linear sum of the four velocities measured at symmetric locations with a different combination of the signs. The following criterion enables the even/even cluster control to mitigate the deterioration in global control performance due to the observability problem:

$$J(x_1, y_1) = |v(x_1, y_1) + v(x_2, y_2) + v(x_3, y_3) + v(x_4, y_4)|^2 + \left| \frac{\partial v(x_1, y_1)}{\partial x} - \frac{\partial v(x_2, y_2)}{\partial x} + \frac{\partial v(x_3, y_3)}{\partial x} - \frac{\partial v(x_4, y_4)}{\partial x} \right|^2 + \left| \frac{\partial v(x_1, y_1)}{\partial y} + \frac{\partial v(x_2, y_2)}{\partial y} - \frac{\partial v(x_3, y_3)}{\partial y} - \frac{\partial v(x_4, y_4)}{\partial y} \right|^2 + \left| \frac{\partial^2 v(x_1, y_1)}{\partial x \partial y} - \frac{\partial^2 v(x_2, y_2)}{\partial x \partial y} - \frac{\partial^2 v(x_3, y_3)}{\partial x \partial y} + \frac{\partial^2 v(x_4, y_4)}{\partial x \partial y} \right|^2. \quad (15)$$

Not only measurement but also excitation of each cluster is possible by taking advantage of the properties of odd and even functions, which are described in equations (10) though (13). Only the even/even cluster can be controlled without spillover among the clusters by driving four point forces at symmetric locations in the same amplitude and phase, which comes from equation (10a) and is referred to as cluster actuation [3].

### 3. Numerical examples

In this section, active vibration control with the criterion including vibration gradients and the one with criterion being only the vibration velocity are compared in terms of their global control performance. The basic vibration control and cluster control are discussed in section 3.1 and 3.2, respectively.

#### 3.1. Active vibration control with vibration gradients

A simply supported steel panel with Young's modulus of 207 GPa, mass density of 7870 kg/m<sup>3</sup>, Poisson's ratio of 0.292, damping loss factor of 0.01, the  $x$  directional length of  $L_x = 0.56$  m, the  $y$  directional length of  $L_y = 0.40$  m, and thickness of 0.0015 m is considered. The frequency band of interest is up to 200 Hz and resonant modes within this frequency band are listed in table 1. A disturbance force is positioned at  $(4L_x/9, 2L_y/9)$ .

**Table 1.** Structural modes the natural frequencies of which are below 200 Hz.

No.	Natural Freq. Hz	Modal Index
1	34.4	(1,1)
2	69.3	(2,1)
3	102.8	(1,2)
4	127.5	(3,1)
5	137.7	(2,2)
6	195.9	(3,2)

Control system with a single measurement point is considered here. The panel is equally divided into 16 elements as shown in figure 1, and a single measurement point is located at the center of one of the elements, in order to verify the dependence of control performance on the measurement location. A single control force is used and its location is  $(L_x/9, L_y/9)$ . Since the criterion defined in equation (5) is the Hermitian quadratic form, the optimal feedforward control force to minimize the criterion can be obtained in a manner such that the derivatives of the criterion with respect to the real and imaginary parts of the control force are set to zero [4]. All the control effects presented in this paper are derived by this method to show the theoretically achievable control performances. In practice, the proposed control method may be implemented by the filtered- $x$  LMS algorithm [5].

For comparison, the conventional feedforward control to minimize only the first term of equation (5), i.e., minimization of vibration velocity, is also conducted.

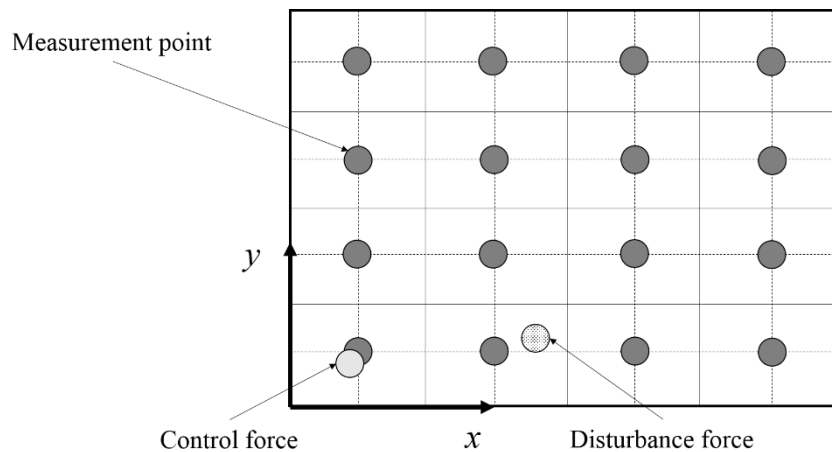
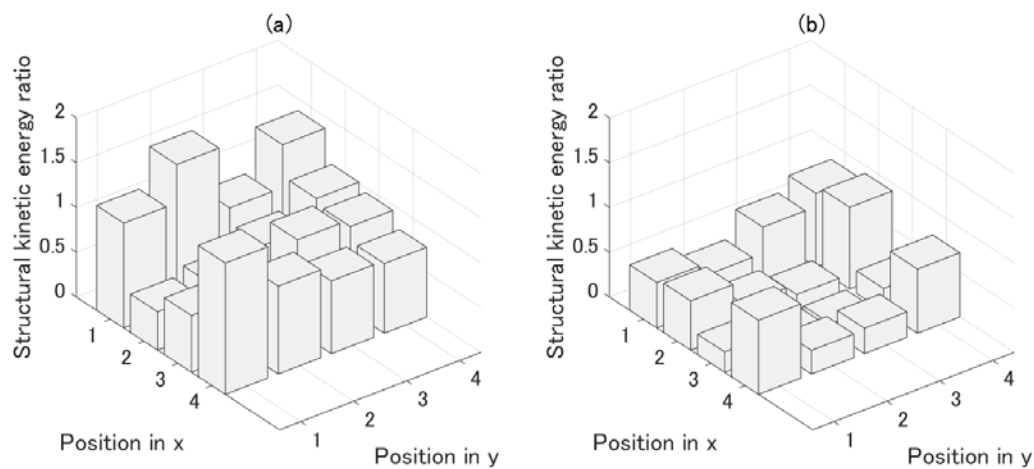
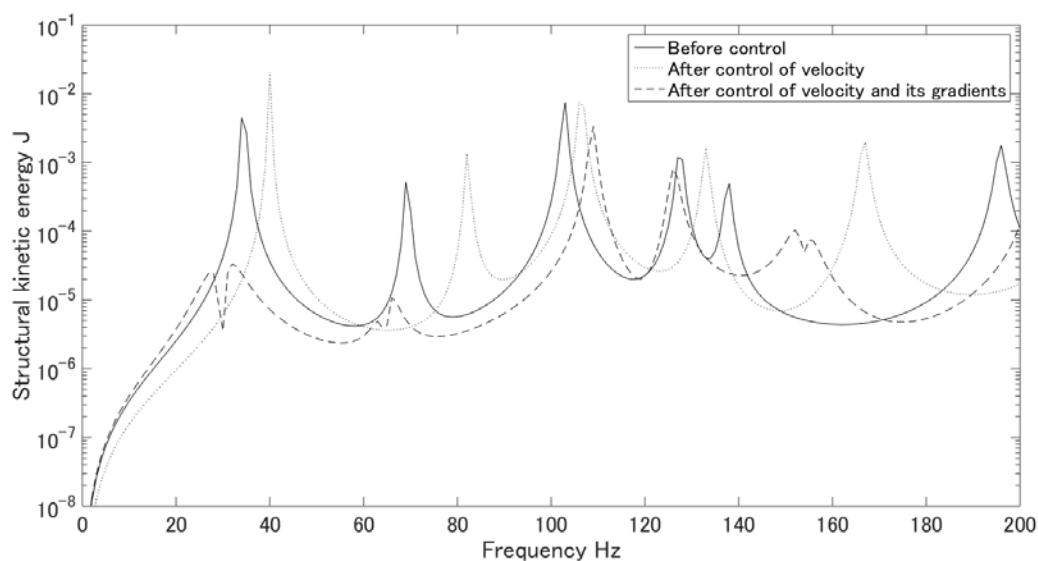
**Figure 1.** Locations of the disturbance force, control force, and measurement point.

Figure 2 shows the global structural kinetic energy averaged over the frequency band up to 200 Hz with minimization of vibration velocity and minimization of vibration velocity and its gradients, where the global structural kinetic energy is normalized by the one without control. It is found from the figure that better control performances are achieved at most measurement locations by the control including the vibration gradients as expected.

Figure 3 shows that the frequency response of the global structural kinetic energy with minimization of vibration velocity and minimization of vibration velocity and its gradients, where the measurement point is located at  $(L_x/8, 2L_y/8)$ . It is found from the figure that the increase in the global structural kinetic energy after control is suppressed by the control including the vibration gradients as expected.



**Figure 2.** Structural kinetic energy after control divided by the one before control for each measurement location: (a), after minimization of vibration velocity; and (b), after minimization of vibration velocity and its gradients.

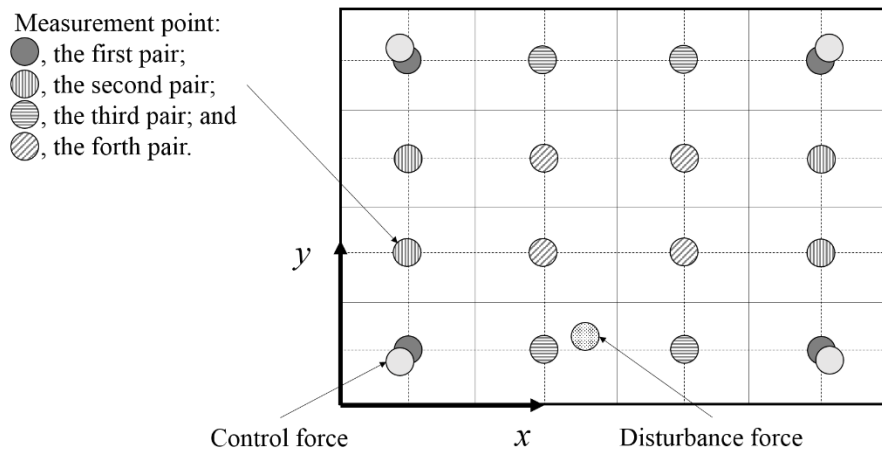


**Figure 3.** Frequency response of the structural kinetic energy before and after control.

### 3.2. Cluster control with vibration gradients

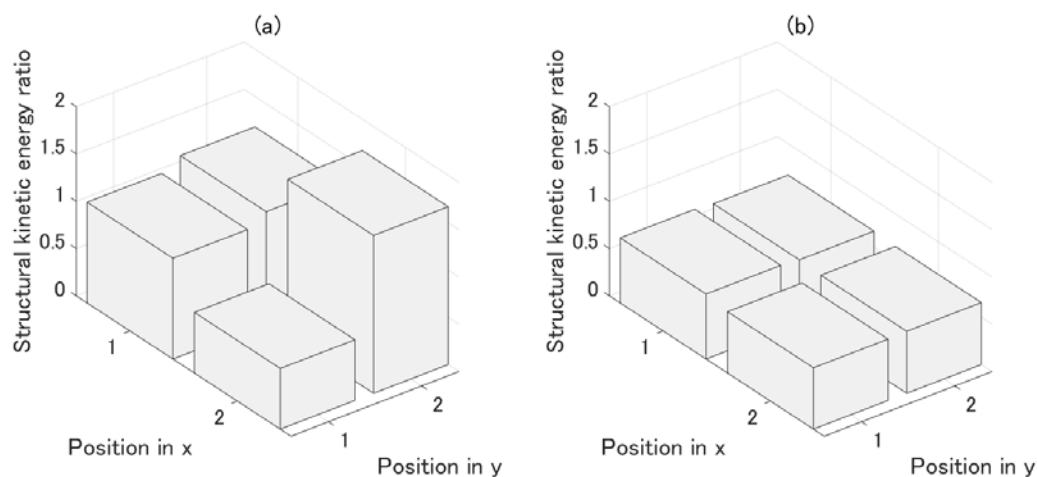
The panel and disturbance are the same as those considered in the previous section. Control system considered here is the one to independently measure and control the even/even cluster. Four measurement points symmetrically located is a pair, and each of four pairs with different locations is considered as shown in figure 4. Four control forces symmetrically located is a pair, and the location of the pair is set as  $(L_x/9, L_y/9)$ ,  $(8L_x/9, L_y/9)$ ,  $(L_x/9, 8L_y/9)$ , and  $(8L_x/9, 8L_y/9)$ . Since the criterion defined in equation (15) is the Hermitian quadratic form, the optimal feedforward control force

to minimize the criterion is derived by the same method as that in the previous section. The conventional cluster control to minimize only the first term of equation (15), i.e., minimization of vibration velocity of the even/even cluster, is also performed for comparison.



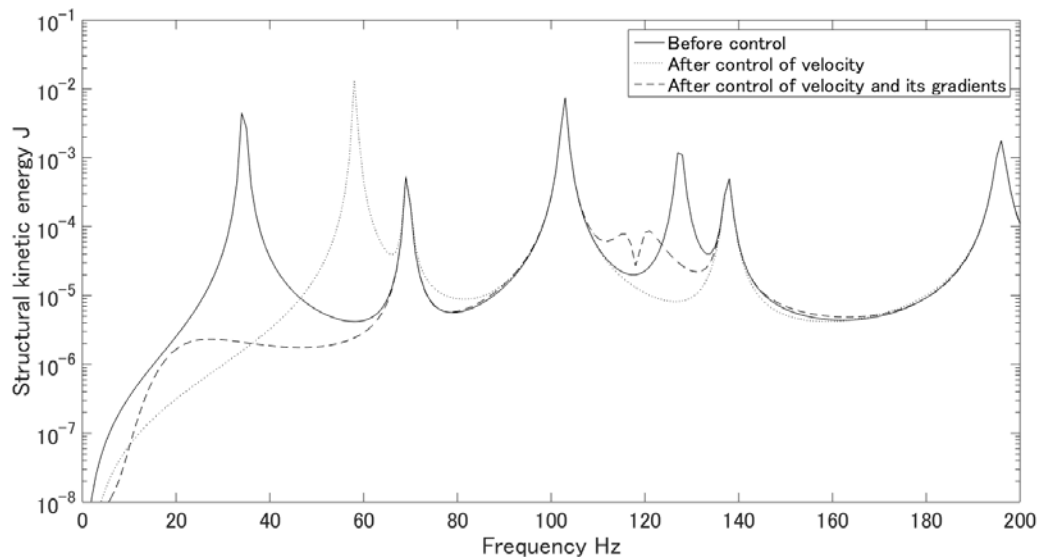
**Figure 4.** Locations of the disturbance force, control force pair, and measurement point pair.

Figure 5 shows the global structural kinetic energy averaged over the frequency band up to 200 Hz with the conventional cluster control or the proposed cluster control including the vibration gradients, where the global structural kinetic energy is divided by the one before control. It is confirmed from the figure that the proposed cluster control performs better than the conventional cluster control at most measurement points as expected.



**Figure 5.** Structural kinetic energy after cluster control divided by the one before control for each measurement location: (a), after minimization of vibration velocity; and (b), after minimization of vibration velocity and its gradients.

Figure 6 shows that the frequency response of the global structural kinetic energy with the conventional cluster control or the proposed cluster control, where the measurement position is  $(L_x/8, 2L_y/8)$ . As shown in the figure, the increase in the global structural kinetic energy after control is mitigated by the proposed method.



**Figure 6.** Frequency response of the structural kinetic energy before and after cluster control.

#### 4. Conclusions

Active vibration control with the criterion being the sum of the squared vibration velocity and the squared vibration gradients at a single measurement point is investigated. Numerical examples indicate that this control method can reduce the global structural kinetic energy of the entire structure more effectively than the conventional control method with the error criterion being the squared velocity at a single measurement point, and the control performance is less dependent on the measurement location in the control method with the error criterion including the vibration gradients. Moreover, the same control strategy is applied to cluster control, and cluster control with the criterion being the sum of the squared vibration velocity and the squared vibration gradients at a pair of symmetric measurement points is formulated. It is found via numerical examples that the global control performance is improved by including the vibration gradients in the control criterion.

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