

Leptonic decays of D -wave vector quarkonia

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Abstract. We give a short and basic introduction to our covariant Dyson-Schwinger-Bethe-Salpeter-equation approach using a rainbow-ladder truncated model of QCD, in which we investigate the leptonic decay properties of heavy quarkonium states in the pseudoscalar and vector channels. Comparing the magnitudes of decay constants, we identify radial 1^{--} excitations in our calculation with experimental excitations of J/Ψ and Υ . Particular attention is paid to those states regarded as D -wave states in the quark model. We predict e^+e^- -decay width of the $\Upsilon(1^3D_1)$ and $\Upsilon(2^3D_1)$ states of the order of ≈ 15 eV or more. We also provide a set of predictions for decay constants of pseudoscalar radial excitations in heavy quarkonia.

1. Introduction

The Dyson-Schwinger-Bethe-Salpeter-equation (DSBSE) approach to hadrons is a modern nonperturbative framework based on QCD as a continuum quantum field theory [1]. Phenomenological DSBSE studies mostly make use of the rainbow-ladder (RL) setup, since this provides excellent value for money or, in other words, an optimal balance of richness and feasibility, e. g., [2–16]. Beyond RL one can explore the infinite set of Dyson-Schwinger equations (DSEs) in symmetry-preserving truncation schemes, see [17–19] and references therein.

2. DSBSE Setup

The Dyson-Schwinger equation (DSE) for the dressed quark propagator S describes how $S(p)$ behaves at quark momentum p depending on the dressed gluon propagator $D(q)$ and the dressed quark-gluon vertex (QGV) $\Gamma(p, q)$ (the bare QGV is denoted by γ), which is depicted in the left panel of Fig. 1. The right panel of this figure shows the definition of the quark self energy $\Sigma(p)$ [20, 21]. S_0 denotes the free quark propagator, which serves as a driving term in this inhomogeneous integral equation and is also used to formulate a renormalization condition for the regularized integral [22]. In all figures, blobs depict dressed quantities, while dots depict the bare counterparts. In particular, the scalar part of S_0 is the current-quark mass m_q . Here and

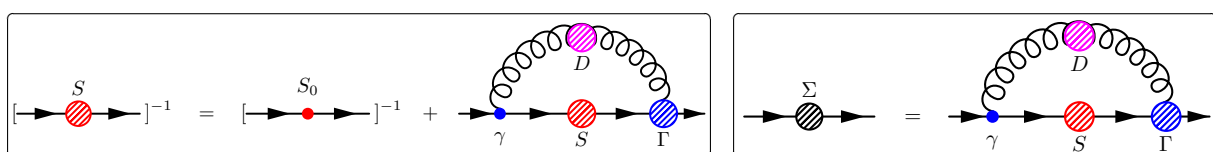


Figure 1. Left panel: quark DSE; Right panel: quark self energy Σ .



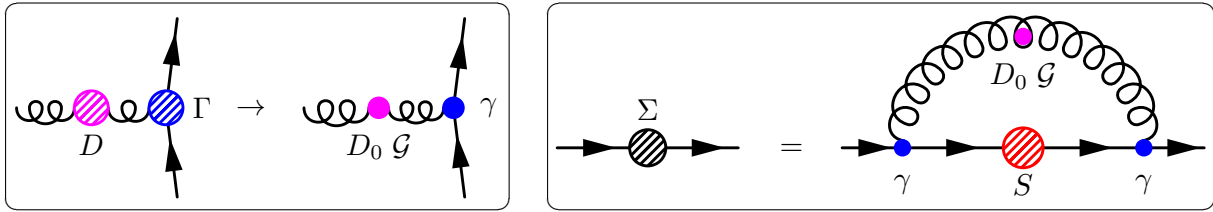


Figure 2. Left panel: RL Ansatz; Right panel: rainbow approximation of Σ in the quark DSE.

in the following, we assume all quantities to be renormalized and omit indices for simplicity. Note that, due to the appearance of the inverses of S and S_0 , the quark DSE is nonlinear.

In RL truncation one makes the *Ansatz* depicted in the left panel of Fig. 2 to replace the product of the dressed gluon propagator and dressed QGV by the product of their bare counterparts multiplied with an effective interaction \mathcal{G} [23]. This function of the gluon momentum squared is then modeled and used to conduct sophisticated covariant studies of hadrons. The right panel of Fig. 2 shows the effect of this *Ansatz* in the quark self energy as defined in the right panel of Fig. 1. Basically, one is left with known quantities and can solve the equation after making an informed choice for \mathcal{G} .

The two-body bound-state equation in a quantum field theory is the Bethe-Salpeter equation (BSE), which is depicted in the left panel of Fig. 3. Ingredients are the dressed quark propagator, which one gets from solving its DSE, and the amputated quark-antiquark scattering kernel K , which is unknown a priori [24]. The RL truncation, inspired and guided by the axial-vector Ward-Takahashi-identity (AVWTI), leads to an effective-gluon exchange ladder kernel as shown in the center panel of Fig. 3. The AVWTI is the manifestation of chiral symmetry in the QCD context, and respecting it guarantees a correct in-principle as well as phenomenological setup of the framework with respect to chiral symmetry and its dynamical breaking. Concretely, RL truncation satisfies the AVWTI and thus one automatically has a massless pion in the chiral limit, if chiral symmetry is dynamically broken by the effective interaction [25, 26].

The solution of the BSE is the Bethe-Salpeter amplitude (BSA) denoted in the left panel of Fig. 3 for the pion by Γ_π . It is the covariant analogue of a wave function and contains all information about the state [27–29]. It is found at a particular value of one of the arguments of Γ_π , the total momentum squared P^2 , which gives the bound-state's mass M via $M^2 = -P^2$.

The ladder-truncated BSE kernel again contains only known objects such that a solution is straightforward. The quark DSE and meson BSE are coupled, since the DSE solution appears as input in the BSE. We solve this coupled set of equations numerically in Euclidean momentum space. The appropriate tools are all at hand and can be found in [30–35]. In order to calculate leptonic decay constants of mesons, one has to compute a diagram as shown in the right panel of Fig. 3, where the appropriate axial-vector current for f_π is depicted as a black-yellow blob and denoted by $\gamma_5\gamma_\mu$ for the pseudoscalar case; the vector decay constant is obtained via γ_μ .

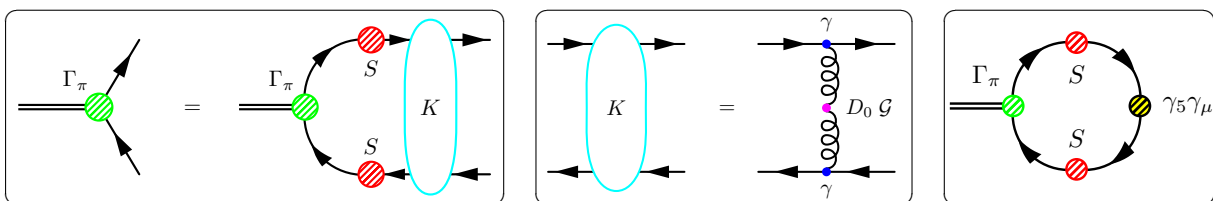


Figure 3. Left panel: meson BSE; Center panel: ladder-truncated BSE kernel; Right panel: diagram to compute the pseudoscalar leptonic decay constant.

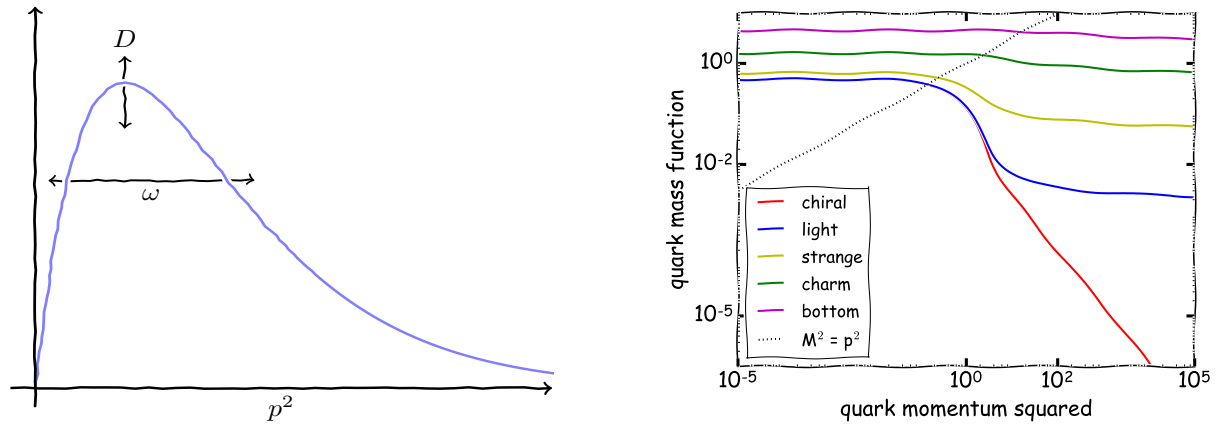


Figure 4. Sketches of: *Left panel:* effective coupling \mathcal{G} ; *Right panel:* typical set of solutions of the gap equation: mass functions from the chiral limit to b quark.

Table 1. Calculated pseudoscalar and vector meson decay constants in MeV compared to experimental data [40, 41], where available. *Left:* charmonium; *right:* bottomonium.

State	J^{PC}	Calc. I	II	Exp.	State	J^{PC}	Calc. I	II	Exp.
Pseudoscalar					Pseudoscalar				
η_c	0^{-+}	401	378	339(14)	η_b	0^{-+}	773	756	—
$\eta_c(2S)$	0^{-+}	244(12)	82	189(50)	$\eta_b(2S)$	0^{-+}	419(8)	285	—
$\eta_c(3S)$	0^{-+}	145(145)	206	—	$\eta_b(3S)$	0^{-+}	534(57)	333	—
$\eta_c(4S)$	0^{-+}	—	87	—	$\eta_b(4S)$	0^{-+}	—	40(15)	—
Vector					Vector				
J/Ψ	1^{--}	450	411	416(5)	Υ	1^{--}	768	707	715(5)
$\Psi(2S)$	1^{--}	30(3)	155	294(4)	$\Upsilon(2S)$	1^{--}	467(17)	393	497(4)
$\Psi(3770)$	1^{--}	118(91)	45	99(3)	$\Upsilon(1^3D_1)$	1^{--}	41(7)	371(2)	—
$\Psi(4040)$	1^{--}	—	188	187(8)	$\Upsilon(3S)$	1^{--}	—	9(5)	430(4)
$\Psi(4160)$	1^{--}	—	1	142(34)	$\Upsilon(2^3D_1)$	1^{--}	—	165(50)	—
$\Psi(4415)$	1^{--}	—	262	161(10)	$\Upsilon(4S)$	1^{--}	—	20(15)	341(18)

3. Results and Discussion

A very successful and typical sophisticated effective interaction was given in [36], which we use also herein. The high-momentum behavior of the interaction is that of perturbative QCD; its shape at intermediate momenta is determined by essentially two parameters: an inverse effective range ω and an overall strength D [37]. Variations of these parameters change the interaction's shape as shown in the left panel of Fig. 4 [38]. The right panel of Fig. 4 shows a characteristic set of solutions of the quark DSE in terms of the quark mass function, where m_q assumes typical values for the masses of the up/down (light), strange, charm, and bottom quarks [39]. In addition to those, we also show the solution in the chiral limit, i. e., with $m_q = 0$ [35].

The most notable feature of this set of mass functions is the transition from the perturbative domain at the right of the plot, where one essentially deals with current quarks, to the left of the plot, where dynamical chiral symmetry breaking ($D\chi SB$) is clearly visible in that the quarks show masses typical for corresponding constituent quarks. The border between these two domains is $\approx 1 \text{ GeV}^2$, a scale set by the model parameters responsible for $D\chi SB$.

After detailed studies of quarkonium spectroscopy in [39, 42] we focus on leptonic decay constants herein. Those were calculated for pseudoscalar and vector mesons already elsewhere, e. g., [42–44], but never systematically or comprehensively for radially excited states. Our results are given in Tab. 1 for two different sets of model parameters and compared to experimental data where available. Set I provides the values obtained for the results as given in [39], i. e., $\omega = 0.7$ GeV and $D = 0.5$ GeV² for charmonium, and $\omega = 0.7$ GeV and $D = 1.3$ GeV² for bottomonium, which were fitted separately for $\bar{c}c$ and $\bar{b}b$ to ground- and low-lying excited quarkonium states [39]. In order to have a common parameter set for both as well as to avoid technical complications [45] which result in a limited number of states in set I, we present also set II, which is uniformly obtained via $\omega = 0.3$ GeV and $D = 1.3$ GeV².

We found that our results for decay constants do not change much from one parameter set to another, so they are robust in terms of their order of magnitude. However, some changes occur. For example, excitations can switch places as seen for the first and second J/Ψ excitation in sets I and II. Also, the smaller the numbers get, the more sensitive they tend to be to parameter changes. Still, on the whole one can identify the relevant patterns and we turn to set II for further analysis. By the size of the decay constants we match our third radial Υ excitation to the missing $\Upsilon(1^3D_1)$ and the fifth to the $\Upsilon(2^3D_1)$, thus reasonably reordering our results.

Then, our prediction for the $\Upsilon(1^3D_1)$ state, at a mass of 10.2 ± 0.1 GeV [39], is a decay width $\Gamma(\Upsilon(1^3D_1) \rightarrow e^+e^-) \approx 15$ eV. For the $\Upsilon(2^3D_1)$ state we obtain $\Gamma(\Upsilon(2^3D_1) \rightarrow e^+e^-) \approx 75$ eV. These predictions for the decay widths are about one order of magnitude larger than those from other approaches, see [46, 47] and references therein. A more in-depth comparison is thus necessary and will be published elsewhere. We also note that in terms of experimental sensitivity, the upper limits for these decays are, in fact, of the order of 40 eV or less [48].

To check the S/D -wave assignment, we investigated the contributions from various covariant structures in the vector-meson BSA to the state's canonical norm. While l is not a Lorentz invariant, we can investigate the situation in the meson rest frame in which we solve the BSE [49]. Upon inspection, the picture holds, i. e., those states with larger leptonic decay constants receive dominant contributions to the norm from S -wave BSA components. In those cases which we compare to D -wave states, the dominant contributions are shifted to D -wave BSA components with small additional contributions from S -wave. We defer a more detailed analysis and discussion of this matter to the future as well.

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