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The eLSM at nonzero density

Francesco Giacosa

Institute of Physics, Jan Kochanowski University, ul. Swietokrzyska 15, 25-406 Kielce, Poland and Institute for Theoretical Physics, J. W. Goethe University, Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany

E-mail: fgiacosa@ujk.edu.pl

Abstract. The extended Linear Sigma Model (eLSM) is an effective model of QCD which includes in the mesonic sector (pseudo)scalar and (axial-)vector quarkonia mesons as well as one dilaton/glueball field and in the baryonic sector the nucleon doublet and its chiral partner in the mirror assignment. The chiral partner of the pion turns out to be the resonance $f_0(1370)$, which is then predominantly a quarkonium state. As a consequence, $f_0(500)$ is predominately not a quarkonium state but a four-quark object and is at first not part of the model. Yet, $f_0(500)$ is important in the baryonic sector and affects nuclear matter saturation, the high-density behavior, and nucleon-nucleon scattering. In these proceedings, we show how to enlarge the two-flavour version of the eLSM in order to include the four-quark field $f_0(500)$ in a chirally invariant manner. We then discuss homogeneous and inhomogeneous chiral restoration in a dense medium.

1. Introduction

Linear Sigma Models (LSMs) are effective models of QCD which contains hadrons (mesons and baryons) as degrees of freedom and which are based on the linear realization of chiral symmetry [1]. As a consequence of the spontaneous breaking of chiral symmetry, in these models the pions emerge as quasi-Goldstone bosons (a relatively small mass is present because of the explicit breaking of chiral symmetry). The chiral partner of the pion, denoted as σ_N, is also an explicit d.o.f. of such models. As various studies show, this meson corresponds to the scalar resonance $f_0(1370)$: this state is then predominantly a quark-antiquark state. Hence, the lightest scalar state listed in the PDG [2], the resonance $f_0(500)$, must be something else: its substructure corresponds to a four-quark state, either as diquark-antidiquark or pion-pion enhancement (see e.g. the recent review on the subject [3]). As such, this resonance should not be (at first) part of a LSM. Yet, as we shall describe later on, $f_0(500)$ is important at nonzero density since it describes a necessary middle-range attraction between nucleons (see also Ref. [4]).

Extensions of LSMs toward the inclusion of (axial-)vector degrees of freedom were performed in Ref. [5]. Quite recently, an as complete as possible LSM, called extended Linear Sigma Model (eLSM), was developed. The eLSM contains from the very beginning (axial-)vector fields and embodies both chiral symmetry and dilatation invariance. Spontaneous and explicit breaking of the former as well as anomalous breaking of the latter are present. As a consequence of chiral symmetry and the dilation invariance, the eLSM contains a *finite* number of terms. The eLSM was first presented for $N_f = 2$ in Refs. [6], then it was enlarged to $N_f = 3$ in Refs. [7] (this is the first version of a chiral model with $N_f = 3$ containing (axial-)vector d.o.f.), and lately it was also studied for charmed mesons ($N_f = 4$, Ref. [8]).

In the baryonic sector, the eLSM was investigated for $N_f = 2$ in Refs. [9] (a first step toward the eLSM at $N_f = 3$ was performed in Ref. [12]). In the eLSM, the mirror assignment, which allows for chirally invariant mass term, is used [10, 11]. As mentioned in Ref. [9], a four-quark field χ corresponding to $f_0(500)$ can be coupled to the eLSM in a chirally invariant way, see Sec. 2. The eLSM with $f_0(500)$ has been investigated at nonzero density in Ref. [14], where the chiral phase transition has been investigated, and later on in Ref. [15], in which inhomogeneous condensation has been studied; these results are here summarized in Sec. 3. The role of the additional, non-conventional meson $f_0(500)$ turns out to be important: it makes a description of the properties of nuclear matter possible (both saturation and compressibility are in agreement with data) and it strongly affects the properties of nuclear matter at high density. Quite interestingly, the resonance $f_0(500)$ was recently investigated in Ref. [16] in the framework of nucleon-nucleon scattering: also in this case, its presence is necessary for a correct description of data.

2. The eLSM

2.1. eLSM without $f_0(500)$

We briefly present the eLSM for $N_f = 2$. (Pseudo)scalar mesons are contained in $\Phi = (\sigma_N + i\eta_N)t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}$, where $t^0 = 1_2/2$, $\vec{t} = \vec{\sigma}/2$, σ_i are the Pauli matrices. In Table 1 we report the identification of the fields with resonances of the PDG [2]. [Note: $\eta_N \equiv \left(u\bar{u} + d\bar{d}\right)/\sqrt{2}$ reads $\eta_N = \cos\varphi_P \eta - \sin\varphi_P \eta'$ with $\varphi_P \approx -44°$ [7]] Because of spontaneous symmetry breaking, σ_N condenses: $\sigma_N \to \sigma_N + \phi$, where ϕ is the chiral condensate. Under $U(2)_R \times U(2)_L$ chiral transformations: $\Phi \to U_L \Phi U_R^\dagger$.

The left-handed and right-handed fields L^μ and R^μ contain the vector states ω^μ and $\vec{\rho}^\mu$ and the axial–vector states f_1^μ and $\vec{a}_1^\mu$: $L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu)\cdot\vec{t}$, $R^\mu = (\omega^\mu - f_1^\mu)t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu)\cdot\vec{t}$, see Table 1. Under chiral transformations: $L^\mu \to U_L \Phi U_L^\dagger$ and $R^\mu \to U_R \Phi U_R^\dagger$.

Table 1: Correspondence of eLSM $\bar{q}q$ fields to PDG [2].

Field	PDG	Quark content	I	J^{PC}	Mass (GeV)
π^+, π^-, π^0	π	$u\bar{d}, d\bar{u}, \frac{u\bar{u}-dd}{\sqrt{2}}$	1	0^{-+}	0.13957
η	$\eta(547)$	$\frac{u\bar{u}+dd}{\sqrt{2}}\cos\varphi_P - s\bar{s}\sin\varphi_P$	0	0^{-+}	0.54786
η'	$\eta'(958)$	$\frac{u\bar{u}+dd}{\sqrt{2}}\sin\varphi_P + s\bar{s}\cos\varphi_P$	0	0^{-+}	0.95778
a_0^+, a_0^-, a_0^0	$a_0(1450)$	$u\bar{d}, d\bar{u}, \frac{u\bar{u}-dd}{\sqrt{2}}$	1	0^{++}	1.474
σ_N	$f_0(1370)$	$\frac{u\bar{u}+dd}{\sqrt{2}}$	0	0^{++}	1.350
ρ^+, ρ^-, ρ^0	$\rho(770)$	$u\bar{d}, d\bar{u}, \frac{u\bar{u}-dd}{\sqrt{2}}$	1	1^{--}	0.77526
ω_N	$\omega(782)$	$\frac{u\bar{u}+dd}{\sqrt{2}}$	0	1^{--}	0.78265
a_1^+, a_1^-, a_1^0	$a_1(1230)$	$u\bar{d}, d\bar{u}, \frac{u\bar{u}-dd}{\sqrt{2}}$	1	1^{++}	1.230
$f_{1,N}$	$f_1(1285)$	$\frac{u\bar{u}+dd}{\sqrt{2}}$	0	1^{++}	1.2819

The mesonic part of the Lagrangian reads

$$\mathcal{L}_{eLSM}^{meson} = \mathrm{Tr}\left[(D^\mu\Phi)^\dagger(D^\mu\Phi)\right] - \mu_0^2\mathrm{Tr}[\Phi^\dagger\Phi] - \lambda_2\mathrm{Tr}[(\Phi^\dagger\Phi)^2] - \frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] +$$
$$\frac{m_1^2}{2}\mathrm{Tr}\left[L^{\mu 2} + R^{\mu 2}\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] + h_2\mathrm{Tr}\left[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger\right] + 2h_3\mathrm{Tr}\left[\Phi R_\mu \Phi^\dagger L^\mu\right] \; ..., \tag{1}$$

where dots refer to large-N_c suppressed terms (including the chiral anomaly). In Refs. [6, 7] it was shown that, thanks to the inclusion of (axial-)vector d.o.f., the eLSM provides a good description of meson phenomenology. An interesting consequence is that the quark-antiquark field σ_N, which represents chiral partner of the pion, is associated to $f_0(1370)$, in agreement with previous phenomenological studies [13]. Hence, $f_0(500)$ must be something else [3].

We now turn to the baryonic sector. For $N_f = 2$ one starts from two nucleon fields Ψ_1 and Ψ_2 with opposite parity which transform mirror-like under chiral transformations: $\Psi_{1,R(L)} \to U(2)_{R(L)}\Psi_{1,R(L)}$, $\Psi_{2,R(L)} \to U(2)_{L(R)}\Psi_{2,R(L)}$. The eLSM Lagrangian is [9]:

$$\begin{aligned}\mathcal{L}_{eLSM}^{baryons} &= \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\ &- \hat{g}_1(\bar{\Psi}_{1L}\Phi\Psi_{1R} + \bar{\Psi}_{1R}\Phi^\dagger\Psi_{1L}) - \hat{g}_2(\bar{\Psi}_{2L}\Phi^\dagger\Psi_{2R} + \bar{\Psi}_{2R}\Phi\Psi_{2L}) \\ &- m_0(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}) \,, \end{aligned} \tag{2}$$

where $D^\mu_{1(2)R(L)} = \partial^\mu - ic_{1(2)}R(L)^\mu$. The fields Ψ_1 and Ψ_2 mix due to the m_0-term and are related to the physical states N and its chiral partner N^* via:

$$\Psi_1 = \frac{1}{\sqrt{2\cosh\delta}}\left(Ne^{\delta/2} + \gamma_5 N^* e^{-\delta/2}\right) \,,\; \Psi_2 = \frac{1}{\sqrt{2\cosh\delta}}\left(\gamma_5 N e^{-\delta/2} - N^* e^{\delta/2}\right) \,, \tag{3}$$

where $\cosh\delta = \frac{m_N + m_{N^*}}{2m_0}$. The field N corresponds to the nucleon $N(939)$ and N^* to $N(1535)$ or $N(1650)$. For the purposes of the present work, the assignment of N^* is marginal, see however [9, 12]. The parameter m_0 represents a chirally invariant mass, which was first discussed in Ref. [10] and further investigated in Refs. [9, 11]. The masses of the nucleon N and its chiral partner N^* are given by:

$$m_{N,N^*} = \sqrt{m_0^2 + \frac{(\hat{g}_1 + \hat{g}_2)^2}{16}\phi^2} \pm \frac{1}{4}(\hat{g}_1 - \hat{g}_2)\phi \,. \tag{4}$$

In the limit $m_0 \to 0$ one obtains the result $m_N = \hat{g}_1\phi/2$, i.e., the nucleon mass is solely generated by the chiral condensate [as in the original Linear Sigma Model [1]].The parameters of the model were determined in Ref. [9], to which we refer for details.

2.2. Inclusion of $f_0(500)$ in the eLSM

We now introduce $\chi \equiv f_0(500)$ with quantum numbers $I(J^{PC}) = 0(0^{++})$ and mass $m_\chi = (0.475 \pm 0.25)$ GeV [2] into the eLSM. This state is regarded as an admixture of $\pi\pi$ and $[u,d][\bar{u},\bar{d}]$ configurations. For $N_f = 2$ it is a singlet under chiral transformations ($\chi \to \chi$). The coupling of χ to baryons is obtained by modifying the m_0-term as:

$$m_0(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}) \to a\chi(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}) \,, \tag{5}$$

where a is now a dimensionless constant. Then, the mass parameter m_0 emerges as a condensation of the four-quark field χ: $m_0 = a\chi_0$. In the mesonic sector, one has too add

$$\mathcal{L}_{eLSM}^{meson} \to \mathcal{L}_{eLSM}^{meson} + \frac{1}{2}\left((\partial_\mu\chi)^2 - m_\chi^2\chi^2\right) + g_{\chi\Phi}\chi Tr[\Phi^\dagger\Phi] + g_{\chi\Phi}\chi Tr[L^{\mu 2} + R^{\mu 2}] + \dots \,, \tag{6}$$

where dots refer to large-N_c suppressed terms. As a consequence, the condensate χ_0 takes the form $\chi_0 = g_{\chi\Phi}\phi^2/m_\chi^2$.

3. Results

3.1. Homogenous condensation

First, we study nuclear matter at nonzero density under the assumption that the condensates are homogenous. Two scalar fields condense: $\langle\sigma_N\rangle = \phi(\mu)$ and $\langle\chi\rangle = \bar{\chi}(\mu)$, where μ is the nuclear

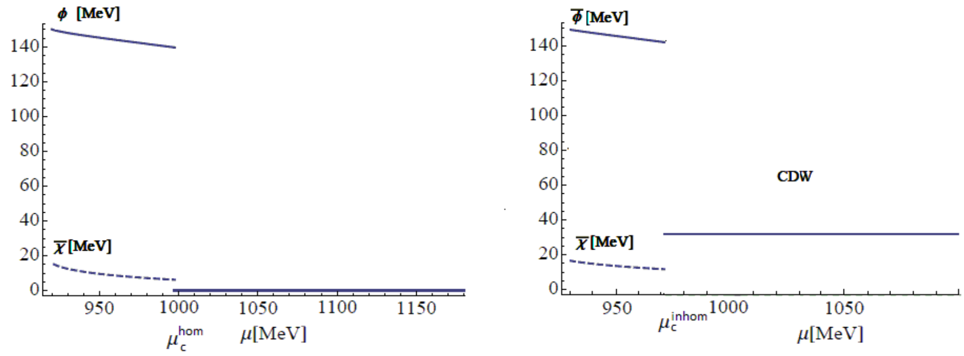

Figure 1: Left: homogenous condensation. The condensates ϕ and $\bar{\chi}$ drop to almost zero at μ_c^{hom}. Right: inhomogeneous condensation. At μ_c^{inhom} a transition to the inhomogeneous condensation of Eq. (7) takes place. This configuration is more favorable than the homogenous case of the left panel.

chemical potential. The results are obtained by minimizing at a given μ the thermodynamical potential Ω w.r.t. ϕ, $\bar{\chi}$, as well as $\langle\omega^0\rangle$. The vacuum's relation $\chi_0 = g_{\chi\Phi}\phi^2/m_\chi^2$ holds approximately also at nonzero density: ϕ slowly decreases as function of μ together with $\bar{\chi}$. Then, at a critical $\mu_c^{\mathrm{hom}} \sim 1$ GeV (corresponding to a density $\rho/\rho_0 \sim 2.7$, where ρ_0 is the nuclear matter saturation density) a first-order phase transition takes place: ϕ and $\bar{\chi}$ drop to to very small (but nonzero) values. Chiral symmetry is restored (see Fig. 1, left panel). In terms of density, there is a long mixed phase between $2.7\rho_0$-$10\rho_0$. The compressibility K lies in the range 200-250 MeV and is therefore in good agreement with the experiment (200-300 MeV). The mass of the nucleon and that of the partner drop to almost zero in the chirally restored phase. For details, see Ref. [14].

3.2. Inhomogeneous condensation

The previous results were obtained under the assumption that the ϕ and $\bar{\chi}$ are homogenous, i.e. are not space-dependent. Yet, various studies have shown (see Ref. [17] and refs. therein) that an inhomogeneous condensation can be favoured. In Ref. [15] the so-called chiral-spiral [18] was investigated:

$$\phi(\mu, z) = \bar{\phi}\cos(2fz), \ \langle\pi^{3\equiv0}\rangle = \bar{\phi}\sin(2fz) \tag{7}$$

with $\bar{\phi} \equiv \bar{\phi}(\mu)$ and $f \equiv f(\mu)$. The homogenous case corresponds to $f = 0$. For $f \neq 0$ one has a condensation of the neutral pion field, which corresponds to a spontaneous breaking of parity at nonzero density. At a given μ, the thermodynamical potential is now minimized for $\bar{\phi}$, f, $\bar{\chi}$, and $\langle\omega^0\rangle$. The results show that below a certain critical chemical potential $\mu_c^{\mathrm{inhom}} \lesssim 1$ GeV one has $f = 0$: homogenous condensation is realized, just as before. However, at μ_c^{inhom} a phase transition takes place: f jumps to a finite value (of about 400 MeV, which then slowly increases with μ), and $\bar{\phi}$ to a lower but finite value, see Fig. 2. Interestingly, for a given parameter set, it turns out that $\mu_c^{\mathrm{inhom}} < \mu_c^{\mathrm{hom}}$: the homogenous chiral phase transition does not occur, but is only a local minimum. Chiral symmetry is only partially restored.

4. Conclusions

The resonance $f_0(500)$ is the lightest scalar listed in the PDG and hence is potentially interesting in hadron phenomenology. Its role has to be investigated case by case. While its condensate may be relevant at nonzero temperature [19], its effect on thermal models turns out to be negligible because of a very subtle and interesting cancellation with the repulsive isotensor channel: for practical purposes, $f_0(500)$ can be neglected in thermal models of heavy ion collisions [20].

At nonzero density, $f_0(500)$ plays indeed a significant role because it mediates a sizable attraction between nucleons: in these proceedings, we have incorporated $f_0(500)$ into the eLSM and reviewed the properties at finite chemical potential. We have shown that inhomogeneous condensation of the chiral-spiral type is favored w.r.t. the homogeneous one. In the future, one should go beyond the chiral-spiral Ansatz and test arbitrary types of inhomogeneous condensation by using the numerical procedure put forward in Ref. [21].

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