

# Nonlinear dynamics of nonequilibrium exciton-polaritons in a periodic potential

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**Abstract.** An exciton-polariton condensate formed in a semiconductor microcavity coupled to an exciton reservoir in the strong coupling regime is studied. The condensate is trapped in one-dimensional periodic potential, and we work in the centre of Brillouin zone. We develop a model for coupled three spatial harmonics of mean field. Using the simplified model we get important analytical relations for polaritonic eigenstates and band-structure. The analytical results are supported by numerical analysis. The strong influence of external potential and nonlinearity is discussed and the feedback induced by the inhomogeneity of the incoherent reservoir on the dynamics of coherent polaritons.

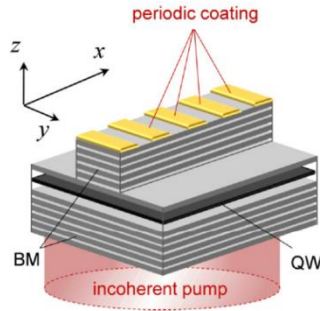
## 1. Introduction

Strongly correlated bosonic particles placed in lattices represent an indispensable tool for fundamental studies of quantum phenomena in modern condensed matter and solid state physics [1]. During the past decade there was a remarkable progress in this area, which was related to exciton polaritons occurring inside a high quality semiconductor microcavity due to the strong light-matter coupling [2–4]. Extremely small effective masses of these composite bosons enable the observation of high temperature nonequilibrium BEC with exciton polaritons [5–7]. An interesting direction is connected with study of polariton condensate placed in a periodic potential [8,9].

In this paper an exciton-polariton nonequilibrium condensate is described confined in a weak-contrast periodic potential embedded into a planar microcavity driven by a homogeneous incoherent pumping [see Figure 1(a)]. Polaritons are subject to rapid radiative decay and their population is maintained due to optical pumping, thus, the condensate of exciton-polaritons demonstrate a nonequilibrium open-dissipative behavior.

The main goal of this work is to explore the macroscopic coherent oscillations of the exciton polaritons (Figure 1) coupled with the exciton reservoir.





**Figure 1.** (Color online) Sketch of the considered system. A one-dimensional microcavity with periodic coating is driven by an incoherent optical pump [9]. Semiconductor quantum well (QW) is placed between two Bragg mirrors (BM).

## 2. The model

### 2.1 Gross-Pitaevskii dissipative model.

Here it is considered an open-dissipative mean-field Gross-Pitaevskii (GP) model that describes the incoherently pumped condensate coupled with an exciton reservoir. The considered system is described by GP-type equation for polaritonic order parameter  $\psi$  and by rate equation for the reservoir density  $n$  [6]

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + g_c |\psi|^2 + i(Rn - \gamma_c)/2 + g_r n \right] \psi, \quad (1)$$

$$\frac{\partial n}{\partial t} = -(\gamma_r + R|\psi|^2)n + P_0, \quad (2)$$

where  $m$  is the effective mass of the polariton, which is much smaller than the free electron mass  $m_e$  ( $m \approx m_e \times 10^{-4}$ ),  $R = 0.01 \text{ ps}^{-1} \mu\text{m}^2$  defines the condensation rate,  $\gamma_c = 0.33 \text{ ps}^{-1}$  and  $\gamma_r = 0.495 \text{ ps}^{-1}$  represent the decay rates of polaritons and reservoir excitons, respectively.  $g_c = 6 \times 10^{-3} \text{ meV } \mu\text{m}^2$  and  $g_r = 2g_c$  characterize the strengths of polariton-polariton and polariton-reservoir interactions, respectively,  $P_0$  characterizes incoherent (nonresonant) homogeneous pumping. Here it is discussed a quasi-one-dimensional polariton condensate [8] in the periodic potential  $V(x) = V_0 \cos(\beta x)$ , where  $\beta = 2\pi/l$  and  $l$  is the period of modulation [see Figure 1(a)].

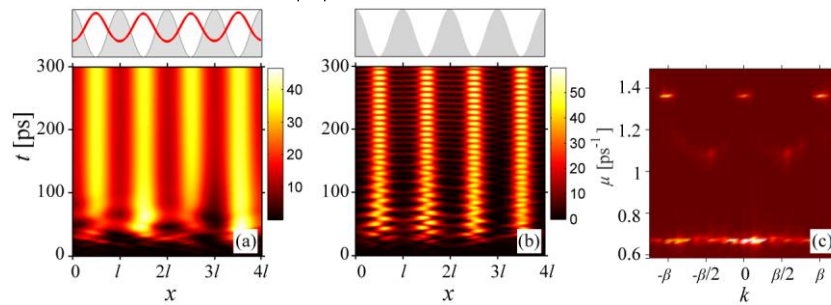
A typical attribute of exciton-polariton condensation dynamics is the presence of the threshold. Above some critical value of pumping, when all the losses are compensated, a non-trivial solutions of the system (1), (2) appears. In the case of homogeneous system ( $V_0 = 0$ ) they are traveling waves

$$\psi^{HS}(x, t) = \psi_0^{HS} e^{-i\mu_0 t + ikx}, \quad (3)$$

where  $\hbar\mu_0 = (\hbar^2/2m)k^2 + g_c |\psi_0|^2 + g_r n_0$ . The threshold value is determined by  $P_{th} = \gamma_c \gamma_r / R$ . The coherent polariton and incoherent reservoir densities are given by  $|\psi_0^{HS}|^2 = (P_0 - P_{th})/\gamma_c$  and  $n_0^{HS} = \gamma_c/R$ , respectively [6]. It is justified to suggest that in the presence of weak periodic potential the threshold value of pumping  $P_{th}$  remains the same as in homogeneous system. Numerical simulations of system (1) and (2) confirm this estimate.

## 2.2 Numerical simulations

It is well known that a rigorous analytical description of the periodic nonlinear system is possible only in the case of a specific shape of potential. Thus, firstly we solved the system (1) numerically. In this paper we are mostly interested in the states of the condensate with a wavevector close to zero. In the case of homogeneous pumping, which corresponds to the wide pumping laser field, and periodical boundary conditions the formation of the state with  $k=0$  from the weak initial noise is the most probable. Simulating the formation of polariton condensate from the small initial noise we observed that the final state of the polariton condensate depends on the system parameters. Two typical scenarios of the evolution of the condensate density  $|\psi|^2$  are presented in Figure 2.



**Figure 2.** (Color online) (a) Formation of the ground state of polariton condensate from a noisy initial conditions under  $V_0/\hbar = 0.1 \text{ ps}^{-1}$  and  $P_0 = 24 \text{ } \mu\text{m}^2\text{ps}^{-1}$ . An upper panel shows the distribution of the condensate mode (red curve) and the shape of periodic potential (shaded region) (b) formation of the oscillating state under the same value of pumping rate but for  $V_0/\hbar = 0.4 \text{ ps}^{-1}$ . Spatio-temporal spectrum of this dynamical solution is shown on the panel (c).

In the first case which is shown in Figure 2(a) the condensate forms in the steady state with a wavevector close to zero. The maxima of the condensate distribution are located at the minima of the potential in this case [see the upper bar of panel (a)]. It means that the condensate is in the state with minimal energy, i.e. in ground state. The analysis of the spatio-temporal spectra of such solutions confirms it. This scenario is realized mostly when the potential depth  $V_0$  is small [for instance  $V_0 = 0.1 \text{ ps}^{-1}$  in Figure 2(a)].

In the case of large modulation the dynamical solution forms which has a form of metastable oscillations of polariton density. After long time (more than several tens of nanoseconds) such a state breaks spontaneously. This scenario is realized for all the values of  $V_0$  greater than some threshold value which is determined by the pumping rate  $P$ . The oscillating nature of the observed solution clearly indicates that two states of the condensate are excited. From the spectrum of the solution, Figure 2(c), it is seen that one of these states is a ground state with zero wavevector and another one is located at the upper band.

To understand the reasons of the formation of such oscillations we use a simplified model which allows determining the spatial structure of the observed states.

## 2.3 Three spatial harmonics model

We search for the solutions in form of Bloch functions  $\psi = \psi(x)e^{ikx}$ , where  $k$  has a sense of quasi-impulse. Since we consider only low contrast lattices the solution  $\psi(x)$  can be constructed as a sum of plane waves. This is equivalent to Fourier expansion of condensate order parameter  $\psi$  and reservoir density  $n$  on the spatial harmonics of the periodic potential. According to numerical simulations the condensate forms only on the lowest energy bands, thus we can use only first three spatial harmonics. In this case an approximate solution of Eqs. (1), (2) is

$$\psi(x, t) \approx (a_0 + a_- e^{-i\beta x} + a_+ e^{i\beta x}) e^{ikx}, \quad (4a)$$

$$n(x, t) \approx n_0 + n_+ e^{i\beta x} + n_- e^{-i\beta x}, \quad (4b)$$

where  $a_0$ ,  $a_-$ , and  $a_+$  are time-dependent amplitudes of the main spatial harmonics of coherent polariton condensate. The modulation of the coherent exciton polaritons evokes a spatial modulation of the reservoir described by the terms with  $n_{\pm}$  in Eqs. (4b). Inserting (4) into the system (1), (2) and neglecting the higher spatial harmonics arising from the nonlinear terms we obtain the set of equations for three condensate components ( $a_0$ ,  $a_-$ , and  $a_+$ ), for the three components of the reservoir ( $n_0$ ,  $n_+$  and  $n_-$ ).

### 3. Steady-state solutions in the frame of three spatial harmonics model

The energy band-structure of the condensate and the spatial structure of the specific states can be obtained numerically by the solution of the steady-stated equations in the frame of the three spatial harmonics model (4). The analytical description is reasonable only in the vicinity of the condensation threshold ( $P_0 \simeq P_{th}$ ) where the nonlinear effects are small and can be neglected. In this case the condensate band structure, i.e. the dependence of the chemical potential  $\mu$  on the wavevector  $k$ , resembles the dispersion of a single particle in the periodic potential, see Figure 3(a). Since pump is homogeneous the reservoir modulation is small just above the threshold, i.e.  $n_0 \simeq n_0^{HS}$ . Thus, the average density of the condensate can be approximated by the value  $|\psi_0|^2$  characterizing the homogeneous system.

Searching for the steady-state solutions of a linear problem in the form  $a_0(t) = a'_0 e^{-i\mu t}$ ,  $a_-(t) = a'_- e^{-i\mu t}$ , and  $a_+(t) = a'_+ e^{-i\mu t}$  one can determine the spatial shapes and the frequencies of the states with  $k = 0$ .

The first solution possessing minimal energy is the *ground state*

$$\begin{pmatrix} a'_0 \\ a'_- \\ a'_+ \end{pmatrix} = \sqrt{\frac{|\psi_0|^2}{2H(H+4K)}} \begin{pmatrix} H+4K \\ -1 \\ -1 \end{pmatrix}, \quad (5)$$

where  $K = \hbar^2 \beta^2 / 8mV_0$  and  $H \equiv \sqrt{16K^2 + 2}$ . In the case of weak modulation and relatively large lattice period we have  $K > 1$ . It means that zero-momentum component  $a'_0$  makes the main contribution in the solution. In this case condensate is periodically modulated on the zero-momentum background.

Solutions corresponding to the excited states have a symmetric and antisymmetric eigenvectors in respect to the signs of the components  $a'_-$  and  $a'_+$ . The lower *antisymmetric state* has a perfect sine shape with  $a'_- = -a'_+$  and vanishing zero-momentum component  $a'_0 = 0$ .

$$\begin{pmatrix} a'_0 \\ a'_- \\ a'_+ \end{pmatrix} = \frac{|\psi_0|}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}. \quad (6)$$

In this case the maxima of the condensate density is located at the maxima of the potential.

An upper *symmetric state* has a symmetric structure ( $a'_- = a'_+$ )

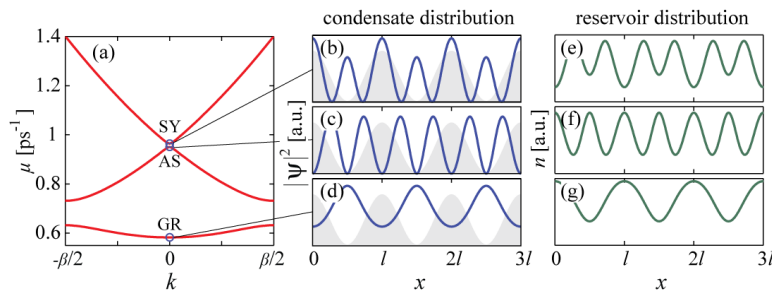
$$\begin{pmatrix} a'_0 \\ a'_- \\ a'_+ \end{pmatrix} = \sqrt{\frac{|\psi_0|^2}{2H(H-4K)}} \begin{pmatrix} H-4K \\ -1 \\ -1 \end{pmatrix}. \quad (7)$$

The above solutions are precise only in the vicinity of the condensation threshold. Under larger pumpings the condensate modulation initiates modulation of the reservoir which results in a spatially

distributed gain proportional to  $Rn(x)$ . In this case steady-state solutions can be obtained numerically. Nevertheless, our simulations show that the states keep their symmetry even in this case.

The typical profiles of polariton condensate at the ground (GR), antisymmetric (AS), and symmetric (SY) states are shown on the panels (b), (c) and (d) of Figure. 3. Corresponding distributions of the reservoir are shown on the panels (e), (f) and (g).

In the next section we show how the spatial structure of the condensate and especially spatially inhomogeneity of the reservoir influence on the condensate dynamics triggering the oscillatory dynamics.



**Figure 3.** (Color online) (a) Energy band structure of the polariton condensate in the vicinity of the condensation threshold  $P_{th}$ . Eigenstates GR, AS and SY are ground, antisymmetric, and symmetric states of polaritons with  $k = 0$ . Other parameters are  $V_0 / \hbar = 0.1 \text{ ps}^{-1}$  and  $l = 2\pi / \beta = 8 \mu\text{m}$ . Panels (b) – (d) show the distribution of condensate in three steady states marked by blue circles in panel (a). The corresponding reservoir profiles are shown on the panels (e)–(g).

#### 4. Oscillation and relaxation dynamics of the condensate

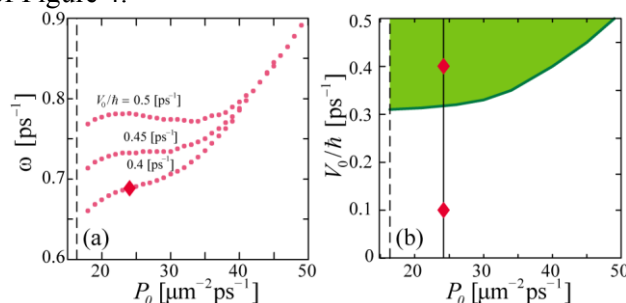
As it was mentioned above in the case of a weak potential the condensate forms in the ground state [compare Figure 3(d) and an upper panel of Figure 2(a)]. Filling of the upper state in the more contrast lattices can be explained by the selective saturation of the gain arising from the spatial modulation of reservoir. With the increase of  $V_0$  the spatial modulation of the condensate becomes deeper since zero-momentum component  $a'_0$  reduces – see Eq. (5). In this case due to the spatial dependency of the reservoir density the gain is saturated mostly in the vicinity of the condensate maxima, i.e., in the minima of the periodic potential. It initiates the growth of the symmetric mode, whose peaks are located near the maxima of the periodic potential [see panel (b)]. Such a phenomenon resembles hole burning effect known from laser physics, but the saturation of the gain occurs in a real space domain and, thus, this effect can be called *spatial hole burning*.

Since oscillations appear due to the temporal beating between the ground and the excited symmetric modes, the oscillation frequency  $\omega$  can be approximated by the difference of their eigenfrequencies  $\mu_{GR}$  and  $\mu_{SYM}$ . The latter can be determined analytically in the frame of the three spatial harmonics model (4) in the linear limit which is valid in the vicinity of the condensation threshold. Using the same approach as in Section 3 one obtains

$$\hbar\omega \approx V_0 \sqrt{4K^2 + 2}. \quad (8)$$

Numerical modeling shows that Eq. (8) accurately predicts the oscillation frequency in the vicinity of the condensation threshold ( $P_0 \approx P_{th}$ ) [Figure 2(c)]. For a stronger pumping the influence of nonlinear effects and reservoir inhomogeneities become significant. As a consequence the oscillation frequency increases with the pump amplitude  $P_0$  until the critical value where the oscillations disappear. The value of oscillation frequencies versus pumping rate for different values of modulational depth is shown on Figure 4. The recovery to the steady state solution occurs because the ground mode of the condensate

stabilizes. This threshold is determined by both the potential depth  $V_0$  and the pumping rate  $P_0$  and is shown on the panel (b) of Figure 4.



**Figure 4.** (Color online) (a) numerically calculated frequency of oscillation for different values of modulation depth. (b) Existence domain of “zero momentum” oscillations as a function of the pump  $P_0$  and the modulation depth  $V_0$  obtained both in the GP model (green shaded area). Red diamonds indicates the points corresponding to Figure 1 (a) (lower) and Figure 1(b) (upper).

## 5. Conclusion

We considered nonlinear dynamics of coherent exciton-polaritons in weak-contrast lattices embedded into a planar microresonator under homogeneous incoherent pumping. Within this approach we developed a simplified mean-field model for three spatial harmonics and found analytical expressions for the relevant eigenstates of the condensate. We have observed that the metastable oscillations between the states locating in the center of the Brillouin zone can form from the weak noise. The numerical results are supported by the analytical analysis. The strong influence of the dissipative effects and the incoherent reservoir on the dynamics of coherent polaritons was also discussed.

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