

Aberration influenced generation of rotating two-lobe light fields

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Abstract. The influence of aberrations on light fields with a rotating intensity distribution is considered. Light fields were generated with the phase masks developed using the theory of spiral beam optics. The effects of basic aberrations, such as spherical aberration, astigmatism and coma are studied. The experimental implementation of the fields was achieved with the assistance of a liquid crystal spatial light modulator HOLOEYE HEO-1080P, operating in reflection mode. The results of mathematical modelling and experiments have been qualitatively compared.

1. Introduction

A problem of an adequate method choice for the precise determination of the location of single fluorescent molecules, quantum spots and other nanostructures arises under investigation of the substance physicochemical properties [1, 2]. And the radiating source coordinates can be changed in time. In particular these methods are of a high demand in science of materials since they permit to study the substance physicochemical properties at a level of single molecules. A precise 3D localization of nano-particles is nowadays an urgent task. Still unsolved is the problem of the precise determination of the longitudinal coordinate of the radiating nano-object in microscopy. Within the solution of this problem a special attention is paid to the use of the light fields where the structure of the intensity distribution changes with changing of, the depth of occurrence. Among these light fields, the two-lobe fields with the intensity distribution in the form of two maxima are the most promissory owing to the simplicity of the shape of their intensity distribution.

We should clarify what the main point is of the method for estimation of the longitudinal coordinate of a nanosize radiating object in a fluorescent microscope by means of the light fields with the intensity rotation. Let a sample of a certain thickness have some nanosize radiating particles (e.g. quantum spots) located at different depth of the sample. The radiation of the nanosize fluorescent objects within the microscope object domain is going through the microscope optical system into a special phase filter. The filter is a diffraction optical element that transforms the field emitted by the nanosize particle to a two-lobe light field. Provided that the radiation emitted by the particle has been transformed by the filter, then two intensity maxima appear within the microscope image plane. And their mutual orientation will determine the longitudinal coordinate of a radiation source [3-6].



Within the present research we study the two-lobe light fields with the intensity rotation, developed using the theory of spiral beam optics. Specifically the effect exerted by basic aberrations (spherical aberration, coma, astigmatism) on the light fields under consideration is evaluated.

1.1. Two-lobe light fields description from the viewpoint of the spiral beams optics

Discovered in 90-ies of the XX century, the spiral beams have been thoroughly studied by now [7-8]. They are structurally stable under a free propagation within the scale and rotation. It was proved that the spiral beams can be formed in a way allowing generation of the field with a specified intensity distribution and desired rotation parameters of the field under its propagation. The expression that determines the rotation angle θ of the spiral beam intensity has the form:

$$\theta(z) = \theta_0 \arctan\left(\frac{2z}{k\rho^2}\right), \quad (1)$$

where k is wave number, ρ is parameter describing the beam size lateral dimension, θ_0 is parameter determining the beam rotation speed under propagation. The full rotation angle of the beam in the Fresnel zone is:

$$\theta = \theta_0 \frac{\pi}{2}. \quad (2)$$

One more property of the spiral beam is that within the initial plane it can be represented as an expansion in Laguerre-Gaussian modes:

$$F(x, y) = \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} c_{nm} LG_{n,m}\left(\frac{x}{\rho}, \frac{y}{\rho}\right), \quad (3)$$

while the indices of these modes satisfy the following condition:

$$2n + |m| + \theta_0 m = const, \quad (4)$$

Thus on selecting the beam rotation parameter, θ_0 we specify the beam rotation speed and determine its functional form.

Within the present research the two-lobe light fields will be studied in the form of Laguerre-Gauss modes superposition with the rotation parameter $\theta_0 = -2$ and $\theta_0 = -4$ [9]. To calculate the phase element forming the two-lobe light field with the rotation parameter $\theta_0 = -2$, the algorithm described in [6] was modified. The Laguerre-Gauss modes superposition was used as zero-order approximation:

$$F = LG_{0,0} + LG_{1,2} + LG_{2,4} + LG_{3,6} + LG_{4,8}, \quad (5)$$

Further on the updating of the intensity distribution of the calculated field is fulfilled in order to discern two operating maxima in the intensity distribution and to reduce noises between them. The adjustment procedure consists in carrying out of two Fresnel transforms, direct and inverse, between N reference planes. One iteration of the adjustment procedure of the intensity distribution consists of N direct Fresnel transforms into each of the selected planes, and N inverse transforms into the phase element plane. We fulfilled nine such transforms, $N=9$. After the direct Fresnel transform, the intensity of the calculated field is changed in accordance with the following rule: if the intensity in the given point is higher than $0.5I_{max}$, where I_{max} is the maximum intensity within the selected plane (i.e. the plane where the Fresnel transform is made to), then the field amplitude remains unchanged. If the condition is not satisfied then the field amplitude in the given point becomes twice lower. The iteration procedure is stopped when the increment of the diffraction effectiveness after the i -th iteration is less than 5 %.

The phase filter for generation of the two-lobe light field with the rotation parameter $\theta_0 = -4$, was obtained when the spiral beam used had the form:

$$F = LG_{0,0} + LG_{3,2} + LG_{6,4}, \quad (6)$$

The phase distributions in the shade of grey gradations and intensity distributions in the phase filter plane are shown in figure 1.

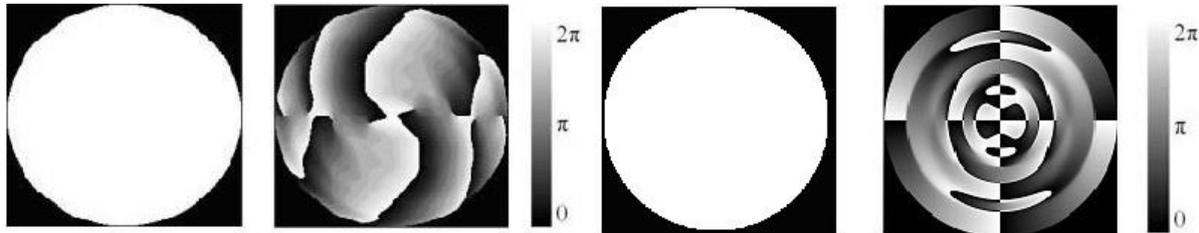


Figure 1. Intensity and phase distributions in the phase filter plane for the fields obtained by means of the development by Laguerre-Gauss modes with the rotation parameter $\theta_0 = -2$ (left) and $\theta_0 = -4$ (right).

1.2. Experimental procedure for the two-lobe light fields generation

Experiments on generating the two-lobe light fields were carried out with the aid of a phase spatial liquid-crystal modulator HOLOEYE HEO-1080. An expanded, collimated, intensity-homogeneous beam from the solid-state laser (power up to 50 mWt, $\lambda=532$ nm) was directed to the modulator HOLOEYE HEO-1080 where the calculated phase distributions of the studied light field was formed. After the light diffraction in the modulator the investigated field was formed which was then focused by the lens inscribed into the phase distribution (in the modulator). The focal length of that lens is 30 cm. The analysis of intensity distribution of the investigated fields in various cross-sections was assisted with the horizontal microscope and digital camera. In figure 2 the experimental set-up layout is shown.

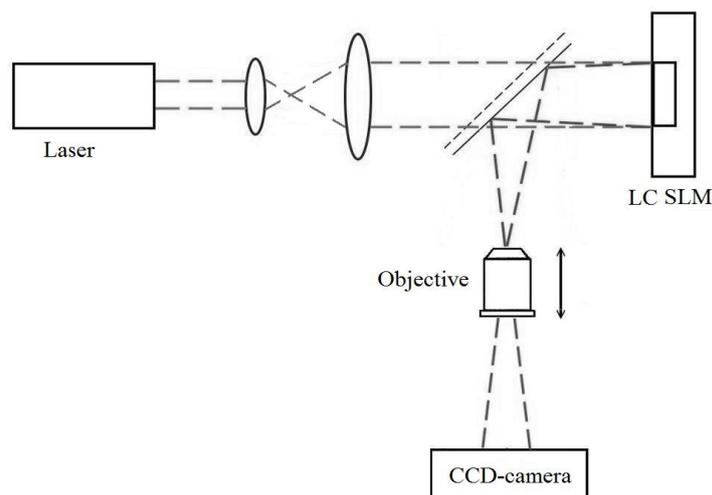


Figure 2. Layout of the experimental set-up.

The developed experimental set-up affords to study cross-sections of the light fields under investigation, perpendicular to the axis of their propagation. The studied field's structure can be judged by the intensity distributions obtained in different sections. Typical patterns of the intensity distribution for the two-lobe light fields are illustrated in figure 3.

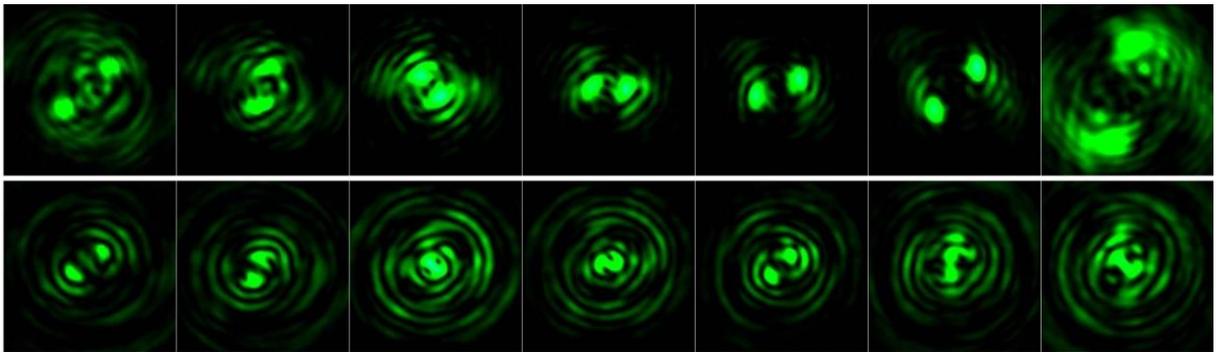


Figure 3. Intensity distribution in sections perpendicular to the axis of propagation of the two-lobe light fields with the rotation parameter $\theta_0 = -2$ (upper row) and $\theta_0 = -4$ (lower row).

The information on the spatial location of the maxima of intensity can be extracted from the experimentally obtained data. First the angle of rotation of the two maxima from the initial state (horizontal line) is measured. Each value obtained for the rotation angle of the intensity distribution has its corresponding distance from the focusing plane to the beam section where this intensity distribution is registered. With the data obtained it is possible to form the relationship between the angle of rotation and the distance to the beam waist. The rotation of the intensity distribution under free spatial propagation is fulfilled counterclockwise (figure 3). Figure 4 shows the experimental points and diagrams (1) for the rotation parameters $\theta_0 = -2$ and $\theta_0 = -4$. It is seen that the experimentally obtained dependencies are well matching the analytical ones.

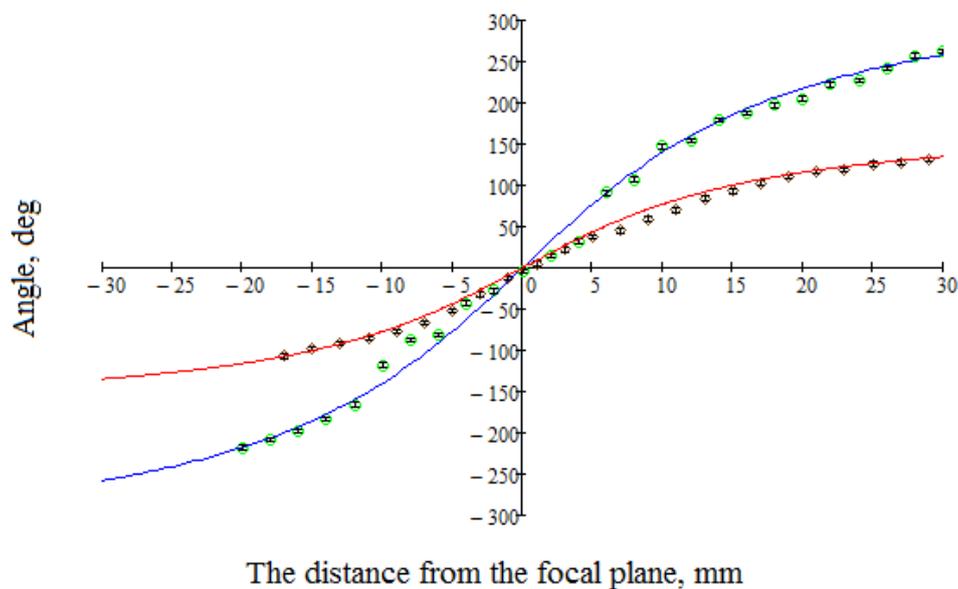


Figure 4. Rotation angle θ of the intensity distribution versus distance from the focusing plane diagram. Red line is the analytical dependency for $\theta_0 = -2$; blue line – for $\theta_0 = -4$. Error of measurement is 5° .

2. Aberrations impact on the two-lobe light fields. Mathematical modeling and experimental data.

While dealing with real optical systems one is inevitably facing the aberrations existing in them. Besides, the radiating nano-particles are located inside a studied sample where some heterogeneities causing the phase distortions can be found. These facts testify that then investigation of the aberrations influence on two-lobe light fields is an important problem. This section of the present work is dedicated to the study on the aberrations influence on a two-lobe light field.

2.1. Simulation of the aberrations influence on two-lobe light fields generation

The aberrations were specified in the representation of the Zernike polynomials [10], in accordance with the OSA standard normalization [11].

$$Z_n^m(r, \theta) = \begin{cases} N_n^m R_n^m(r) \cos(m\theta), & \text{for } m \geq 0; \\ -N_n^m R_n^m(r) \sin(m\theta), & \text{for } m < 0. \end{cases} \quad (7)$$

$$R_n^m(r) = \sum_{s=0}^{(n-|m|)/2} \frac{(-1)^s (n-s)!}{s! \left(\frac{n+|m|}{2} - s\right)! \left(\frac{n-|m|}{2} - s\right)!} r^{n-2s}; \quad (8)$$

$$N_n^m = \sqrt{\frac{2(n+1)}{1+\delta_{m0}}}; \quad (9)$$

where r and θ are polar coordinates.

The computational modeling was carried out to simulate the influence of astigmatism (Z_2^2), coma (Z_3^1) and spherical aberrations (Z_4^0) on the studied light fields with the rotation of the intensity distribution. The phase transmission of the calculated mask was multiplied by the phase transmission adequate to the specified aberration type. The aberration value ranged from $\lambda/32$ to λ . Then the intensity distributions were calculated for the sections perpendicular to the axis of the light field propagation within the area of focusing by a lens with the focal distance of 300 mm, in cross sections spaced one from another by 1mm. It was revealed that when the value of the spherical aberration of $\lambda/16$, coma $\lambda/16$ and astigmatism $\lambda/8$ in the intensity distribution of the light field with the rotation parameter $\theta_0 = -2$ it is still possible to discern two spots with the maximum illumination. Though for the astigmatism of $\lambda/8$ there is an area inside the range where the two spots are not clearly cut. The simulation results for the mentioned cases are given in figure 5. For the light field with the rotation parameter $\theta_0 = -4$ the aberration value allowing to discern two maxima of the intensity distribution is $\lambda/16$. This result is illustrated in figure 6. It should also be mentioned that with the rise of the aberration value, the area where the two maxima of the intensity distribution can be discerned, is reducing.

Table 1 contains the data obtained by the computational modeling results processing. This data confirm that the rotation of two well-separated intensity maxima can be observed under low aberration values. The problem of evaluating the depth of occurrence of the radiating nano-object requires that two-lobe intensity distribution rotated by 180° . On assuming this condition we'll consider the aberration influence insignificant provided that the angle of intensity distribution rotation is 180° . By using table 1 and on assuming this condition we can conclude that the aberrations influence is nonessential for the value of $\lambda/16$, both for the field with the rotation parameter $\theta_0 = -2$ and for field with the rotation parameter $\theta_0 = -4$.

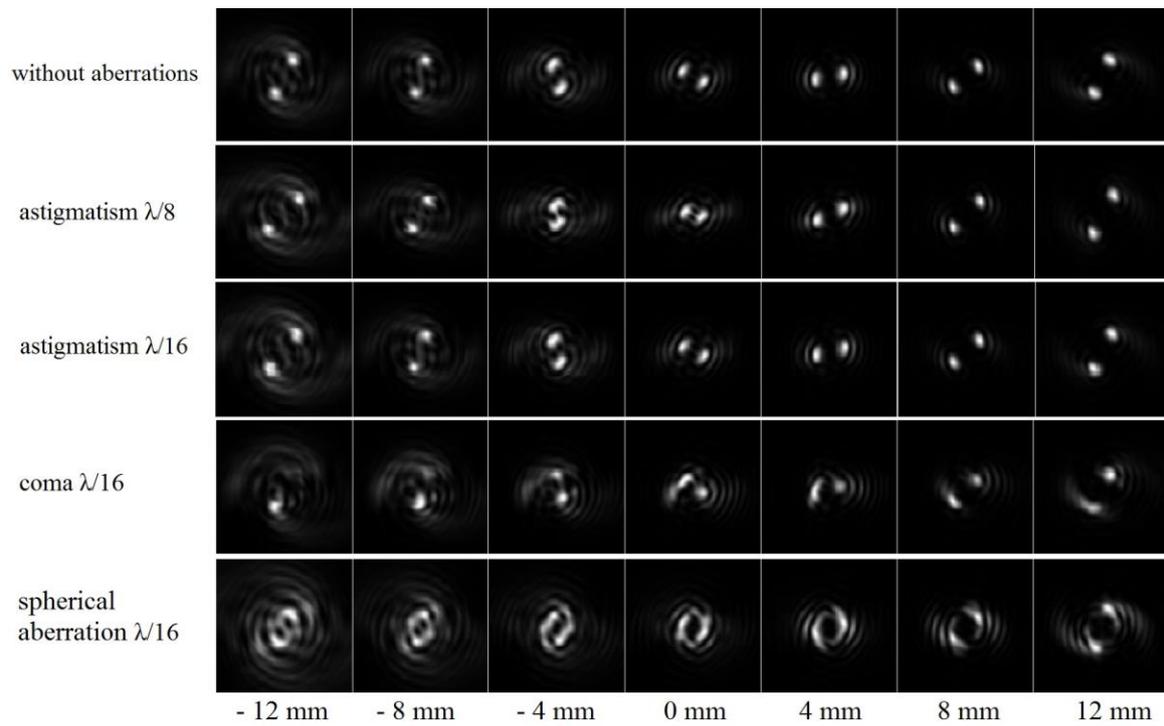


Figure 5. Results of aberrations influence modeling for the light field with the rotation parameter $\theta_0 = -2$.

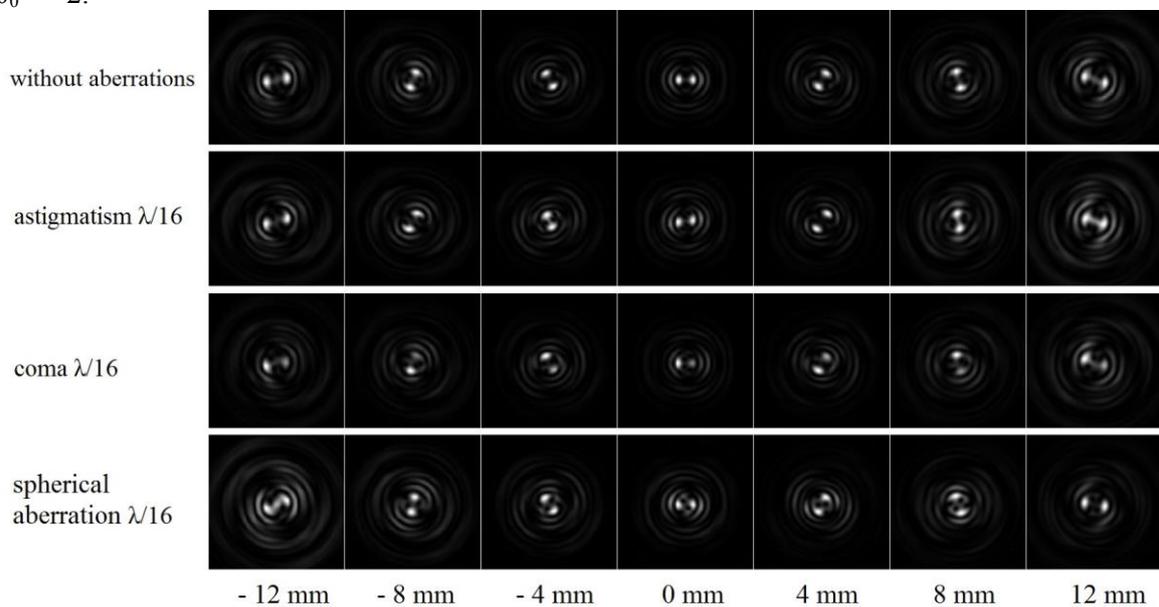


Figure 6. Results of aberrations influence modeling for the light field with the rotation parameter $\theta_0 = -4$.

Table 1. Full rotation angle of two maxima in the intensity distribution of the light fields being researched under aberrations.

The light field with the rotation parameter $\theta_0=-2$			The light field with the rotation parameter $\theta_0=-4$		
Astigmatism	$\lambda/32$	214°	Astigmatism	$\lambda/32$	370°
	$\lambda/16$	212°		$\lambda/16$	285°
	$\lambda/8$	30°, 98°		$\lambda/8$	250°
	$\lambda/4$	-		$\lambda/4$	-
	$\lambda/2$	-		$\lambda/2$	-
	λ	-		λ	-
Coma	$\lambda/32$	208°	Coma	$\lambda/32$	370°
	$\lambda/16$	185°		$\lambda/16$	372°
	$\lambda/8$	143°		$\lambda/8$	15°
	$\lambda/4$	-		$\lambda/4$	-
	$\lambda/2$	-		$\lambda/2$	-
	λ	-		λ	-
Spherical aberration	$\lambda/32$	223°	Spherical aberration	$\lambda/32$	365°
	$\lambda/16$	216°		$\lambda/16$	291°
	$\lambda/8$	20°		$\lambda/8$	-
	$\lambda/4$	-		$\lambda/4$	-
	$\lambda/2$	-		$\lambda/2$	-
	λ	-		λ	-

2.2. Experimental study on the aberrations influence on two-lobe light fields

A number of experiments on the two-lobe light fields generation under aberrations (spherical aberration, coma, astigmatism) of various values ($\lambda/32$, $\lambda/16$, $\lambda/8$, $\lambda/4$, $\lambda/2$, λ) in accordance with the OSA standard normalization. Under the experiments the phase distributions formed by the spatial light modulator represents a superposition of the phase distribution of the studied field and the initial aberration of a certain value. The aberrations were specified in the representation of the Zernike polynomials in the form: Z_2^2 – astigmatism, Z_3^1 – coma, Z_4^0 – spherical aberration. The variable r was ranging within $[0, 1]$. During the phase mask calculation the scaling of radial coordinate was carried out, so that the aperture radius was equal to 1.

The registration of the obtained field intensity distribution was fulfilled within the planes perpendicular to the axis of the light field propagation. In the course of a visual analysis of the experimentally obtained intensity distributions of the researched light fields under aberrations it was revealed that at the value $\lambda/16$ of spherical aberration, coma and astigmatism it is feasible to extract, basing on the intensity distribution, the information on the maxima location for the considered fields. This can be clearly seen in figures 7 and 8. The data obtained are coinciding with the mathematical modeling results for the initial aberrations influence on the light fields under investigation.

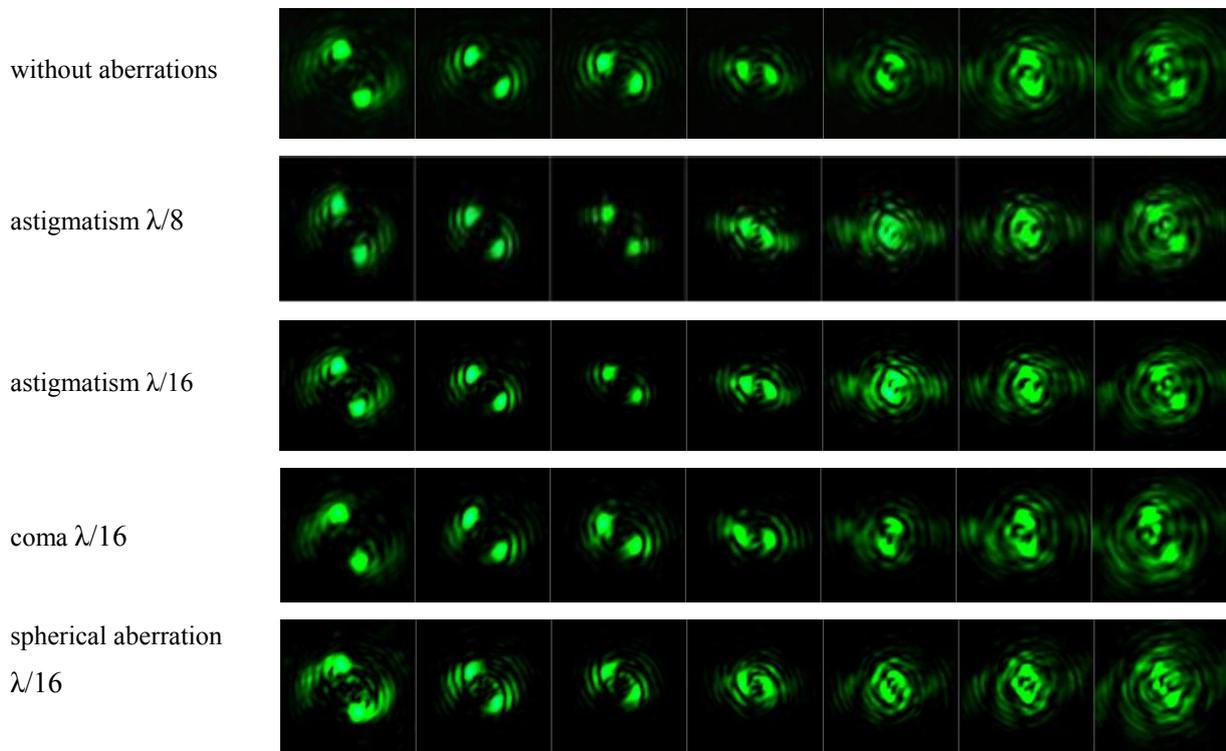


Figure 7. Results of experiments on the aberrations influence on the light field with the rotation parameter $\theta_0 = -2$

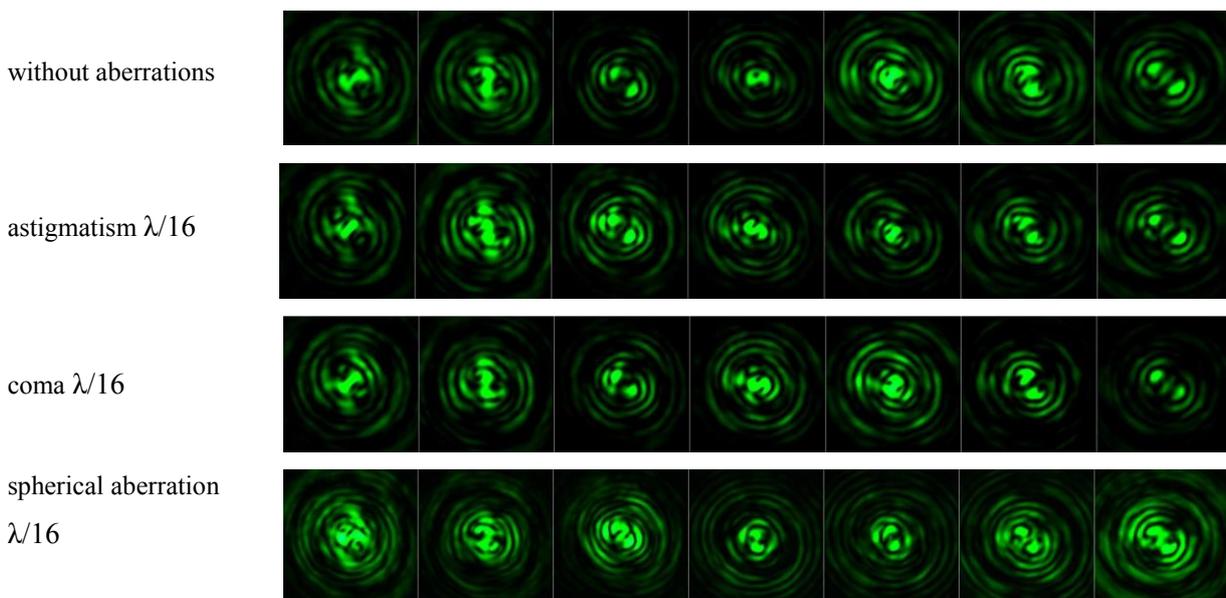


Figure 8. Results of experiments on the aberrations influence on the light field with the rotation parameter $\theta_0 = -4$

3. Results

The paper presents the results obtained for the generation of two-lobe light fields with the rotation of intensity distributions. The light fields have been obtained on the basis of the superposition of Laguerre-Gaussian modes, with the rotation parameters $\theta_0 = -2$ и $\theta_0 = -4$. The experimentally obtained relationships of the rotation angle and the distance agree well with the theoretic ones (figure 4).

First studies of the aberrations (astigmatism, coma, spherical aberration) influence on the generation of two-lobe light fields have been fulfilled. The computational modeling of the aberrations influence showed that with the rise of the aberration value the range of rotation angles of the two discernible maxima in the intensity distribution is decreased (table 1). It is caused by the distortion of the light field intensity distribution due to the aberration value increase, as a result it becomes impossible to discern the two maxima. A quantitative analysis of the numerical and life-size modeling permits to conclude that from the intensity distribution pattern, the information on the angular location of the maxima can be obtained for the studied fields with the rotation parameter $\theta_0 = -2$ and $\theta_0 = -4$ at the spherical aberration $\lambda/16$, coma $\lambda/16$ and astigmatism $\lambda/16$.

Note that normalization of the Zernike polynomials by the OSA standards and that used in [9] are different. In order to transfer to the aberrations described in [9] one should multiply the astigmatism value under the OSA standard by $\sqrt{6}$, coma – by $2\sqrt{2}$, astigmatism – by $\sqrt{10}$. On making this transition experimentally observed admissible aberration values are obtained for astigmatism $0,153\lambda$, for coma $0,177\lambda$ and for spherical aberration – $0,198\lambda$.

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