

# Dirac Particles Emission from Reissner-Nordstrom-Vaidya Black Hole

**Yuant Tiandho and Triyanta\***

Theoretical High Energy Physics and Instrumentation Division,  
Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung,  
Jl. Ganesha No. 10, Bandung 40132, Indonesia

\*Corresponding author: triyanta@fi.itb.ac.id

**Abstract.** Using Hamilton-Jacobi method, we study the Dirac particles emission from Reissner-Nordstrom-Vaidya (RNV) black hole. The Dirac particles are described by Dirac equation in curved spacetime and emission process is defined as tunneling effect. The probability of Dirac particles emission is related to the Hawking temperature and we obtain that this temperature is equal to temperature that derived through spinless particles emission. Furthermore, we also show that the mass of Dirac particles does not affect to the Hawking temperature.

## 1. Introduction

Using quantum mechanics theory, Hawking proposed that a black hole has a temperature, meaning that the black hole can emit particles not as stated in the classical theory [1-3]. In the first explanation of the black hole temperature, Hawking considered the Schwarzschild black hole. Its Hawking temperature is [3],

$$T_H = \frac{\hbar c}{8\pi kM} \quad (1)$$

where  $k$  is the Boltzmann constant and  $M$  is the black hole mass. It turns out that the temperature of black holes is inversely proportional to its mass.

Calculation of the Hawking temperature at different types of black holes with various methods is a hot topic in a recent decade [4-6]. In this work, we chose the Reissner-Nordstrom-Vaidya (RNV) black hole. Compared with the Schwarzschild black hole, the Vaidya black hole is more realistic because its mass depends on space and time [7-8]. Previously, one of us has studied the Hawking temperature of Vaidya black hole [9] and RNV black hole [10] for spinless and massless particles emission. Therefore, in this paper we would like to extend this study for RNV black hole with Dirac particles (spin  $\frac{1}{2}$  particles) emission.

In this work we use semi classical Hamilton-Jacobi method or complex path method [11] to calculate the Hawking temperature. In this method, the wave function which is defined as function of the action is substituted into the Dirac equation with the RNV spacetime as background. This detailed discussion is in Section 2. Through the Hamilton-Jacobi method we can obtain the action for calculate the probability of Dirac particles emission. By using the balanced principle, we will know that the probability correspond to the Hawking temperature. Furthermore, in this work we also investigated



the influence of particles mass that are emitted to the temperature. Behaviour of the massive tunneling particles is showed in Section 3. In the last section we give a conclusion of our work.

## 2. Dirac equation in RNV spacetime

The line interval of RNV spacetime is defined by,

$$ds^2 = -f dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (2)$$

where  $f = 1 - 2(M+p)/r + Q^2/r^2$ . In this paper  $M$  and  $Q$  correspond to mass and charge of RNV black hole,  $v$  is Eddington time coordinate, and  $p$  is arbitrary function of mass and charge,  $p(M, Q)$ . By using Eddington coordinate transformation the above metric can be written as,

$$ds^2 = f dt^2 + f^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (3)$$

From that metric, it is clear that the RNV black hole has two event horizons,

$$r_{EH} = (M+p) \pm \sqrt{(M+p)^2 - Q^2} \quad (4)$$

where plus (minus) sign correspond to outer (inner) event horizon and single singularity is reached for neutral black hole. However, this form of metric does not give information on the velocity of massive particle. Accordingly, we use Painleve coordinates by transform,  $t \rightarrow t - \int \sqrt{\frac{1-f}{f^2}} dr$  and the metric in eq. (3) can read as.

$$ds^2 = -f dt^2 + 2\sqrt{1-f^{-1}} dr dt + dr^2 + r^2 d\Omega^2 \quad (5)$$

Dirac particles is described by the Dirac equation equation in curved spacetime,

$$i\gamma^\mu D_\mu \psi + \frac{m}{\hbar} \psi = 0 \quad (6)$$

where  $m$  is mass of emission particles. The covariant derivative is given by  $D_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\sigma_{ab}$ , where  $\sigma_{ab}$  correspond to commutator of Minkowskian spacetime gamma matrices  $\sigma_{ab} = \frac{1}{2}[\gamma_a, \gamma_b]$  and  $\omega_\mu^{ab}$  is spin connection. We use the flat spacetime gamma matrices  $\gamma^a$  as,

$$\gamma^0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \quad (7)$$

The flat gamma matrices  $\gamma^a$  and the curved gamma matrices  $\Gamma^\mu$  are related by  $\Gamma^\mu = e_a^\mu \gamma^a$  and those are specified by definition  $\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu} \times I$ . There are several different expression of gamma matrices  $\Gamma^\mu$  and in this work we choose the representation for Dirac matrices to be [12],

$$\begin{aligned} \Gamma^0 &= \frac{1}{\sqrt{f}} \begin{pmatrix} 0 & 1 + \sqrt{1+f}\sigma^3 \\ -1 + \sqrt{1-f}\sigma^3 & 0 \end{pmatrix}, & \Gamma^1 &= \sqrt{f} \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \\ \Gamma^2 &= \frac{1}{r} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, & \Gamma^3 &= \frac{1}{r \sin \theta} \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix} \end{aligned} \quad (8)$$

where  $\sigma^k$  are Pauli matrices. Spinor wave function  $\psi$  has two spin states: spin-up and spin-down. Because RNV black hole is spherically symmetric so the Hawking radiation depends on  $r$  and  $t$  only. Thus, the functions for spin-up and spin-down particles respectively satisfy,

$$\psi_{\uparrow} = \begin{pmatrix} a(t,r)\xi_{\uparrow} \\ b(t,r)\xi_{\uparrow} \end{pmatrix} \exp\left[\frac{i}{\hbar}S_{\uparrow}(t,r)\right] = \begin{pmatrix} a(t,r) \\ b(t,r) \\ 0 \end{pmatrix} \exp\left[\frac{i}{\hbar}S_{\uparrow}(t,r)\right] \quad (9)$$

$$\psi_{\downarrow} = \begin{pmatrix} c(t,r)\xi_{\downarrow} \\ d(t,r)\xi_{\downarrow} \end{pmatrix} \exp\left[\frac{i}{\hbar}S_{\downarrow}(t,r)\right] = \begin{pmatrix} 0 \\ c(t,r) \\ 0 \\ d(t,r) \end{pmatrix} \exp\left[\frac{i}{\hbar}S_{\downarrow}(t,r)\right] \quad (10)$$

where  $S_{\uparrow}$  and  $S_{\downarrow}$  are action of emission particles for spin-up particles and spin-down particles. However, we only analyse the spin-up case since the spin-down case is just analogous. By substituting eq. (9) into eq. (6) and recalling that the Planck constant is very small we have,

$$\begin{aligned} -b \left( \frac{(1+\sqrt{1-f})\partial_t S_{\uparrow}}{\sqrt{f}} + \sqrt{f}\partial_r S_{\uparrow} \right) + ma &= 0 \\ a \left( \frac{(1-\sqrt{1-f})\partial_t S_{\uparrow}}{\sqrt{f}} - \sqrt{f}\partial_r S_{\uparrow} \right) + mb &= 0 \end{aligned} \quad (11)$$

Through the Hamilton-Jacobi method, the action can be expressed in two parts: the time part which has the form of  $Et$  and the part that relates to radial expressed as  $R(r)$ ,

$$S_{\uparrow} = \int_0^t E(t')dt' + R(r,t) \quad (12)$$

The term  $\int_0^t E(t')dt'$  is a generalization of  $Et$  because energy can vary in time. Substituting eq. (12) into eq. (11) we obtain,

$$\begin{aligned} -b \left( \frac{(1+\sqrt{1-f})}{\sqrt{f}} (E(t) + \partial_t R(r,t)) + \sqrt{f}\partial_r R(r,t) \right) + ma &= 0 \\ a \left( \frac{(1-\sqrt{1-f})}{\sqrt{f}} (E(t) + \partial_t R(r,t)) - \sqrt{f}\partial_r R(r,t) \right) + mb &= 0 \end{aligned} \quad (13)$$

The two equations above have two possible solutions of  $R$ ,

$$\begin{aligned} a=0 \rightarrow \partial_r R(r)_- &= -\frac{(1+\sqrt{1-f})}{f} (E(t) + \partial_t R(r,t)) \\ b=0 \rightarrow \partial_r R(r)_+ &= \frac{(1-\sqrt{1-f})}{f} (E(t) + \partial_t R(r,t)) \end{aligned} \quad (14).$$

### 3. Behaviour of the massive Dirac particles

The equation of motion between massive particles and massless particles are different. When we consider tunneling of massless particles we may use radial null geodesic method but for massive particles the method is not valid. Since the world line of massive particles is not light-like. By using

Landau theory of the coordinate clock synchronization and the definition of phase velocity of de Broglie wave, Wen can obtain the velocity of massive particles [13],

$$\dot{r} = -\frac{1}{2} \frac{g_{00}}{g_{01}} \quad (15)$$

Substituting  $g_{00}$  and  $g_{01}$  (eq. (5)) into eq. (15), we obtain,

$$\dot{r} = \frac{1}{2} \frac{f}{\sqrt{1-f}} \quad (16)$$

Accordingly, solution of two equation in eq. (14) can be written as,

$$\frac{dR}{dr} = \frac{\partial R}{\partial r} + \frac{\partial t}{\partial r} \frac{\partial R}{\partial t} \quad (17)$$

where  $\partial t / \partial r$  can be obtained from eq. (16). Near the event horizon, the metric coefficient  $f$  can be expressed by Taylor series. Since we only need their approximation values for short distances from event horizon, we can apply the Taylor expansion at a fixed time,

$$f(r_{EH}, t) \Big|_t \simeq f'(r, t) \Big|_t (r - r_{EH}) + O(r - r_{EH})^2 \Big|_t \quad (18)$$

Considering a slowly varying  $R$ , the function  $R$  in the near event horizon may be obtained by integrating eq. (17) with respect to  $r$ . Notice that  $\partial_r R_+$  has a pole at horizon but  $\partial_r R_-$  does not have a pole and well defined limit at the event horizon. Thus we may conclude that the solution of  $R_-$  function is zero and  $R_+$  function is,

$$R_+ = \int \frac{(1 - \sqrt{1-f})E}{f'(r_{EH})} dr = \frac{2\pi i E}{f'(r_{EH})} \quad (19)$$

Finally, the complete expression of action is,

$$S_- = \int_0^t E(t') dt'; \quad S_+ = \int_0^t E(t') dt' + \frac{2\pi i E(t)}{f'(r_{EH})} \quad (20)$$

In tunneling process, the energy of an emitted particle is less than barrier potential. Accordingly, the particle's momentum and the action function are imaginary. Thus  $\int E dt$  can be written as  $i \text{Im}(\int E dt)$ .

Probabilities of ingoing and outgoing particles respectively are,

$$P_{in} = \exp \left[ \frac{2}{\hbar} \left( \text{Im} \int E(t') dt' \right) \right] \quad (21)$$

$$P_{out} = \exp \left[ \frac{2}{\hbar} \left( \text{Im} \int E(t') dt' - \frac{2\pi E(t)}{f'(r_{EH}, t)} \right) \right]$$

If all ingoing particles absorbed by black hole or  $P_{in} = 1$ , the probability of outgoing particle is,

$$P_{out} = \exp \left[ -\frac{4\pi}{\hbar} \left( \frac{E(t)}{f'(r_{EH}, t)} \right) \right] \quad (22)$$

The tunneling probability can be expressed in a Boltzmann factor and its energy  $P_{out} = \exp[-E\beta]$ . Thus the Hawking temperature due to Dirac massive particles emission from the RNV black hole is,

$$T_H = \frac{\hbar}{4\pi k} f'(r_{EH}, t) \quad (23)$$

Recalling the explicit form of metric coefficient  $f$  the Hawking temperature for the RNV black hole becomes,

$$T_{H\pm} = \frac{\hbar}{4\pi k} \left( -\frac{2(M' + p')}{r_{EH\pm}} + \frac{2(M + p) + 2QQ'}{r_{EH\pm}^2} - \frac{2Q^2}{r_{EH\pm}^3} \right) \quad (24)$$

The plus (minus) sign correspond to the temperatur in outer (inner) event horizon. The above expression is exactly the same with that in [10]. The above expression also does not contain particle mass. It can be concluded that the Hawking temperature does not depend on particle spin and particle mass. It is clear that for  $Q = 0$ ,  $p = 0$  and  $M$  is a constant, the Hawking temperature above corresponds that for the Schwarzschild black hole.

### Conclusions

We have successfully extended our consideration about Hawking temperature of RNV black hole by using a tunneling method for fermion particles. The analysis has showed that the Hawking temperature in this work has same value as that for the RNV black hole when analysed through spinless particles emission. In addition, the mass of emitted particles also does not affect to the temperature. The black hole temperature is inversely proportional to its mass or directly proportional to derivative of the metric coefficient.

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