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Modeling and Calculation of Optical Amplification in One Dimensional Case of Laser Medium Using Finite Difference Time Domain Method

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Abstract. Finite Difference Time Domain (FDTD) method has been much employed for studying light propagation in various structures, from simple one-dimensional structures up to three-dimensional complex structures. One of challenging problems is to implement this method for the case of light propagation in amplifying medium or structures, such as optical amplifier and lasers. The implementation is hindered by the fact that the dielectric constant becomes a complex number when optical gain parameter is involved in the calculation. In general, complex dielectric constant is related to complex susceptibility, in which the imaginary part is related to optical gain. Here, we then modify the formulation for updating electric field in the calculation algorithm. Using this approach, we then finally can calculate light amplification in laser active medium of Nd^{3+} ion doped glass. The calculation result shows an agreement with the result from the calculation using differential equation for intensity. Although this method is more time consuming, the method seem promising for optical complex micro- and nano-structures, such quantum dot lasers, micro-ring lasers, etc.

1. Introduction

Finite Difference Time Domain (FDTD) is a numerical method for calculating the propagation of electromagnetic wave. This method has been much employed for various studies because of its easiness to be applied for various complicated structures [1]. In most cases, this method is applied for transparent dielectric materials with optically lossless characteristics. However, in particular cases, the medium is optically absorbing medium or optically amplifying medium, such as in lasers and optical amplifiers. In such cases, the medium permittivity is no longer real number, but it is a complex number. Direct implementation of complex permittivity in the FDTD calculations leads to numerical errors. Therefore, many approaches to solve this problem have been proposed [2, 3].

In a laser medium, the medium is irradiated with a light pumping to produce a population inversion, in which more valence electrons are in the excited state rather than in the ground state. These electrons will then proceed in a stimulated emission emitting coherent photon. This event can be triggered by the presence of the signal light entering from the outside medium. The process producing laser emission is called as photo-pumped laser. The medium for this purpose is commonly rare-earth based laser medium, such as Nd^{3+} in crystals or glasses. Such laser mediums are often used for diode pumped solid state laser systems, optical amplifiers etc [4].

2. Optical amplification

For the 4-level system, which is commonly used for rare-earth based laser medium, the schematic diagram of the electronic levels involved in the process is shown in Figure 1. The rates of electron population change at those electronic levels are dependent on the rates of upward transition (excitation) and downward transition (relaxation), including stimulated transition. The optical transition stimulated optical transition W_{ij} is dependent on the local light intensity (I), which is given by $W_{ij} = \dfrac{\sigma I_{ij}}{h\nu_{ij}}$, where ij indicates the transition from the i-th level to j-th level in the above diagram. It is supposed that the non-radiative transition (γ) is much faster in comparison to that of the radiative transitions. In such case, therefore, only the rates of electron population changes at the 1st level and 3rd level are needed to be considered. The rates under steady state condition are given by

$$\begin{aligned}
\frac{dN_1}{dt} &= -W_{14}N_1 + \left(\frac{1}{\tau} + W_{32}\right)N_3 \\
\frac{dN_3}{dt} &= +W_{14}N_1 - \left(\frac{1}{\tau} + W_{32}\right)N_3 \\
N_1 + N_3 &= N_{total}
\end{aligned} \tag{1}$$

The spontaneous emission rate, hence $A = 1/\tau$, can be considered much smaller under lasing condition.

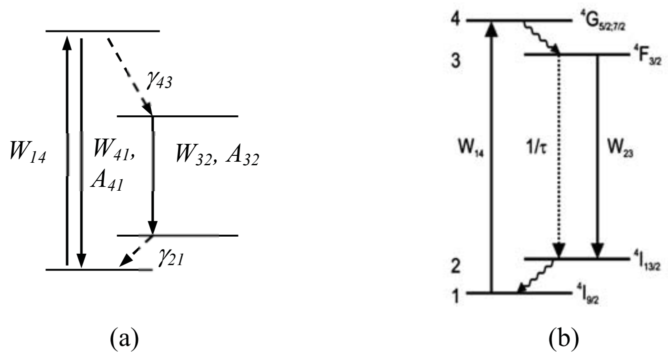

Figure 1. (a) The general 4-levels system and (b) the 4-levels model for Nd^{3+} ions.

In order to find the distribution of the amplified signal intensity or lasing intensity inside the medium, the simplest way is by solving the first order of differential equations for both the excitation (pumping) beam and the amplified (signal) beam. When the light source is a laser, for a simplification, we may consider that both the pumping beam and the amplified signal beam are propagating coaxially. In such case, we then have a set of differential equations of one dimensional case

$$\begin{aligned}
\frac{dI_p}{dz} &= -\sigma_{abs}N_1 I_p - \alpha_p I_p \\
\frac{dI_s}{dz} &= \sigma_{em}N_3 I_s - \alpha_s I_s
\end{aligned} \tag{2}$$

where I_p and I_s are the pumping beam and signal intensity, respectively. We may see that to solve this set of equation, we must also solve the set of equation (2). It can be solved easily by numerical computation. The optical amplification or gain can then be found from

$$g_{tot}\left(dB/cm\right)=\frac{10}{L(cm)}\log\left(\frac{I_s(L)}{I_s(0)}\right) \tag{3}$$

3. Finite Difference Time Domain method

Finite Difference Time Domain (FDTD) is a method to calculate electromagnetic propagation by solving numerically the differential form of Maxwell's equation. This method firstly introduced by Yee, who propose an algorithm of numerical calculation for those differential forms of Maxwell's equation. If we see the Ampere and Faraday-Lenz laws in the Maxwell's equation, namely

$$\begin{aligned}\nabla\times\vec{H}&=\varepsilon_0\varepsilon_r\frac{\partial\vec{E}}{\partial t}\\ \nabla\times\vec{E}&=\mu\frac{\partial\vec{H}}{\partial t}\end{aligned} \tag{4}$$

We may notice that those equations connect the electric field and magnetic field in different domain. On the left side is in the spatial domain, while on the right side is the time domain. Therefore, by forming a discretization in both spatial and time domain, as illustrated in Figure 2, the above equations can be written as

$$\begin{aligned}E_x^{n+1/2}(k)&=E_x^{n-1/2}(k)+\frac{\Delta t}{\varepsilon_r\varepsilon_0\,\Delta z}\left(H_y^n(k-1/2)-H_y^n(k+1/2)\right)\\ H_y^{n+1}(k+1/2)&=H_y^n(k+1/2)+\frac{\Delta t}{\mu_0\,\Delta z}\left(E_z^{n+1/2}(k)-E_z^{n+1/2}(k+1)\right).\end{aligned} \tag{5}$$

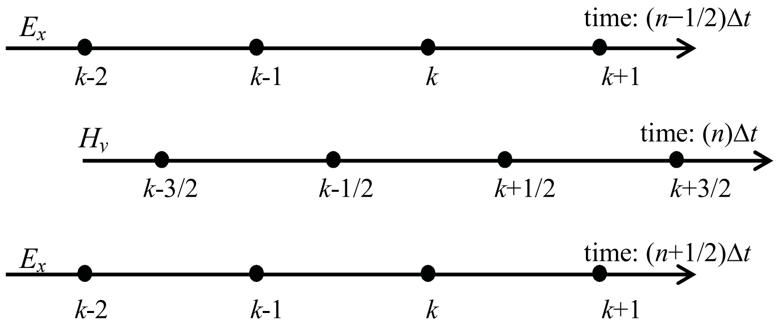

Figure 2. Spatial and time discretization to be implemented in the FDTD method.

We may notice here that those equations contains material parameters only in the electric permittivity (ε) and magnetic permeability (μ). Those equations are commonly be implemented for dielectric constants with real number, that is for simple transparent dielectric materials.

4. A simple scheme for employing FDTD in an amplifying medium

When a medium exhibit optically absorbing or amplifying characteristics, the electric permittivity becomes a complex number, which can written as

$$\tilde{\varepsilon}_r\left(\omega\right)=\varepsilon'\left(\omega\right)+i\varepsilon''\left(\omega\right) \tag{6}$$

The real part is related to the refractive index of the material, while the imaginary part is related to the optical loss (absorption) or optical amplification gain (stimulated emission). The positive value is for absorption, while the negative value for amplification. This permittivity is related to the complex susceptibility by the following relationship

$$\tilde{\varepsilon}_r(\omega) = \left(1 + \tilde{\chi}(\omega)\right) = (1 + \chi'(\omega) + i\chi''(\omega)) \tag{7}$$

It should be noted that equation (5) will produce a numerical error if the electric permittivity is a complex number. Therefore, we need to modify equation (5) so that it can accommodate the complex number of permittivity dielectric. If we see equation (5) and equation (6), we may also notice that the permittivity is frequency dependence. In such characteristics, the time response will be also varies depend on the frequency so that the implementation of FDTD is not straightforward.

In order to simplify the calculation, we made some assumptions. The first assumption is that the whole process is under steady conditions. The second one is that the propagating wave is a plane wave with single wavelength. Those assumptions, however, are consistent with a particular lasing mode, that is, the cw operating mode with a very sharp single wavelength emission. With those assumptions, the Ampere law is the can be written as

$$\begin{aligned} \vec{\nabla} \times \vec{H} &= \frac{\partial}{\partial t}\left(\varepsilon_0(1+\tilde{\chi})\vec{E}\right) \\ &= \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \varepsilon_0 \tilde{\chi} \frac{\partial \vec{E}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \varepsilon_0(\chi' + i\chi'')\frac{\partial \vec{E}}{\partial t} \end{aligned} \tag{8}$$

where χ is the susceptibility, which is related to complex permittivity by the relationship:

$$\tilde{\varepsilon}_r = \left(1 + \tilde{\chi}\right) = (1 + \chi' + i\chi''). \tag{9}$$

For the 4-level model, the complex susceptibility can be written as

$$\varepsilon_r'' = \chi_{abs}'' = \frac{nc}{\omega}\left[N_1\sigma_{abs}(\omega) - N_3\sigma_{em}(\omega)\right], \text{ for upward transition} \tag{10.a}$$

and

$$= \chi_{em}'' = \frac{nc}{\omega}\left[N_3\sigma_{em}(\omega) - N_1\sigma_{abs}(\omega)\right], \text{ for downward transition} \tag{10.b}$$

Substituting eq. (10) into eq. (8) results in

$$\varepsilon_0\varepsilon_r' \frac{\partial \vec{E}}{\partial t} + i\varepsilon_0 \frac{nc}{\omega}\langle N\sigma\rangle \frac{\partial \vec{E}}{\partial t} = \vec{\nabla} \times \vec{H} \tag{11}$$

where $\langle N\sigma\rangle = \left[N_1\sigma_{14} - N_3\sigma_{32}\right]$ is for net upward transition (1→4), while $\langle N\sigma\ \rangle = \left[N_3\sigma_{32} - N_1\sigma_{14}\right]$ is for net downward transition. Therefore, the FDTD formula for the electric field can be written as

$$E_x^{n+1/2}(k) = E_x^{n-1/2}(k) + \frac{\Delta t}{\Delta z}\frac{1}{\varepsilon_0\varepsilon_r'}\left(H_y^n(k+1/2) - H_y^n(k-1/2)\right) + \frac{\Delta t}{\varepsilon_r'}\frac{c}{4}\langle N\sigma\rangle E_x^{n-1/2} \tag{12}$$

By performing calculation of the electric field and magnetic field from the one edge of the medium to the other one edge position, we can get the intensities at those edges and optical gain from equation (3).

5. Calculation results and Discussions

In order to see the results of the above approximation methods, the calculations have been performed for a case of Nd^{3+} ions doped in glass laser medium. Figure 3 shows the optical amplification by the first method for various concentrations of Nd^{3+} ions. The lifetime parameter used in these calculations is 200 μs with the total length is 3 cm. Other parameters used in the calculations are listed in Table. 1. At the ion concentration of 5×10^{20} cm^{-3}, the optical amplification increases quickly and saturates at about 0.3 cm from the origin edge. As the concentration become smaller, the optical amplification raises slowly. Figure 3(b) shows the optical gain at various pumping intensities. At pumping intensities of 800 mW, the optical gain up to about 4.3 dB/cm can be obtained.

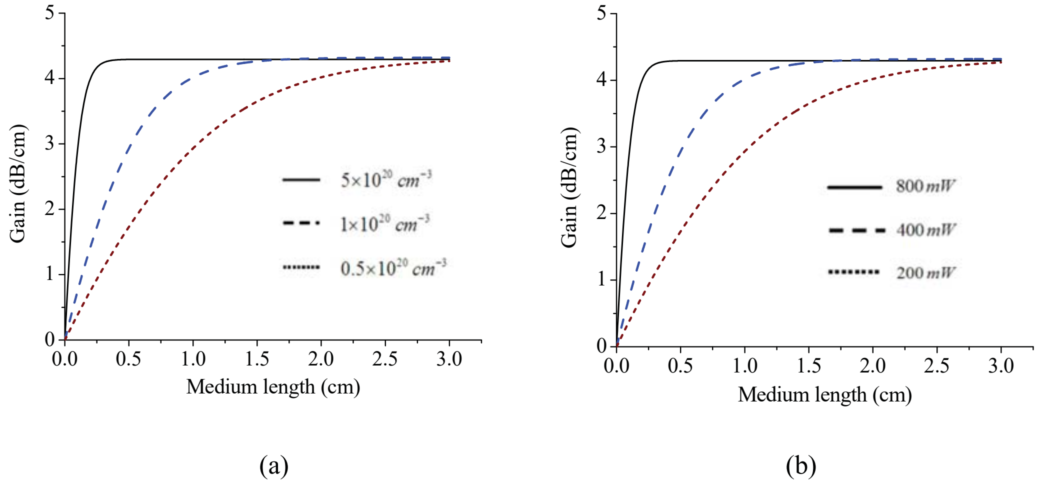

Figure 3. (a) Optical gain at different Nd^{3+} ion concentrations and (b) at various light pumping intensities.

Table 1. The parameters used in the calculations.

$\sigma_{abs}(cm^2)$	$\sigma_{emi}(cm^2)$	Δ	$Signal\,(mW)$	$L\,(cm)$	$\lambda_{pump}(nm)$	$\lambda_{signal}(nm)$
3×10^{-20}	5×10^{-20}	0.002	40	3	808	1064

Figure 4 shows the calculation results by FDTD method, indicated by the square symbol (■). The thin solid line shows the calculation result of the first method as comparison. Because the FDTD method required much longer time calculation, the calculations in this work was limited for medium length up to 0.2 mm only. However, as evident in Figure 4, the calculation results show a high agreement between this FDTD method and the first method.

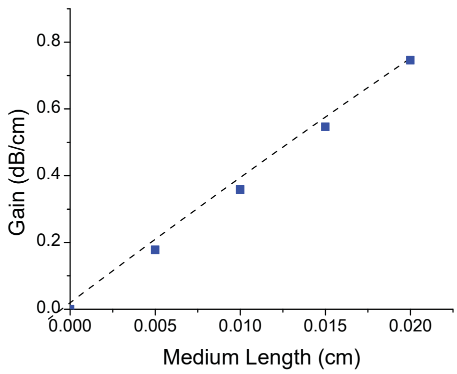

Figure 4. The optical calculation results by FDTD method (square symbol) and the conventional method by solving numerically eq.(2) (dash line).

6. Conclusions

We have performed optical gain calculations by FDTD method with a modification in its electric field formula to accommodate the complex permittivity of the medium. The calculation results were compared with the calculation results from the common method based on the differential equation for intensities. The simulation results shows about 4.3 dB/cm can be obtained in lasing medium containing Nd^{3+} ions of about 5×10^{20} cm^{-3}, under light pumping intensities of 800 mW. The agreement with the first method shows the validity of approximation applied for simplifying the FDTD calculations. Although this method requires a much longer time, this method may offer much flexibility and easiness for calculating complicated micro- and nano-structures, such quantum dot lasers, micro-ring lasers, etc.

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