

# Approximate analytical solution of the Dirac equation for pseudospin symmetry with modified Pöschl-Teller potential and trigonometric Scarf II non-central potential using asymptotic iteration method

B N Pratiwi<sup>1</sup>, A Suparmi<sup>1,2</sup>, C Cari<sup>1,2</sup>, A S Husein<sup>2</sup> and M Yuniarto<sup>1</sup>

<sup>1</sup> Physics Department, Faculty of Mathematics and Science, Sebelas Maret University, Jl.Ir. Sutami 36A Kentingan Surakarta 57126, Indonesia

<sup>2</sup> Physics Department, Graduate Program, Sebelas Maret University

Email: namakubetanurpratiwi@gmail.com

**Abstract.** We applied asymptotic iteration method (AIM) to obtain the analytical solution of the Dirac equation in case exact pseudospin symmetry in the presence of modified Pöschl-Teller potential and trigonometric Scarf II non-central potential. The Dirac equation was solved by variables separation into one dimensional Dirac equation, the radial part and angular part equation. The radial and angular part equation can be reduced into hypergeometric type equation by variable substitution and wavefunction substitution and then transform it into AIM type equation to obtain relativistic energy eigenvalue and wavefunctions. Relativistic energy was calculated numerically by Matlab software. And then relativistic energy spectrum and wavefunctions were visualized by Matlab software. The results show that the increase in the radial quantum number  $n_r$  causes decrease in the relativistic energy spectrum. The negative value of energy is taken due to the pseudospin symmetry limit. Several quantum wavefunctions were presented in terms of the hypergeometric functions.

## 1. Introduction

Schrodinger equation plays important roles for describing the behaviors of a particle at the microscopic scale for non relativistic system. For relativistic system will be used Klein-Gordon equation or the Dirac equation. Klein-Gordon equation for spin-0 particles and the Dirac equation for spin-1/2 particles[1,2]. The Dirac equation describes the motion of a relativistic particle with spin  $\frac{1}{2}$  which is widely used in solving problems of nuclear physics and high energy physics[3]. Spin symmetry in framework of the Dirac equation occurs when the scalar potential  $S(r)$  is equal to the vector potential  $V(r)$  and pseudospin (p-spin) symmetry occurs when  $S(r) = -V(r)$ [4]. The concept of spin symmetry and pseudospin symmetry have been recognized in and hadronic spectroscopies[5]. Spin symmetry has been applied to the spectra of mesons and antinucleon[6] while pseudospin symmetry concept is used to explain the quasi-degeneracy of the nucleon doublets[7], exotic nuclei[8], superdeformation in nuclei[9], and to establish an affective nuclear shell-model scheme[10]. Recently, has been studied about hidden pseudospin and spin symmetries and their origins in atomic nuclei. On the recent progress on the pseudospin and spin symmetries have been studied in various systems and potentials, including extensions of the pseudospin symmetry study from stable to exotic nuclei, from



non-confining to confining potentials, from local to non-local potentials, from central to tensor potentials, from bound to resonant states, from nucleon to anti-nucleon spectra, from nucleon to hyperon spectra, and from spherical to deformed nuclei[11].

The Dirac equation with various potentials have been solved, including the Hulthen potential and type-Coulomb tensor potential[12],the Woods-Saxon potential[13], the Pöschl-Teller trigonometric potential and tensor Coulomb potential[3], the Deng-Fan potential and the Coulomb potential[14], the Pöschl-Teller potential plus Manning Rosen potential[15], the modified Pöschl-Teller non-central potential[16], q-deformed Scarf II potential plus Coulomb-type tensor[17], the generalized Pöschl-Teller potential plus trigonometric Pöschl-Teller non-central potential[18],q-deformed hyperbolic Pöschl-Teller potential and trigonometric Scarf II non-central potential[2], Scarf potential with new tensor coupling potential[19], the Deng-Fan and Eckart potentials with Coulomb-like and Yukawa-like tensor interactions[20], tensor coupling on theMie-type potential[21],and others. The various methods have been used , included the Laplace transformation method[21], hypergeometric method[15], SUSY quantum mechanics[18], Romanovski polynomials[19], the Nikiforov-Uvarov method[2,16,20], asymptotic iteration method[22,23], and others.

In this study, will be solved the Dirac equation in case pseudospin symmetry for modified Pöschl-Teller potential plus trigonometric Scarf II potential using asymptotic iteration method. The basic strategy to obtain the solutions is reducing the Dirac equation to the hypergeometric type equation with suitable changes of variables. Then the eigenvalue and eigenfunction can be obtained using asymptotic iteration method.

The asymptotic iteration method will be briefly reviewed in Section 2. In Section 3, we review the modified Pöschl-Teller potential and Scarf II trigonometric potential, give a brief introduction to the Dirac equation for pseudospin symmetry, and apply the separation of variables in spherical coordinates. In Section 4, we solve the radial part and angular part of the Dirac equation with modified Pöschl-Teller potential combined with trigonometric Scarf II non-central potential and obtain the relativistic energyspectrum and wavefunction via asymptotic iteration method. In Section 5, we present graphically some wavefunctions of the Dirac equation, present several relativistic energy spectra, and discuss some consequences of the results obtained. Finally, the last section presents a brief conclusion.

## 2. Asymptotic Iteration Method (AIM)

Asymptotic Iteration Method (AIM) is used to solve second order differential equation in terms:

$$y_n''(x) - \lambda_0(x)y_n'(x) - s_0(x)y_n(x) = 0 \quad (1)$$

where  $\lambda_0(x) \neq 0$  and  $s_0(x)$  are coefficients of the differential equation and are well defined functions as well as sufficiently differentiable. The one-dimensional Dirac equation can be reduced into hypergeometric or confluent hypergeometric type differential equation by suitable changes of variables, and then changes it into the differential equation which has the form in equation (1). The solution of equation (1) can be obtained by using iteration of  $\lambda_i$  and  $s_i$ ,

$$\begin{aligned} \lambda_i(x) &= \lambda_{i-1}' + \lambda_{i-1}\lambda_0 + s_{i-1} \\ s_i(x) &= s_{i-1}' + s_0\lambda_{i-1} \\ i &= 1, 2, 3, \dots \end{aligned} \quad (2)$$

Eigenvalues can be obtained using equation (3): [24]

$$\lambda_i(x)s_{i-1}(x) - \lambda_{i-1}(x)s_i(x) = 0 = \Delta_i, i = 1, 2, 3 \dots \quad (3)$$

On the other hand, equation (1) can be written in term:

$$y''(x) = 2 \left( \frac{tx^{N+1}}{1-bx^{N+2}} - \frac{c+1}{x} \right) y'(x) - \frac{Wx^N}{1-bx^{N+2}} \quad (4)$$

Equation (4) is AIM-type differential equation which is solved by using equation (5) [25,26]

$$y_n(x) = (-1)^n C'(N+2)^n {}_2F_1(-n, p+n, \sigma, bx^{N+2}) \quad (5)$$

where,

$$(\sigma)_n = \frac{\Gamma(\sigma+n)}{\Gamma(\sigma)} \quad (6)$$

$$\sigma = \frac{2c+N+3}{N+2} \quad (7)$$

$$p = \frac{(2c+1)b+2t}{(N+2)b} \quad (8)$$

Here  $C'$  is normalization constant and  ${}_2F_1$  is hypergeometric function. Equation (5) is eigenfunction of AIM-type differential equation in equation(4). Equation (5) is used to obtain wavefunctions of the Dirac equation. The relativistic energy equation can be formulated by equating eigenvalue equation by using equation (3).

### 3. The Dirac Equation with Pseudospin Symmetry for Modified Pöschl-Teller Potential Combined with Trigonometric Scarf II Non-Central Potential

The Dirac equation for a single particle with mass  $M$  in a scalar potential  $S(r)$  and vector potential  $V(r)$  can be given as ( $\hbar = 1, c = 1$ ),

$$\{\vec{\alpha} \cdot \vec{p} + \beta(M + S(\vec{r}))\}\psi(\vec{r}) = \{E - V(\vec{r})\}\psi(\vec{r}) \quad (9)$$

which  $E$  is relativistic energy of system and  $\vec{p}$  is momentum operator ( $\vec{p} = -i\nabla$ ), while  $\vec{\alpha}$  and  $\beta$  is matrix in term:

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad (10)$$

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (11)$$

which  $I$  is identity matrix  $2 \times 2$  and  $\vec{\sigma}$  is Pauli matrix,

$$\vec{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \vec{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \vec{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (12)$$

In the Pauli-Dirac representation, let[25]

$$\psi_{nk}(\vec{r}) = \begin{pmatrix} f_{nk}(\vec{r}) \\ g_{nk}(\vec{r}) \end{pmatrix} \quad (13)$$

we have,

$$\vec{\sigma} \cdot \vec{p} g_{nk}(\vec{r}) = [E - V(\vec{r}) - M - S(\vec{r})] f_{nk}(\vec{r}) \quad (14a)$$

$$\vec{\sigma} \cdot \vec{p} f_{nk}(\vec{r}) = [E - V(\vec{r}) + M + S(\vec{r})] g_{nk}(\vec{r}) \quad (14b)$$

For pseudospin symmetry ( $S(\vec{r}) = -V(\vec{r})$ ), equation (14) becomes

$$f_{nk}(\vec{r}) = \frac{\vec{\sigma} \cdot \vec{p} g_{nk}(\vec{r})}{[E - M]} \quad (15a)$$

$$\vec{\sigma} \cdot \vec{p} f_{nk}(\vec{r}) = [E - 2V(\vec{r}) + M] g_{nk}(\vec{r}) \quad (15b)$$

Substituting equation (15a) into equation (15b) yields

$$[p^2 + 2(E - M)V(r)]f_{nk}(\vec{r}) = [E^2 - M^2]g_{nk}(\vec{r}) \quad (16)$$

In spherical coordinates, modified Pöschl-Teller potential combined with trigonometric Scarf II non-central potential is defined as

$$V(r, \theta) = \mu^2 \left( \frac{k(k-1)}{\sinh^2 \mu r} - \frac{\eta(\eta+1)}{\cosh^2 \mu r} \right) + \frac{1}{r^2} \left( \frac{b^2 + a(a-1)}{\sin^2 \theta} - \frac{2b(a-\frac{1}{2}) \cos \theta}{\sin^2 \theta} \right) \quad (17)$$

where  $\kappa, \eta, b^2 + a(a-1), b(2a-1)$  are positive real numbers and  $\mu$  shows reach of potential. The Pöschl-Teller potential is used to explain spectrum vibration and interaction of atomic system.[27] The Scarf potential is applied to explain the atomic or molecular force[28]. Putting equation (17) into equation (16) and simplifying the equation, and let

$$f_{nk} = \frac{U(r)}{r} \Theta(\theta) \Phi(\varphi) \quad (18)$$

we have,

$$\left[ \left( \frac{r^2}{U(r)} \frac{d^2 U(r)}{dr^2} + \frac{1}{\Theta(\theta)} \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left( \sin\theta \frac{\partial\Theta(\theta)}{\partial\theta} \right) + \frac{1}{\phi(\varphi)} \frac{1}{\sin^2\theta} \frac{\partial^2 \phi(\varphi)}{\partial\phi^2} \right) + \left( r^2 \mu^2 \left( \frac{k(k-1)}{\sinh^2 \mu r} - \frac{\eta(\eta+1)}{\cosh^2 \mu r} \right) + \left( \frac{b^2+a(a-1)}{\sin^2\theta} - \frac{2b\left(a-\frac{1}{2}\right)\cos\theta}{\sin^2\theta} \right) (E-M) \right) \right] = -[E^2 - M^2]r^2 \quad (19)$$

Separating the variables in equation (19), we obtain

$$\frac{r^2}{U(r)} \frac{d^2 U(r)}{dr^2} + (E-M)r^2 \mu^2 \left( \frac{k(k-1)}{\sinh^2 \mu r} - \frac{\eta(\eta+1)}{\cosh^2 \mu r} \right) + [E^2 - M^2]r^2 - l(l+1) = 0 \quad (20a)$$

$$\frac{\sin\theta}{\Theta(\theta)} \frac{\partial}{\partial\theta} \left( \sin\theta \frac{\partial\Theta(\theta)}{\partial\theta} \right) + (E-M) \sin^2\theta \left( \frac{b^2+a(a-1)}{\sin^2\theta} - \frac{2b\left(a-\frac{1}{2}\right)\cos\theta}{\sin^2\theta} \right) + l(l+1) \sin^2\theta - m^2 = 0 \quad (20b)$$

$$-\frac{1}{\phi(\varphi)} \frac{\partial^2 \phi(\varphi)}{\partial\phi^2} = m^2 \quad (20c)$$

Equation (20c) is well known with its solution

$$\phi_m = \frac{1}{\sqrt{2\pi}} e^{im\varphi}, \quad m = 0, \pm 1, \pm 2, \dots \quad (21)$$

#### 4. Analytical solution of Radial and Angular Parts of the Dirac equation

##### 4.1. Solution of the radial part

The radial part of the Dirac equation in equation (20a) can be solved with approximation,

$$\frac{1}{r^2} \approx \frac{\mu^2}{\sinh^2 \mu r} = \frac{\mu^2}{w-1} \quad (22)$$

Substituting equation (22) into equation (20a) and simplifying the equation by substituting variable  $\cosh^2 \mu r = w$ , we have:

$$w(1-w) \frac{d^2 U(r)}{dw^2} + \left( \frac{1}{2} - w \right) \frac{dU(r)}{dw} + \left\{ (E-M) \left( \frac{k(k-1)}{4(1-w)} + \frac{\eta(\eta+1)}{4w} \right) + \frac{[E^2-M^2]}{4\mu^2} + \frac{l(l+1)}{4(1-w)} \right\} U(r) = 0 \quad (23)$$

by substituting,

$$U(r) = w^\delta (1-w)^\gamma p(w) \quad (24)$$

into equation (23), yields

$$w(1-w)p'' + p' \left( \left( 2\delta + \frac{1}{2} \right) - (2\delta + 2\gamma + 1)w \right) + p \left( \frac{(E^2-M^2)}{4\mu^2} - (\delta + \gamma)^2 \right) = 0 \quad (25)$$

where,

$$\delta = \frac{1}{4} - \frac{1}{2} \sqrt{(M-E)(\eta(\eta+1)) + \frac{1}{4}} \quad (26)$$

$$\gamma = \frac{1}{4} + \frac{1}{2} \sqrt{(M-E)(k(k-1)) - l(l+1) + \frac{1}{4}} \quad (27)$$

Equation (25) can be transform to differential equation type AIM,

$$p'' + p' \left( \frac{\left( (2\delta + \frac{1}{2}) - (2\delta + 2\gamma + 1)w \right)}{w(1-w)} \right) + p \left( \frac{\left( \frac{(E^2 - M^2)}{4\mu^2} - (\delta + \gamma)^2 \right)}{w(1-w)} \right) = 0 \quad (28)$$

From equation (28), we have

$$\lambda_0 = \frac{\left( (2\delta + 2\gamma + 1)w - (2\delta + \frac{1}{2}) \right)}{w(1-w)} \quad (29)$$

$$s_0 = \frac{\left( (\delta + \gamma)^2 - \frac{(E^2 - M^2)}{4\mu^2} \right)}{w(1-w)} \quad (30)$$

To have eigenvalue of equation.(28), further iterations  $\lambda_i$  and  $s_i$ , which  $i$  is iteration numbers. By using equation (3) and using Matlab 2011 software, energy eigenvalue can be obtained, with  $\varepsilon = \frac{(E^2 - M^2)}{4\mu^2}$ ,

$$\Delta_0 \quad \varepsilon_0 = (\delta + \gamma)^2$$

$$\Delta_1 \quad \varepsilon_1 = (2\delta + 2\gamma + 1) + (\delta + \gamma)^2 = (\delta + \gamma + 1)^2$$

$$\Delta_2 \quad \varepsilon_2 = (4\delta + 4\gamma + 4) + (\delta + \gamma)^2 = (\delta + \gamma + 2)^2$$

$$\Delta_3 \quad \varepsilon_3 = (6\delta + 6\gamma + 9) + (\delta + \gamma)^2 = (\delta + \gamma + 3)^2$$

$$\Delta_i \dots \dots \dots$$

Relativistic energy eigenvalue can be obtained by generalized  $\Delta_i$ , which yields

$$\varepsilon = (\delta + \gamma + n_r)^2 = \left( -\frac{1}{2} \sqrt{(M - E)(\eta(\eta + 1) + \frac{1}{4})} + \frac{1}{2} \sqrt{(M - E)(k(k - 1) - l(l + 1) + \frac{1}{4} + n_r + \frac{1}{2})} \right)^2 \quad (31)$$

where  $n_r$  is radial quantum numbers ( $n_r = 0, 1, 2, \dots$ ),  $l$  is orbital quantum numbers which is obtained from angular part solution. And then, radial wavefunction can be obtain by using equations (5-8), we have:

$$c = \frac{2\delta - \frac{3}{2}}{2}, N = -1, t = \frac{2\gamma + \frac{1}{2}}{2}, b = 1, \text{ so, } \sigma = \frac{2c + N + 3}{N + 2} = 2\delta + \frac{1}{2} \text{ and } p = \frac{(2c + 1)b + 2t}{(N + 2)b} = 2\delta + 2\gamma.$$

From equation (5), we have,

$$p(w) = (-1)^{n_r} C_2(1)^{n_r} \left( 2\delta + \frac{1}{2} \right)_{n_r} {}_2F_1 \left( -n_r, 2\delta + 2\gamma + n_r, 2\delta + \frac{1}{2}, w \right) \quad (32)$$

By substituting equation (32) to equation (24), we have radial wavefunction,

$$U(w) = w^\delta (1 - w)^\gamma (-1)^{n_r} C(n_r)(1)^{n_r} {}_2F_1 \left( -n_r, 2\delta + 2\gamma + n_r, 2\delta + \frac{1}{2}, w \right) \quad (33)$$

which  $w = \cosh^2 \mu r$ , so

$$U(r) = (\cosh^2(\mu r))^\delta (-\sinh^2(\mu r))^\gamma (-1)^{n_r} C(n_r) (1)^{n_r} \left(2\delta + \frac{1}{2}\right)_{n_r} {}_2F_1\left(-n_r, 2\delta + 2\gamma + n_r, 2\delta + \frac{1}{2}, \cosh^2(\mu r)\right) \quad (34)$$

where  $C(n_r)$  is radial normalization constant,  ${}_2F_1$  is hypergeometric function and  $\left(2\delta + \frac{1}{2}\right)_{n_r}$  is Pochamer symbol.

#### 4.2. Solution of the angular part

For angular part in equation (20b), can be obtain by using AIM to find orbital quantum number. By multiplying equation (20b) with  $\frac{\Theta(\theta)}{\sin \theta}$ , we have

$$\frac{d}{d\theta} \left( \sin \theta \frac{d\Theta(\theta)}{d\theta} \right) + (E - M) \sin \theta \left( \frac{b^2 + a(a-1)}{\sin^2 \theta} - \frac{2b\left(a - \frac{1}{2}\right) \cos \theta}{\sin^2 \theta} \right) \Theta(\theta) + l(l+1) \sin \theta \Theta(\theta) - m^2 \frac{\Theta(\theta)}{\sin \theta} = 0 \quad (35)$$

Equation (35) must be simplified by using parameter  $\Theta = \frac{H(\theta)}{\sqrt{\sin \theta}}$ ,  $\cos \theta = 1 - 2u$  and by simplifying it, yields,

$$u(1-u) \frac{d^2 H(\theta)}{du^2} + \left(\frac{1}{2} - u\right) \frac{dH(\theta)}{du} + \left\{ (E - M) \left( \frac{b^2 + a(a-1)}{4u(1-u)} - \frac{2b\left(a - \frac{1}{2}\right)(1-2u)}{4u(1-u)} \right) + \frac{m^2 - 1/4}{4u(1-u)} \right\} H(\theta) + \left( l(l+1) + \frac{1}{4} \right) H(\theta) = 0 \quad (36)$$

by using

$$H(\theta) = u^\alpha (1-u)^\beta q(u) \quad (37)$$

and simplifying it, equation (36) can be transform to hypergeometric differential equation:

$$u(1-u)q'' + q' \left( \left(2\alpha + \frac{1}{2}\right) - (2\alpha + 2\beta + 1)u \right) + q \left( \left(l + \frac{1}{2}\right)^2 - (\alpha + \beta)^2 \right) = 0 \quad (38)$$

where,

$$\alpha = \frac{1}{4} + \frac{1}{2} \sqrt{(M - E) \left( \left( \left( a - \frac{1}{2} \right) - b \right)^2 - \frac{1}{4} \right) - m^2 + \frac{1}{2}} \quad (39)$$

$$\beta = \frac{1}{4} + \frac{1}{2} \sqrt{(M - E) \left( \left( \left( a - \frac{1}{2} \right) + b \right)^2 - \frac{1}{4} \right) - m^2 + \frac{1}{2}} \quad (40)$$

$m$  is magnetic quantum number.

Equation (38) is hypergeometric differential equation, so we must transform it to AIM type equation by divide equation (38) with  $u(1-u)$ , yields

$$q'' + q' \frac{\left( \left(2\alpha + \frac{1}{2}\right) - (2\alpha + 2\beta + 1)u \right)}{u(1-u)} + q \frac{\left( \left(l + \frac{1}{2}\right)^2 - (\alpha + \beta)^2 \right)}{u(1-u)} = 0 \quad (41)$$

From equation (41), we have

$$\lambda_0 = \frac{\left( (2\alpha + 2\beta + 1)u \right) - \left( 2\alpha + \frac{1}{2} \right)}{u(1-u)} \quad (42)$$

$$s_0 = \frac{\left( (\alpha + \beta)^2 - \left( l + \frac{1}{2} \right)^2 \right)}{u(1-u)} \quad (43)$$

To have eigenvalue of this equation, further iterations  $\lambda_i$  and  $s_i$ , which  $i$  is iteration numbers. By using equation (3) and Matlab software, eigenvalue can be obtained,

$$\Delta_0 = s_0 \lambda_1 - s_1 \lambda_0 = 0, \text{ yield } (\alpha + \beta)^2 = \left( l + \frac{1}{2} \right)^2$$

$$\Delta_1 = s_1\lambda_2 - s_2\lambda_1 = 0, \text{ yield } (\alpha + \beta + 1)^2 = \left(l + \frac{1}{2}\right)^2$$

$$\Delta_2 = s_2\lambda_3 - s_3\lambda_2 = 0, \text{ yield } (\alpha + \beta + 2)^2 = \left(l + \frac{1}{2}\right)^2$$

$$\Delta_i \dots \dots \dots \dots \dots$$

Eigenvalue can be obtained by generalized  $\Delta_i$ , which yields

$$l = (\alpha + \beta + n_l - \frac{1}{2}) \tag{44}$$

Where  $l$  is orbital quantum number and  $n_l$  is angular quantum number.

And then, angular wavefunction can be obtained by using equations (5-8), we have,

$$c = \frac{2\alpha - \frac{3}{2}}{2}, N = -1, t = \frac{2\beta + \frac{1}{2}}{2}, b = 1, \text{ so, } \sigma = \frac{2c + N + 3}{N + 2} = 2\alpha + \frac{1}{2}, p = \frac{(2c + 1)b + 2t}{(N + 2)b} = 2\alpha + 2\beta$$

From equation (5), we have

$$q(u) = (-1)^{n_l} C_2(1)^{n_l} \left(2\alpha + \frac{1}{2}\right)_{n_l} {}_2F_1\left(-n_l, 2\alpha + 2\beta + n_l, 2\alpha + \frac{1}{2}, u\right) \tag{45}$$

By substituting equation (45) into equation (37), yields

$$H(u) = u^\alpha(1 - u)^\beta (-1)^{n_l} C_2(1)^{n_l} {}_2F_1\left(-n_l, 2\alpha + 2\beta + n_l, 2\alpha + \frac{1}{2}, u\right)$$

which  $u = -\frac{1}{2}(\cos \theta - l)$ , so the angular part wavefunction can be obtained, as follows

$$H(\theta) = \left(\frac{1}{2} - \frac{1}{2}\cos \theta\right)^\alpha \left(\frac{1}{2} + \frac{1}{2}\cos \theta\right)^\beta (-1)^{n_l} C_{n_l}(1)^{n_l} \left(2\alpha + \frac{1}{2}\right)_{n_l} {}_2F_1\left(-n_l, 2\alpha + 2\beta + n_l, 2\alpha + \frac{1}{2}, \left(\frac{1}{2} - \frac{1}{2}\cos \theta\right)\right) \tag{46}$$

$C_{n_l}$  is angular normalization constant.

### 5. Result And Discussion

In this section, we discuss several results which were obtained in the previous section. From relativistic energy equation in Eq.(31) and orbital quantum number equation in Eq.(44), and by using Matlab software we have numeric solution of relativistic energy are listed in Table 1 with parameters  $\kappa = 2, \eta = 1.5, a = 0.35, b = 0.65$  and  $M = 5 \text{ fm}^{-1}$ , the negative value of relativistic energy is taken due to the pseudospin symmetric limit[19]. By inspecting Table 1, show that increase of value  $\mu$  and  $n_r$  in the same quantum state causes decrease energy eigenvalue. The parameter  $\mu$  has a dimension inverse of distance in space that describes the reach of Pöschl-Teller potential. If  $\mu$  is enlarged, physically means that the potential range is smaller in a space, it causes decrease energy.

**Table 1.** Relativistic energy corresponding to several sates of a particle under the influence of modified Pöschl-Teller potential and trigonometric Scarf II potential.

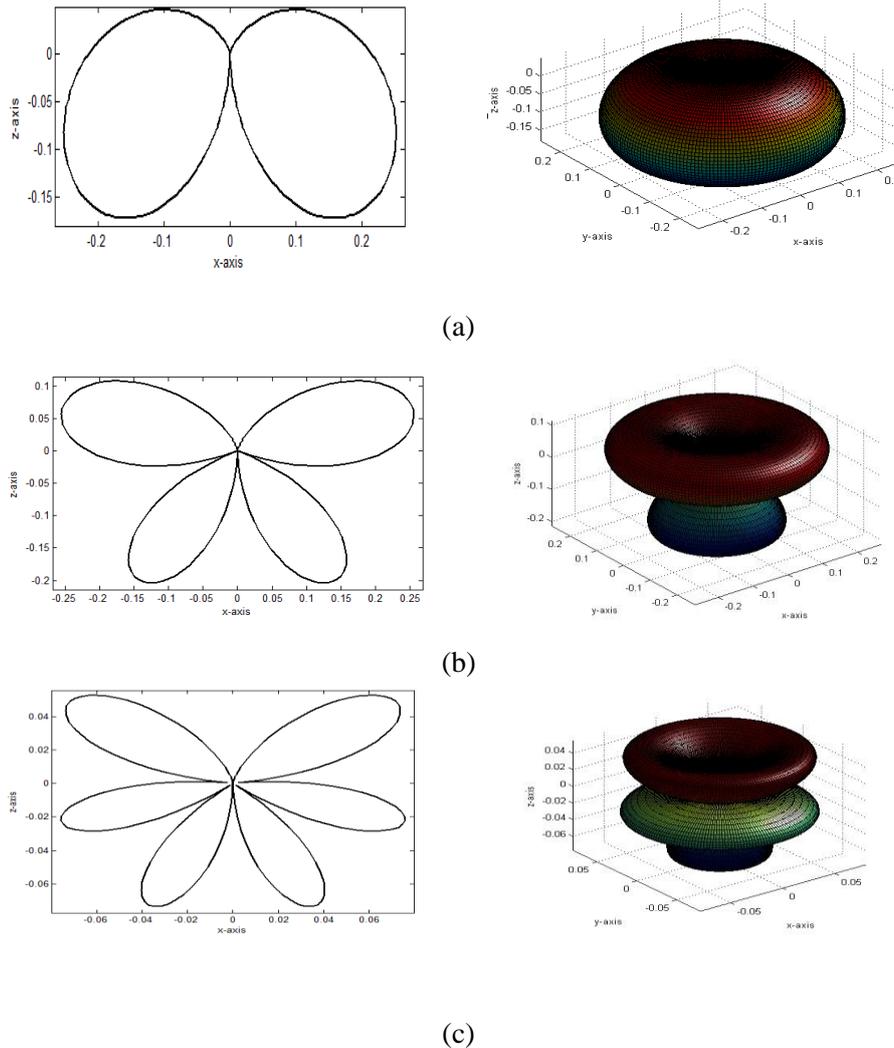
$n_r$	$n_l$	$m$	$E_{n_r n_l m} (\text{fm}^{-1})$		
			$\mu = 0,3 \text{ fm}^{-1}$	$\mu = 0,6 \text{ fm}^{-1}$	$\mu = 0,9 \text{ fm}^{-1}$
1	0	0	-5.0084	-5.0332	-5.0739
1	1	0	-5.0010	-5.0039	-5.0088
1	2	0	-5.0065	-5.0257	-5.0575
2	0	0	-5.0781	-5.3020	-5.6450
2	1	0	-5.0486	-5.1902	-5.4142
2	2	0	-5.0119	-5.0477	-5.1073

By varying parameter which corresponding value  $\delta$  and  $\gamma$ , some of the radial wavefunctions are listed in Table 2. Radial wavefunctions for particle under the influence of modified Pöschl-Teller potential and Scarf II potential are affected by potential constants  $\eta$ ,  $a$ ,  $b$  and by  $\mu$ . The parameter  $\mu$  has a dimension inverse of distance in space that describes the reach of Pöschl-Teller potential. If  $\mu$  is enlarged, physically means that the potential reach is smaller in a space. By inspecting Table 2 due to the increase in the value of  $\mu$  causes particles move further away from the nucleus and show that change in radial wavefunctions are affected of potential constants  $\kappa$ ,  $\eta$ ,  $a$  and  $b$ .

**Table 2.** Radial wavefunctions with  $n_l = 2, m = 0, M = 5$  for particle under the influence of modified Pöschl-Teller potential and trigonometric Scarf II potential variation  $\kappa, \eta, a$  and  $b$ .

$n_r$	$\mu(\text{fm}^{-1})$	$\kappa$	$\eta$	$a$	$b$	$E$	$l$	$U(r)C_{nr}$
1	0.9	2	1.5	0.35	0.65	-5.0575	3.4050	$5.1616(\cosh^2 \mu r)^{-2.8308}(-\sinh^2 \mu r)^{1.4082}(1 - 0.36 \cosh^2 \mu r)$
2	0.9	2	1.5	0.35	0.65	-5.1073	3.4073	$21.62(\cosh^2 \mu r)^{-2.8384}(-\sinh^2 \mu r)^{1.4170}(1 - 0.33 \cosh^2 \mu r - 0.00613(\cosh^2 \mu r)^2)$
3	0.9	2	1.5	0.35	0.65	-5.7618	3.4372	$79.21(\cosh^2 \mu r)^{-2.9361}(-\sinh^2 \mu r)^{1.5269}(1 + 0.101 \cosh^2 \mu r + 0.027(\cosh^2 \mu r)^2 + 0.00591(\cosh^2 \mu r)^3)$
2	0.3	2	1.5	0.35	0.65	-5.0119	3.4029	$21.35(\cosh^2 \mu r)^{-2.8239}(-\sinh^2 \mu r)^{1.4001}(1 - 0.329 \cosh^2 \mu r - 0.00605(\cosh^2 \mu r)^2)$
2	0.6	2	1.5	0.35	0.65	-5.0477	3.4046	$21.4526(\cosh^2 \mu r)^{-2.8293}(-\sinh^2 \mu r)^{1.465}(1 - 0.3278 \cosh^2 \mu r - 0.0061(\cosh^2 \mu r)^2)$
1	0.9	2.5	2	0.35	0.65	-5	3.4024	$6.762(\cosh^2 \mu r)^{-3.6310}(-\sinh^2 \mu r)^{2.6360}(1 - 0.146 \cosh^2 \mu r)$
1	0.9	3	2.5	0.35	0.65	-5.0105	3.4029	$8.3724(\cosh^2 \mu r)^{-4.4362}(-\sinh^2 \mu r)^{3.6164}(1 - 0.0764 \cosh^2 \mu r)$
1	0.9	2	1.5	0.25	0.75	-5.2033	3.7811	$5.2058(\cosh^2 \mu r)^{-2.8529}(-\sinh^2 \mu r)^{1.0528}(1 - 0.50 \cosh^2 \mu r)$
1	0.9	2	1.5	0.5	0.5	-5.0002	2.7071	$5.1442(\cosh^2 \mu r)^{-2.8221}(-\sinh^2 \mu r)^{1.8480}(1 - 0.18 \cosh^2 \mu r)$

For the angular solution of the wavefunction in Eq.(46), we compare spinor wavefunctions for variation  $n_l$ . From Eq.(46) with  $n_r = 2, m = 0, \mu = 0.9 \text{ fm}^{-1}, a = 0.25, b = 0.75, k = 2, \eta = 1.5$ , we have angular wave function  $H(\theta)$  is shown in figure 1a for  $n_l = 0$ , figure 1b for  $n_l = 1$  and figure 1c for  $n_l = 2$ . By inspecting figure 1, the variation of  $n_l$ , influences the shape of the orbital probability distribution in space. In figure 1 indicates that the increase  $n_l$  cause to rise in wave that are formed.



**Figure 1.** Two-dimensional and three dimensional angular wavefunctions with variations  $n_l$  (a)  $n_l = 0$ , (b)  $n_l = 1$ , (c)  $n_l = 2$ .

## 6. Conclusion

In this paper, we study the Dirac equation for particle spin-1/2 in the modified Pöschl–Teller potential combined with the trigonometric Scarf II non-central potential under the condition pseudospin symmetry. The radial part of the wavefunction is obtained approximately from Eq.(34) and the angular part in Eq.(46). The results show that the disturbance of modified Pöschl–Teller Potential and trigonometric Scarf II potential change in the wave function of the radial part and the angular part. Relativistic energy equation can be obtained via AIM in Eq.(31) and equation of orbital quantum number  $l$  in Eq.(44), where both are interrelated between quantum numbers. Relativistic energy also is

solved numerically using Matlab software, where the increase in the radial quantum number  $n_r$  causes a decrease in the energy spectrum. The negative value of energy is taken due to the pseudospin symmetry limit.

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