

# Dynamics of massive interacting scalar fields in Bekenstein-Sandvik-Barrow-Magueijo theory

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**Abstract.** We examine the dynamical characteristics of massive interacting scalar field introduced in the Bekenstein-Sandvik-Barrow-Magueijo (BSBM) theory. Based on the classification of matter- and radiation-dominated cosmological eras, we consider three possible physical scenarios that allowed by the theory. We present the analysis of its dynamics in terms of phase space of the corresponding massive scalar field. The results demonstrate distinct characteristics between those eras. Characteristic of the related fine structure constant is also discussed.

## 1. Introduction

The problem of fine-structure constant variation has been mainly of interest due to recent observation of accelerating universe. There are many high-redshift observations of quasar absorption spectra [1, 2, 3, 4] showing that this constant might be varying in space and time coordinates. For example, the problems of time variation of this constant, was first theoretically considered by Jordan [5], Teller [6], Gamow [7], Dicke [8], and Stanyukovich [9]. Another attempt to explain such phenomenon has also been conducted recently in Bekenstein-Sandvik-Barrow-Magueijo (BSBM) theory, by assuming that the variation of the corresponding constant is due to the present of scalar field coupled to electromagnetic field kinetic term in the associated action [10-13]. The recent experiments on Large Hadron Collider for detecting the Higgs particles [14] and BICEP2 experiment for detecting the primordial gravitational waves [15] also suggested that the scalar fields may play a significant role as one of nature's building blocks and show significant effect on cosmological scale, at least in the early universe. Nevertheless, effort to detect the presence of scalar particles is still a challenge due to their large masses and weak interactions to other fields.

It has been proposed recently in ref. [16] that the BSBM theory integrated with the Brans-Dicke scalar-tensor theory of gravity can be a promising candidate to explain the corresponding fine-structure constant variation. The presence of scalar Brans-Dicke scalar field can be used to explain the present observational fine-structure variation as redshift function. Based on this fact, we propose another possible scenario to explain the aforementioned variation by considering the inclusion of massive interacting scalar field to the BSBM theory without considering the Brans-Dicke scalar field.

In this report, we discuss our preliminary results of the corresponding scenario. Conducted within the dynamical system approach, we demonstrate the variation of the fine-structure constant with respect to the



dynamics of scalar field. We found that there are three critical points which represent the local minima of the potential function. We consider three different scenarios in terms of cosmological eras of matter and radiation.

## 2. BSBM theory

### 2.1. Action

The action for the BSBM theory is given as follows:

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + L_m - \frac{\omega}{2} \partial_\mu \psi \partial^\mu \psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} e^{-2\psi} - V(\psi) \right] \quad (1)$$

The symbol  $L_m$  denotes the lagrangian for matter, while  $R$  represents the Ricci scalar and  $\psi$  is the scalar field with  $\omega > 0$  is its coupling constant, while  $f^{\mu\nu} = F^{\mu\nu} \exp(\psi)$  with  $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$  is electromagnetic tensor. We choose the potential function  $V(\psi)$  to be in the following form:

$$V(\psi) = \frac{1}{2} m^2 \psi^2 + \frac{1}{4} \lambda \psi^4 \quad (2)$$

which describes interacting massive scalar field, with  $m^2 < 0$  is the mass parameter and  $\lambda$  is a real positive parameter denoting the interaction strength. Based on this theory it is assumed that the unit of electric charge evolves according to  $e = e_0 \exp(\psi)$ , such that the corresponding fine-structure constant variation is given by relation:

$$\alpha = \alpha_0 \exp(2\psi) \quad (3)$$

where the subscript “0” denote the present time values. Note that in all calculations we used the unit of  $c = 1$ .

### 2.2. Equation of motion

By varying with respect to metric we found the following Einstein field equation from the BSBM action given by Eq. (1):

$$G_{\mu\nu} = 8\pi G \left[ T_{\mu\nu}^m + T_{\mu\nu}^\psi + T_{\mu\nu}^{em} e^{-2\psi} \right] \quad (4)$$

where  $G_{\mu\nu}$  is the Einstein tensor, and  $T_{\mu\nu}^m$  and  $T_{\mu\nu}^{em}$  are the stress-energy tensor for matter and electromagnetic field given by the following expression under the perfect fluid assumption:

$$T_{\mu\nu}^m = (\rho_m + p_m) u_\mu u_\nu - p_m g_{\mu\nu} \quad (5)$$

$$T_{\mu\nu}^{em} = (|\zeta| \rho_m + \rho_r + p_m + p_r) u_\mu u_\nu \exp(-2\psi) - (p_m + p_r) g_{\mu\nu} e^{-2\psi} \quad (6)$$

Here,  $p_m$  and  $p_r$  are the matter and radiation pressure and  $\rho_m$  and  $\rho_r$  are the corresponding matter- and radiation- densities. The symbol  $u_\mu$  is a four velocity, while  $|\zeta| \leq 1$  denote the ratio of electromagnetic lagrangian with matter-density. The  $T_{\mu\nu}^\psi$  is the stress-energy tensor for scalar which is given in the following expression:

$$T_{\mu\nu}^\psi = \omega \left( \partial_\mu \psi \partial_\nu \psi - \frac{1}{2} g_{\mu\nu} g^{\sigma\rho} \partial_\sigma \psi \partial_\rho \psi \right) + \frac{1}{2} g_{\mu\nu} \left( \frac{1}{2} m^2 \psi^2 + \frac{1}{4} \lambda \psi^4 \right) \quad (7)$$

From the Eq. (4), one can derive the following Friedmann Equation for Friedmann-Lemaitre-Robertson-Walker (FLRW) metric:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8G\pi}{3} \left[ \rho_m (1 + |\zeta| e^{-2\psi}) + \rho_r e^{-2\psi} + \frac{\omega}{2} \dot{\psi}^2 - \frac{1}{2} \left( \frac{1}{2} m^2 \psi^2 + \frac{1}{4} \lambda \psi^4 \right) \right] - \frac{\kappa}{R^2} + \frac{\Lambda}{3} \quad (8)$$

where  $\kappa$  and  $\Lambda > 0$  are spacetime curvature and cosmological constant, respectively, while  $R$  is the scale factor. The dots in  $R$  and  $\psi$  indicate the time derivative. The conservation of stress-energy tensor yields the following matter and radiation evolution energy:

$$\dot{\rho}_m + 3H\rho_m = 0 \quad (9)$$

$$\dot{\rho}_r + 4H\rho_r = 2\dot{\psi}\rho_r \quad (10)$$

where  $H = \dot{R}/R$  represents the Hubble constant. In the mean time, by varying the BSBM action with respect to scalar field yields the dynamical equation for the scalar field as follows:

$$\ddot{\psi} + 3H\dot{\psi} = \frac{2}{\omega} |\zeta| \rho_m e^{-2\psi} + \frac{1}{\omega} (m^2 \psi + \lambda \psi^3) \quad (11)$$

To this end, it is clear that the dynamics of BSBM theory can be investigated using Eqs. (8)–(11), which is discussed in the following section.

### 3. Dynamics of massive scalar field and fine-structure constant

We focus our ensuing discussion to the case of flat spacetime i.e.  $\kappa = 0$ . In this case, the Hubble constant  $H$  given by Eq. (8) can be considered independently, such that we can restrict our attention only to Eqs. (9)–(11). By defining new functions:  $\psi_1 = \psi$  and  $\psi_2 = \dot{\psi}$ , the Eq. (11) can then be written into the following set of ordinary differential equations (ODE):

$$\dot{\psi}_1 = \psi_2 \quad (12)$$

$$\dot{\psi}_2 = -3H\psi_2 + \frac{2}{\omega} |\zeta| \rho_m e^{-2\psi_1} + \frac{1}{\omega} (m^2 \psi_1 + \lambda \psi_1^3) \quad (13)$$

Together with Eq. (9) and Eq. (10) they form a set of ODE of  $(\psi_1, \psi_2, \rho_m, \rho_r)$  that can be analyzed using the dynamical system approach. The corresponding critical points  $(\psi_1^0, \psi_2^0, \rho_m^0, \rho_r^0)$  can be found by setting up  $(\dot{\psi}_1 = 0, \dot{\psi}_2 = 0, \dot{\rho}_m = 0, \dot{\rho}_r = 0)$  condition simultaneously for the given set of ODE and yields the following three physically allowed critical points:

$$(0, 0, 0, 0) \quad (14)$$

$$\left( \sqrt{-m^2/\lambda}, 0, 0, 0 \right) \quad (15)$$

$$\left( -\sqrt{-m^2/\lambda}, 0, 0, 0 \right) \quad (16)$$

Next, by conducting linearization on the associated ODE around the first critical point given by Eq. (14), we found the following four eigenvalues:

$$\Omega_1^0 = \frac{4\sqrt{3}\Lambda}{3} \quad (17)$$

$$\Omega_2^0 = \sqrt{3\Lambda} \quad (18)$$

$$\Omega_{3,4}^0 = \frac{-\omega\sqrt{3\Lambda} \pm \sqrt{3\Lambda\omega^2 + 4m^2\omega}}{2\omega} \quad (19)$$

While for the second and third critical points, the eigenvalues are given by:

$$\Omega_1 = \frac{4}{3}\sqrt{3C} \quad (20)$$

$$\Omega_2 = \sqrt{3C} \quad (21)$$

$$\Omega_{3,4}^\pm = \frac{-\omega\sqrt{3C} \pm \sqrt{3C + 4m^2\omega}}{2\omega} \quad (22)$$

where  $C = (G\pi m^4 + \Lambda\lambda)/\lambda > 0$ .

To analyze the dynamics of the corresponding massive scalar field, we consider specific values of the related parameters as follows:  $\zeta = 1$ ,  $\omega = 1$ ,  $\Lambda = 0.01$ ,  $\lambda = 0.25$ ,  $m^2 = -1$  and  $8G\pi/3 = 1$ . For these values, we found that the eigenvalues  $\Omega_{1,2}^0$  are real and positive, while  $\Omega_{3,4}^0$  are complex with negative real parts since the discriminant in Eq. (19) i.e.  $3\Lambda\omega^2 + 4m^2\omega < 0$ . Thus, it is clear that  $\Omega_{1,2}^0$  are related to unstable manifold, whereas  $\Omega_{3,4}^0$  are related to stable manifold around the corresponding critical point. On the other hand, we found that the eigenvalues  $\Omega_{1,2}^\pm$  are real positive, while  $\Omega_{3,4}^\pm$  are real negative, indicating the existence of unstable and stable manifolds, respectively, around the associated critical points of saddle type. It should be emphasized that the variable “time” used in this discussion should not be strictly considered as the cosmological scale time, since we are only interested in the dynamical system analysis of the associated ODE system.

Given in Fig. 1 is the trajectories on  $(\psi_1, \psi_2)$  phase plane at which  $(\rho_m, \rho_r) = (0, 0)$ . These trajectories are calculated by solving the corresponding ODE numerically. It is shown that the origin is a stable focus, where all trajectories around are heading spirally toward the critical point denoted by solid black circle, whereas the other critical points, represented by solid red circles, are unstable, such that all trajectories are going out from them. Therefore, it can be interpreted that the critical point at the origin is related to the future time.

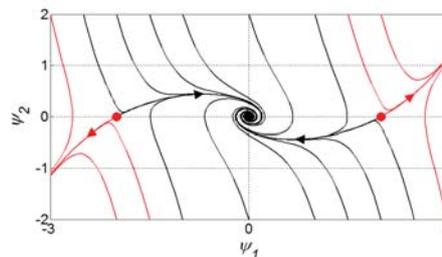
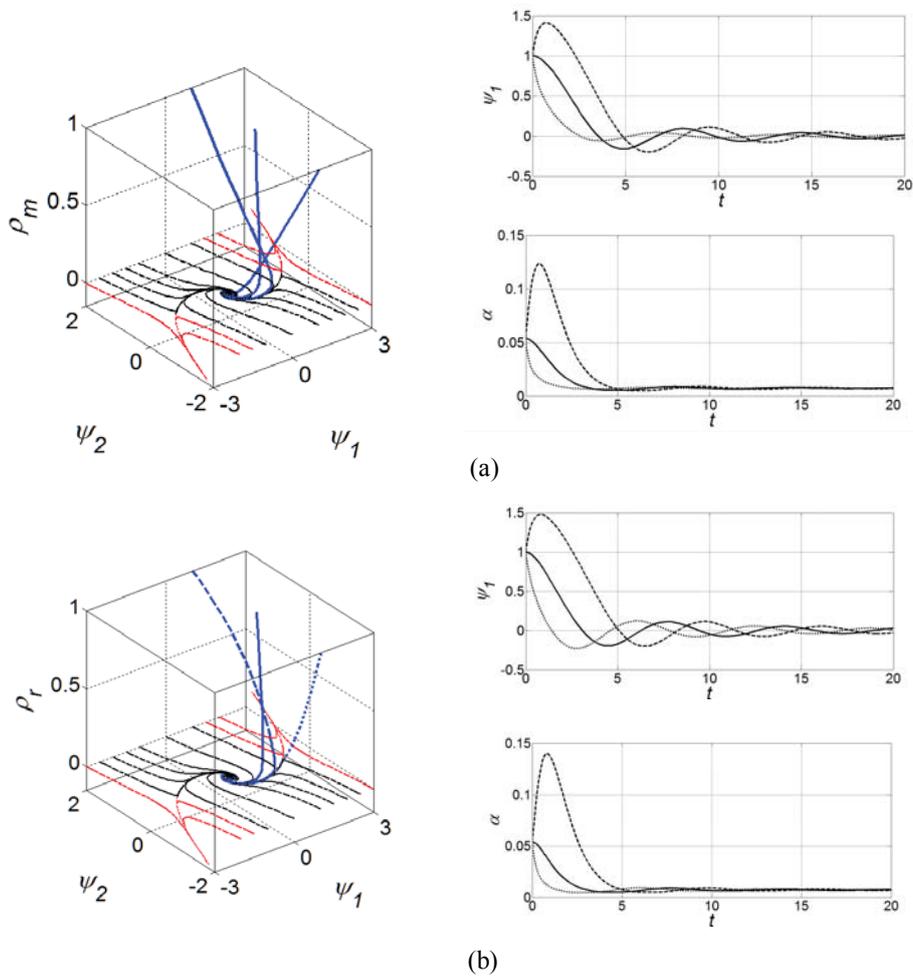


Fig. 1. Scalar field phase plane at  $\rho_m = \rho_r = 0$ .

Depicted in Fig. 2 are the trajectories in three-dimensional (3-D) phase space  $(\psi_1, \psi_2, \rho_m$  or  $\rho_r)$  along with the corresponding evolution of scalar field and fine-structure constant given by Eq. (3) as function of time. We classify the dynamics of scalar field into three different eras namely: (i) matter-dominated era (ii)

radiation-dominated era, and (iii) balance era where the ratio between matter- and radiation-density  $\approx 1$ . For each era, we consider three different combinations of  $(\psi_1, \psi_2)$  namely:  $(1, 0)$  and  $(1, \pm 2)$ , where the associated trajectories in the Figs. 2(a) and 2(b) are given by solid, dashed and dotted curves, respectively. For the balance era given by Fig. 2(c), the blue and red curves on the 3-D phase space are correspond to  $\rho_m$  and  $\rho_r$ , respectively. For the sake of clarity, we include the phase plane on Fig. 1 to the 3-D phase space on Fig. 2. Here, we only consider the combinations that lead to trajectories in the stable manifolds, since the unstable one may lead to the unphysical behavior of matter- and/or radiation-density evolution namely it can go to infinity.



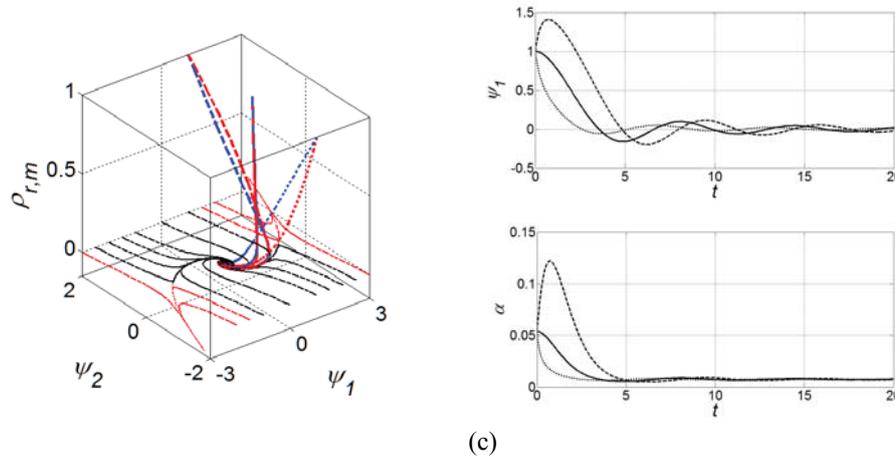


Fig. 2. (left panel) Trajectories in 3-D phase space, (right panel) evolution of scalar field (top) and fine-structure constant (bottom) for (a) matter-dominated era with  $\rho_m(0)=1$  and  $\rho_r(0)\approx 0$  (b) radiation-dominated era with  $\rho_r(0)=1$  and  $\rho_m\approx 0$ , (c) balance era with  $\rho_m(0)=\rho_r(0)\approx 1$ . Here  $\alpha_0 = 1/137$ .

Obviously, the initial slope of  $\psi_1$  evolution depends on  $\psi_2$ . It is demonstrated in Fig. 2, that in a short period from initial time of particular era, different value of  $\psi_2$  leads to different evolution pattern. At longer time, however, the evolution tends to form similar modulated patterns for all eras. It is also interesting to note from the Fig. 2(b) and 2(c), that the balance era exhibits a very similar pattern for both field and fine-structure constant evolutions with radiation-dominated era. This phenomenon suggests that for the considered case, the radiation dominates the evolution of scalar field and the fine-structure constant as well.

Back to eigenvalues given by Eqs. (20)–(22), it is obvious that another set of parameter values may lead to a different stability conditions. By varying the value of  $m^2 < 0$  for instance, it is possible to get the condition of  $3C + 4m^2\omega < 0$ . A transcritical bifurcation from the previous condition can occur when  $3\Lambda\omega^2 + 4m^2\omega > 0$  in Eq. (19) satisfied. Because the critical point at the origin will change stability namely from a critical point with two real and two complex eigenvalues into a critical point with four real eigenvalues, and vice versa, the other two critical points will have two real and two complex eigenvalues from the previous four real eigenvalues. Obviously, this bifurcation yields bit different characteristics of scalar field and fine-structure constant variation at least at the future time, the scalar field is non-zero.

Finally, it should be emphasized that the propose scenario have to be further examined by considering the real data from cosmological observation, especially that of related to the variation of the fine-structure constant. This subject still under consideration and will be reported elsewhere.

#### 4. BSBM theory

We have discussed the characteristics of interacting massive scalar field in BSBM theory using dynamical system analysis of three different cosmological eras namely the matter-dominated, radiation dominated and balance eras. Despite the evolution of scalar field and the corresponding fine-structure variation are

not so different for particular cosmological era, the results, however, show that our proposal potentially can be considered as an alternative scenario to explain the dynamical variation of fine-structure constant. Further test should be conducted using realistic data of cosmological observation.

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