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Asymptotically AdS Solutions of Five Dimensional Gravity-Dilaton Theory

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Abstract. We search for Asymptotically AdS solutions of the background metric in which dilaton back reacts to gravity in five-dimensional gravity-dilaton theory. The five-dimensional gravity-dilaton theory generally appears in the context of the low energy effective action of closed string theory in the Einstein frame. In particular, we consider dilaton which are minimally coupled to gravity in which the potential for dilation is taken to be simple and contain only one exponential term parametrized by a constant a. We solve analytically and show for a simple constant dilaton potential it appears there are no black hole solutions if we turn on the dilaton contribution. On the other hand, the exponential dilaton potential has black hole solutions but they are not in general Asymptotically AdS. We argue that there are some possible Asymptotically AdS black hole solutions in the range of $|a| < \frac{4}{\sqrt{6}}$.

1. Introduction

It is believed that QGP(Quark Gluon Plasma) produced in RHIC(Relativistic Heavy Ion Collision) at Brookhaven and also the current running experiments in LHC(Large Hadron Collider) are in the region where quarks and gluons loose their identity but still they interact strongly each other. In this region, we can not trust perturbative calculation and one start looking at non-perturbative method such as lattice QCD to compute some properties in QGP. The more elegant way to solve this problem is using AdS/QCD correspondence where we can do a perturbative calculation in string theory to get some correlation function of QGP in strong coupling [7, 8].

AdS/QCD model has been one of many models in an attempt to describe QCD as low effective theory of string theory [1, 2, 5, 11, 12]. The correspondence between field theory, in particular QCD, and string theory under Anti-de Sitter background is not trivial and it comes as a relation of the coupling constants [6]. For years, people have known that there is a correspondence between strongly couple gauge theory and weakly coupled string theory while the vice versa is still not clear.

The dilaton has been one of important ingredients in attempts of building the AdS/QCD model [4, 10]. In this article, we would like to consider the effective five-dimensional gravity-dilaton theory, with negative cosmological constant, descended from the string theory. Our main focus is the dilaton potential that would take an exponential form parametrized by a constant a. At first, we will discuss the solutions in which the dilaton potential is a constant which contributes as cosmological constant term. Unfortunately, this potential does not have a black hole solution that would corresponds to the finite temperature field theory as shown by QGP produced at RHIC and LHC [9]. Then, we generalize the dilaton potential to an exponential form that still inherits the cosmological constant term of the previous case. We will show that there exist some black hole solutions in this case.

2. From String to Einstein Frame

Consider the following action in the string frame[1]

$$S \sim \int d^D x\sqrt{-g}e^{-2\Phi}\left[R + 4(\partial\Phi)^2 - \frac{1}{4}F_{\mu\nu}^2\right] + \cdots. \tag{1}$$

We can go from the string frame to the Einstein frame using a Weyl transformation $g_{\mu\nu}^E = e^{-\frac{4}{D-2}\Phi}g_{\mu\nu}$, such that, in the Einstein frame, the action (1) becomes

$$S_E = \int d^D x\sqrt{-g_E}\left(R_E - \frac{4}{D-2}(\partial\Phi)^2 - \frac{1}{4}e^{-\frac{4}{D-2}\Phi}F_{\mu\nu}^2\right) + \cdots. \tag{2}$$

Assume that we can find all classical solution of the fields except for gravity and dilaton. Plugging back those solutions into the action, we can write the effective action in terms of gravity and dilaton as[2]

$$S_E = \int d^D x\sqrt{-g_E}\left(R_E - \frac{1}{2}(\partial\Phi)^2 + V(\Phi)\right). \tag{3}$$

In our case, we will just consider the potential which gives asymptotic AdS solution to the metric. Therefore we take the potential to be of the form $V = \frac{12}{b^2} + \mathcal{O}(\Phi)$, where b is meant to be the AdS radius.

3. BPS-like Forms

The method that we are going to use here is based on the one used by Miller et al [3]. It is basically using BPS-like equations which are in first order differential equations to solve equations of motion which normally take the form of second order differential equations. Despite the fact that BPS-like equations are simpler to solve, it is not an easy job to get the BPS-like form from the action. One has to able to write all terms in the action as a sum of squares plus a total derivative then one can trust their BPS-like solutions to satisfy the equations of motion.

Now, we are going to show how do these BPS-like forms satisfy the equations of motion. Lets, consider a theory in D-dimensions with N fields ϕ_i and assume that we can write its action into BPS-like forms plus a total derivative terms,

$$S = \int dx^D \sum_{i=1}^{N} A_i[\vec{\phi}, \partial\vec{\phi}]^2 + \int dx^{D-1}B[\vec{\phi}, \partial\vec{\phi}], \tag{4}$$

where A_i are the BPS-like forms, B is the total derivative terms, and $\vec{\phi} \equiv (\phi_1, \ldots, \phi_N)$. Neglecting the boundary terms, we can find the equations of motions by taking a variation of the action above to zero,

$$\delta S = \int dx^D \sum_{i=1}^{N} 2A_i[\vec{\phi}, \partial\vec{\phi}]\delta A_i[\vec{\phi}, \partial\vec{\phi}] = 0. \tag{5}$$

Here, we can see the simplest solution of these BPS-like forms that satisfy the equations of motion is when $\vec{A}[\vec{\phi}, \partial\vec{\phi}] = 0$ and these are our BPS-like equations that we are going to use in many of our calculations.

[1] Here, we only consider fields (gravity, dilaton, and $U(1)$ gauge field) that are needed in our calculation.
[2] We have scaled the dilaton $\Phi \to \sqrt{\frac{D-2}{8}}\Phi$ and assumed that the potential term is just a function of dilaton.

4. Constant Dilaton Potential

As a first attempt, we take the dilaton potential to be a constant in order to make sure that near the boundary (infinity) the resulting metric is Asymptotically AdS. Let us consider the action that is given by the effective five-dimensional action of gravity-dilaton, with cosmological constant term,

$$S = \int d^5x\sqrt{-g}\left(R - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{12}{b^2}\right). \tag{6}$$

We write the following general ansatz for the metric and dilaton in Poincare patch coordinates of AdS that preserve less symmetry(three-dimensional rotation) than four-dimensional Lorentz invariance,

$$\begin{aligned} ds^2 &= e^{2n_4}\left[-dt^2 + e^{2(n_3-(3/2)n_4)}\left(dx_3^2 + e^{2n_1}du^2\right)\right] \\ \Phi &= \Phi(u), \end{aligned} \tag{7}$$

with u is the fifth dimension. Substituting these ansatz back to the action (6), we obtain an effective $(1+1)$-dimensional action, after factoring out an overall three-dimensional volume,

$$S_{eff} = \int dt\ \mathcal{L}_{eff}, \tag{8}$$

with the effective Lagrangian is then given by

$$\mathcal{L}_{eff} = \int du\ e^{2n_3-n_1}\left[6(n_3')^2 - (3/2)(n_4')^2 + \frac{12}{b^2}e^{2n_1+2n_3-n_4} - \frac{1}{2}(\Phi')^2\right] + \int du\frac{d}{du}\left(e^{2n_3-n_1}(n_4' - 6n_3')\right). \tag{9}$$

Note that there is no n_1'-quadratic terms in the Lagrangian, so we can eliminate it by a field redefinition, or equivalently taking a coordinate transformation of u. In order to remove the overall exponential factor in the first integral of the right hand side effective action (9), we choose $n_1 = 2n_3$. It turns out to be much better if we would rather substitute $n_3 = \frac{1}{2}n_1$ and get a nicer Lagrangian,

$$\mathcal{L}_{eff} = \int du\left[\frac{3}{2}(n_1')^2 - \frac{3}{2}(n_4')^2 + \frac{12}{b^2}e^{3n_1-n_4} - \frac{1}{2}(\Phi')^2\right] + boundary. \tag{10}$$

Here the degree of freedom is n_1 instead of n_3. From now on, we will neglect the boundary terms.

4.1. Non-Black Hole Solutions

For non-black hole solutions, we make an identification $n_1 = 3n_4$ to preserve four-dimensional Lorentz invariance and so the metric becomes

$$ds^2 = e^{2n_4}(-dt^2 + dx_3^2) + e^{8n_4}du^2. \tag{11}$$

It is clear that this metric, parametrized by one field n_4, does not have characteristic of black hole solutions.

4.1.1. Trivial Dilaton Solution For trivial dilaton solution, we may write the effective Lagrangian in BPS-like forms as follows

$$\mathcal{L}_{eff} = \int du\left[12\left(n_4' + \frac{1}{b}e^{4n_4}\right)^2 - \frac{1}{2}\left(\Phi'\right)^2\right], \tag{12}$$

with BPS-like equations are given by

$$n_4' = -\frac{1}{b}e^{4n_4}, \qquad \Phi' = 0, \tag{13}$$

where the solution for dilaton is just a constant. In order to get an AdS solution, we make another identification such that $e^{2n_4} = \frac{b^2}{r^2}$ and the first BPS-like equation of (13) becomes

$$\frac{dr}{du} = -\frac{1}{b\, n_4'}e^{4n_4} = \frac{b^3}{r^3}. \tag{14}$$

So, the solution for the metric is just the usual AdS metric

$$ds^2 = \frac{b^2}{r^2}\left(-dt^2 + dx_3^2 + dr^2\right). \tag{15}$$

4.1.2. Non-trivial Dilaton Solution The non-trivial dilaton solution can be obtained by setting the solution of dilaton as a linear function of u. This is true because from the effective Lagrangian (10) we can derive the equation of motion of dilaton that is a linear function of coordinate u such that the first derivative of dilaton is a constant C, $\Phi' = C$. To get the BPS-like forms, we can not just substitute this constant into the effective Lagrangian[3] since this will not satisfy the original second order differential equations of motion. What we should do is writing the effective Lagrangian in BPS-like forms while keeping one of its BPS-equations to be the same as the equation of motion of dilaton. The satisfying effective Lagrangian is given by

$$\mathcal{L}_{eff} = \int du \left[12\left(n_4' + \sqrt{\frac{1}{b^2}e^{8n_4} + \frac{C^2}{24}}\right)^2 - \frac{1}{2}\left(\Phi' - C\right)^2 - 24n_4'\sqrt{\frac{1}{b^2}e^{8n_4} + \frac{C^2}{24}}\right]. \tag{16}$$

Here we assume the last term in the first integral can be written as an additional contribution to the total derivative term[4]. The BPS-like equations are given by

$$n_4' = -\sqrt{\frac{1}{b^2}e^{8n_4} + \frac{C^2}{24}}, \qquad \Phi' = C. \tag{17}$$

Now, we choose an identification such that the metric becomes asymptoticaly AdS and the choice is $e^{4n_4}du = \frac{b}{r}dr$. The first BPS-like equation of (17) has solution

$$e^{2n_4} = \pm\frac{e^{2\sqrt{6}c_1}}{2\sqrt{6}r^2}\sqrt{1 - 6C^2b^2e^{-8\sqrt{6}c_1}r^8}, \tag{18}$$

where c_1 is an integration constant. The fact that we have two solutions of the first order differential equation might be a reminiscence of second order differential equation of motion. The explicit form of u in term of r is

$$u = \frac{\sqrt{6}}{C}\operatorname{arctanh}\left(Cb\sqrt{6}e^{4\sqrt{6}c_1}r^4\right) + \text{constant}. \tag{19}$$

[3] One may think that there is no difference by doing so but actually it will give at least a different sign in the Lagrangian.

[4] In more detail, one can confirm with a paper by Miller et al [3] in the section where they discuss about nonextremal BPS solutions.

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We can set c_1 such that $b^2 = \frac{1}{2\sqrt{6}} e^{2\sqrt{6}c_1}$ and so the solution for metric and dilaton in terms of r are

$$ds^2 = \frac{b^2}{r^2}\left(\sqrt{1-\frac{C^2}{96b^6}r^8}(-dt^2+dx_3^2)+dr^2\right), \qquad \Phi(r) = \sqrt{\frac{3}{2}}\ln\left(\frac{1+\frac{C}{4\sqrt{6}b^3}r^4}{1-\frac{C}{4\sqrt{6}b^3}r^4}\right)+\phi_0. \quad (20)$$

By defining $r_c^4 = \frac{4\sqrt{6}b^3}{C}$, we are celebrating the Csaki-Recee solution [4]. In this solution, we can see clearly how dilaton changes the metric solution and indeed if we turn-off the dilaton by setting $C = 0$, we get back the usual AdS solution.

4.2. Black Hole Solutions

For black hole solutions, we need to use some general form of a asymptotic AdS black hole solution as the following [5]

$$ds^2 = \frac{b^2}{r^2}f(r)\left(-g(r)dt^2 + dx_3^2 + \frac{1}{g(r)}dr^2\right), \quad (21)$$

with $f(r)$ and $g(r)$ are regular function and $f(0) = g(0) = 1$ and also $g(r)$ is zero at some finite $r > 0$. So, the metric ansatz (7) can be rewritten to mimic the general black hole from as below

$$ds^2 = e^{n_1-n_4}\left(-e^{3n_4-n_1}dt^2 + dx_3^2 + e^{2n_1}du^2\right). \quad (22)$$

Here we can identify $e^{n_1-n_4} = \frac{b^2}{r^2}f(r)$ and $e^{2n_1}du^2 = e^{n_1-3n_4}dr^2$, with $e^{3n_4-n_1} = g(r)$.

4.2.1. Trivial Dilaton Solution The effective Lagrangian (10) can be written in the BPS-like forms

$$\mathcal{L}_{eff} = \int du\left[\frac{3}{2}\left(n_1' - e^{n_1+n_4} - \frac{2}{b^2}e^{2(n_1-n_4)}\right)^2 - \frac{3}{2}\left(n_4' + e^{n_1+n_4} - \frac{2}{b^2}e^{2(n_1-n_4)}\right)^2 - \frac{3}{2}(\Phi')^2\right]. \quad (23)$$

The BPS-like equations are

$$(n_1' + n_4') = \frac{4}{b^2}e^{2(n_1-n_4)}, \qquad (n_1' - n_4') = 2e^{n_1+n_4}, \qquad \Phi' = 0. \quad (24)$$

Define $p = n_1 + n_4$ and $q = n_1 - n_4$, so that the first two BPS-like equations of (24) are now

$$p' = \frac{4}{b^2}e^{2q}, \qquad q' = 2e^p. \quad (25)$$

The first BPS-like equation of (24) can be solved as

$$e^p = \frac{1}{b^2}e^{2q} + C, \quad (26)$$

with C is an integration constant. Now, let us identify $e^q = \frac{b^2}{r^2}$ so that the metric can be written in terms of functions p and q as follows

$$ds^2 = -e^{p-q}dt^2 + e^q dx_3^2 + e^{2q+p}du^2 = -\left(\frac{1}{r^2} + \frac{C}{b^2}r^2\right)dt^2 + \frac{b^2}{r^2}dx_3^2 + \frac{b^4}{r^4}\left(\frac{b^2}{r^4} + C\right)du^2. \quad (27)$$

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Using $\frac{du}{dr} = \frac{1}{\frac{b^2}{r^4}+C}\left(-\frac{1}{r}\right)$, the metric becomes

$$ds^2 = \frac{b^2}{r^2}\left[-\left(1+\frac{C}{b^2}r^4\right)\frac{1}{b^2}dt^2 + dx_3^2 + \frac{1}{\left(1+\frac{C}{b^2}r^4\right)}dr^2\right]. \tag{28}$$

This metric has a horizon at $r_h^4 = -\frac{b^2}{C}$, with $C < 0$ and this is the usual AdS-Schwarzschild(black hole) solution after rescaling the time coordinate.

4.2.2. Non-trivial dilaton solution We can get another solution by redefining $8p = 3n_1 - n_4$ and $8q = n_1 - 3n_4$ and writing the Lagrangian (10), in BPS-like form, as the following

$$\begin{aligned}\mathcal{L}_{eff} &= \int du\left[12\left(p' - \sqrt{\frac{1}{b^2}e^{8p}+\frac{C^2}{24}}\right)^2 - 12\left(q' - \frac{B}{2\sqrt{6}}\right)^2 - \frac{1}{2}(\Phi' - A)^2\right.\\ &\quad \left. +24p'\sqrt{\frac{1}{b^2}e^{8p}+\frac{C^2}{24}} - 2\sqrt{6}q'B - \Phi' A\right],\end{aligned} \tag{29}$$

where A, B and C are real constant and they are constrained by $C^2 = A^2 + B^2$. The BPS-like equations are given by

$$p' = \sqrt{\frac{1}{b^2}e^{8p}+\frac{C^2}{24}}, \qquad q' = \frac{B}{2\sqrt{6}}, \qquad \Phi' = A. \tag{30}$$

The metric can be written in terms of p and q,

$$ds^2 = -e^{2p-6q}dt^2 + e^{2p+2q}dx_3^2 + e^{8p}du^2. \tag{31}$$

Now, we have to find an identification that gives us asymptotic AdS metric with an event horizon. The easiest way to do it is by identifying $e^{8p} = f(r)$. So from the first BPS-like equation of (30), we obtain

$$\frac{dr}{du} = \frac{8f(r)}{f'(r)}\sqrt{\frac{f(r)}{b^2}+\frac{C^2}{24}}. \tag{32}$$

As a simple case take $f(r) = \frac{b^8}{r^8}$ which give solutions

$$\begin{aligned} ds^2 &= \frac{b^2}{r^2}\left[\left(\sqrt{\frac{C^2r^8}{24b^6}}+\sqrt{1+\frac{C^2r^8}{24b^6}}\right)^{-\frac{B}{2C}}\left(-\left(\sqrt{\frac{C^2r^8}{24b^6}}+\sqrt{1+\frac{C^2r^8}{24b^6}}\right)^{\frac{2B}{C}}dt^2 + dx_3^2\right) + \frac{dr^2}{\left(1+\frac{C^2}{24b^6}r^8\right)}\right],\\ \Phi(r) &= -\frac{A}{C}\sqrt{\frac{3}{2}}\ln\left(\sqrt{\frac{C^2r^8}{24b^6}}+\sqrt{1+\frac{C^2r^8}{24b^6}}\right)+\phi_0.\end{aligned} \tag{33}$$

Here we have fixed all constants to give us asymptotic AdS solution (15) near $r \to 0$.

Curvature scalar in terms of u coordinate, using the BPS-like equations, is given by

$$R = e^{-8p}\left[12\left(\frac{1}{b^2}e^{8p}+\frac{C^2}{24}\right) - 12\left(\frac{B^2}{24}\right) - 8\left(\frac{4}{b^2}e^{8p}\right)\right] \tag{34}$$

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and in terms of r coordinate it becomes

$$R = \frac{A^2}{2f(r)} - \frac{20}{b^2}. \tag{35}$$

Here, we can see that naked singularity happens when $f(r) = 0$, or in our example is at $r \to \infty$, and so at $r^8 = -\frac{24b^6}{C^2}$ we have a horizon, with $C^2 < 0$ if we define $0 \le r < \infty$. If we turn off the dilaton contribution, $A = 0$, we will get back the negative curvature scalar of AdS metric[5]. We can also see that the first term in the curvature scalar is coming from the matter contribution and in this case is the dilaton. This solution in some sence is not a good one because it gives us imaginary number in C if we expect the horizon to be at some finite positive number of r.

The difficulty of finding a correct solutions for metric and dilaton in terms of real functions could arise as the fact that in the corresponding non-black hole solution we obtain a naked singularity at some finite r. In general, we will show that in the black hole solutions even though there is a singularity at some finite r, this singularity turns out to be a naked singularity instead of a horizon. The ansatz for a black hole (31) can be written as

$$ds^2 = e^{2(p+q)} \left(-e^{-8q} dt^2 + dx_3^2 + e^{6p-2q} du^2 \right). \tag{36}$$

From general form of asymptotic AdS black hole (21), if there is a horizon at some finite $r = r_H$ then we would expect that $e^{-8q(r_H)} = 0$ or $u = \pm\infty$, from the second BPS-like solution(30). The first BPS-like equation (30) will give us

$$e^{-8p} = \frac{12}{C^2 b^2} \cosh\left(\frac{4}{\sqrt{6}} C u + C b c_1 \right) - \frac{12}{C^2 b^2} \tag{37}$$

and this function is infinite at $u = \pm\infty$. So, as we can see from (34) at $r = r_H$, the curvature scalar will blow up and we can conclude that this singularity is a naked singularity. A more detail study on the non-existence of black hole solutions in this theory can be read in [13].

5. Exponential Dilaton Potential

In general, we can add more dilaton potential terms into the action (6). Dilaton potential usually comes in terms of exponential form. Here, we are going to consider the action with one exponential form,

$$S = \int d^5x \sqrt{-g} \left(R - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{12}{b^2} e^{a\Phi} \right), \tag{38}$$

where a is a constant real number.

5.1. Non-Black Hole Solutions

Using the non-black hole ansatz (11), the effective Lagrangian becomes

$$\mathcal{L}_{eff} = \int du \left[12(n_4')^2 + \frac{12}{b^2} e^{8n_4 + a\Phi} - \frac{1}{2}(\Phi')^2 \right]. \tag{39}$$

Furthermore, we can rewrite our fields in a more simple Lagrangian by defining $l = 8n_4 + a\Phi$ and $m = m_1 n_4 + m_2 \Phi$. The constants m_1 and m_2 can be determined by requiring that the Lagrangian should not contain any mixing term of fields l and m, $a = \frac{m_1}{3m_2}$, and there should be inversion operation to the fields n_4 and Φ, $a \neq \frac{8m_2}{m_1}$. Both these requirements give a restriction

[5] For Einstein-Hilbert action in AdS, we have $S = \int \sqrt{-g}\,(R - 2\Lambda)$ such that the field equation is given by $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0$. In our case, we have $\Lambda = -\frac{6}{b^2}$ so that the curvature scalar $R = -\frac{20}{b^2}$.

to dilaton potential such that $a \neq \pm\frac{4}{\sqrt{6}}$. Substituting those fields into the action, we obtain a simple action in BPS-like form

$$\mathcal{L}_{eff} = \int du \left[\frac{3}{2(8-3a^2)} \left(l' - \sqrt{\frac{8(8-3a^2)}{b^2} e^l + C^2} \right)^2 - \frac{4}{m_2^2(8-3a^2)} (m' - \frac{\sqrt{6}}{4} m_2 C)^2 \right], \quad (40)$$

where C is a constant. The BPS-like equations are

$$l' = \sqrt{\frac{8(8-3a^2)}{b^2} e^l + C^2}, \qquad m' = \frac{\sqrt{6}}{4} m_2 C, \quad (41)$$

and the solutions in terms of u coordinate are given by

$$l(u) = \ln \left(-\frac{C^2 b^2}{8(8-3a^2)} \mathrm{sech} \left(\frac{1}{2} C(u + bc_1) \right)^2 \right), \qquad m(u) = \frac{\sqrt{6}}{4} m_2 C u + c_2, \quad (42)$$

where c_1 and c_2 are constant. So, the solution of the original fields (dilaton and metric) is given by

$$n_4(u) = \frac{1}{8-3a^2} \left(l(u) - \frac{a}{m_2} m(u) \right), \qquad \Phi(u) = \frac{1}{8-3a^2} \left(\frac{8}{m_2} m(u) - 3al(u) \right). \quad (43)$$

One can check that this solution has no naked singularity in the curvature scalar with restriction $a > \frac{4}{\sqrt{6}}$ and $C > 0$ or $a < -\frac{4}{\sqrt{6}}$ and $C < 0$. This is because the solution has a nice finite secant hyperbolic function, in coordinate u, unlike sinus hyperbolic function of previous Csaki-Reece solution. This could also be a sign the existence of black hole solutions.

5.2. Black Hole Solutions

Defining $8p = 3n_1 - n_4$ and $8q = n_1 - 3n_4$ and using the black hole ansatz (21) and (22), we can rewrite the effective Lagrangian to be

$$\mathcal{L}_{eff} = \int du \left[12 \left((p')^2 - (q')^2 \right) + \frac{12}{b^2} e^{8p + a\Phi} - \frac{1}{2} (\Phi')^2 \right]. \quad (44)$$

Using another fields definition, $l = 8p + a\Phi$ and $m = m_1 p + m_2 \Phi$, the effective Lagrangian written in BPS-like form now becomes

$$\mathcal{L}_{eff} = \int du \left[\frac{3}{2(8-3a^2)} \left(l' - \sqrt{\frac{8(8-3a^2)}{b^2} e^l + C^2} \right)^2 - \frac{4\left(m' - \frac{\sqrt{6}}{4} m_2 B\right)^2}{m_2^2(8-3a^2)} - 12 \left(q' - A \right)^2 \right], \quad (45)$$

where A, B, and C are real constant and they are constrained by $C^2 - B^2 = 8(8-3a^2)A^2$. The BPS-like equations are

$$l' = \sqrt{\frac{8(8-3a^2)}{b^2} e^l + C^2}, \qquad m' = \frac{\sqrt{6}}{4} m_2 B, \qquad q' = A. \quad (46)$$

The solution to (46) in terms of u coordinate is given by

$$l(u) = \ln \left(-\frac{C^2 b^2}{8(8-3a^2)} \mathrm{sech} \left(\frac{1}{2} C(u + bc_1) \right)^2 \right), \qquad m(u) = \frac{\sqrt{6}}{4} m_2 B u + c_2 \qquad q(u) = Au + c_3, \quad (47)$$

where c_1, c_2, and c_3 are constant. So the solution of the original fields (dilaton and metric) is given by

$$p(u) = \frac{1}{8-3a^2}\left(l(u) - \frac{a}{m_2}m(u)\right), \qquad \Phi(u) = \frac{1}{8-3a^2}\left(\frac{8}{m_2}m(u) - 3al(u)\right), \qquad q(u) = Au + c_3. \tag{48}$$

Let us compute the area of horizon, which can be read off from the metric,

$$e^{2p+2q} = \left(\frac{4C^2b^2e^{-bCc_1-\frac{a}{m_2}c_2-(3a^2-8)c_3}}{8(3a^2-8)}\right)^{-\frac{2}{3a^2-8}} \left(1+e^{-C(u+bc_1)}\right)^{\frac{4}{3a^2-8}} e^{\frac{4(3a^2-8)A+4C+a\sqrt{6}B}{2(3a^2-8)}u}. \tag{49}$$

In order to get a finite horizon area, at $u = \infty$, we impose the last exponential term equal to one by setting $B = -\frac{a\sqrt{6}}{4}C$ and $C = 8A$, with $C, a > 0$. Unfortunately, we will show later on, this solution does not give an AdS metric at $u = 0$ unless $a = 0$. For the moment, we would expect that the area is infinity at $u = 0$ where in this case we fix the constant $e^{-Cbc_1} = -1$ and $a^2 < \frac{8}{3}$ hence the horizon area becomes

$$e^{2p+2q} = \left(\frac{4C^2b^2e^{-\frac{a}{m_2}c_2+(8-3a^2)c_3}}{8(8-3a^2)}\right)^{\frac{2}{8-3a^2}} \left(1-e^{-Cu}\right)^{-\frac{4}{8-3a^2}} = \frac{b^2}{r^2}f(r), \tag{50}$$

where $f(r)$ is a regular function at the horizon and equal to one at $r = 0$. So the entropy density and temperature for this solution are given by

$$s = \lim_{u\to\infty}\frac{2\pi}{\kappa_5^2}e^{3p+3q} = \frac{2\pi}{\kappa_5^2}\left(\frac{4C^2b^2e^{-\frac{a}{m_2}c_2+(8-3a^2)c_3}}{8(8-3a^2)}\right)^{\frac{3}{8-3a^2}}, \tag{51}$$

$$T = \lim_{u\to\infty}\frac{e^{(p+q)-(3p-5q)}\left|\left(e^{-8q}\right)'\right|}{4\pi} = \frac{|-8q'|}{4\pi}e^{-2(p+q)} = \frac{8A}{4\pi}\left(\frac{4C^2b^2e^{-\frac{a}{m_2}c_2+(8-3a^2)c_3}}{8(8-3a^2)}\right)^{-\frac{2}{8-3a^2}} \tag{52}$$

where $A > 0$.

Now, let compute curvature scalar for this solution. We can write p in terms of q, with $c_3 = 0$ for simplification,

$$e^{2p} = \left(\frac{4C^2b^2e^{-\frac{a}{m_2}c_2}}{8(8-3a^2)}\right)^{\frac{2}{8-3a^2}} \left(1-e^{-8q}\right)^{-\frac{4}{8-3a^2}} e^{-2q} \tag{53}$$

and its derivatives

$$p' = \left(-q' - \frac{16e^{-8q}}{(8-3a^2)(1-e^{-8q})}q'\right), \qquad p'' = \frac{2^7e^{-8q}}{(8-3a^2)(1-e^{-8q})^2}(q')^2. \tag{54}$$

From formula (34), we can write curvature scalar,

$$R = e^{-8p}\left[12(p')^2 - 12(q')^2 - 8p''\right], \qquad R(u\to\infty) = \left(\frac{4C^2b^2e^{-\frac{a}{m_2}c_2}}{8(8-3a^2)}\right)^{-\frac{8}{8-3a^2}} \frac{-5C^2}{2(8-3a^2)}, \tag{55}$$

while at $u = 0$ one can show the curvature scalar vanishes for $a \neq 0$ means that the solution is a black hole but not asymptotically AdS. If $a = 0$, the curvature scalar will be finite both at $u = 0$ and at $u \to \infty$ which gives us back AdS-Schwarzchild. To find the exact form of this black hole, we need to use identification (50) such that we can write u in terms of r,

$$u = -\frac{1}{C} \ln \left[1 - \left(\frac{r^2}{b^2 f} \right)^{\frac{8-3a^2}{4}} \left(\frac{4C^2 b^2 e^{-\frac{a}{m_2} c_2}}{8(8-3a^2)} \right)^{1/2} \right]. \tag{56}$$

The metric is given by

$$\begin{aligned} ds^2 &= \frac{b^2}{r^2} f \left(- \left[1 - \left(\frac{r^2}{b^2 f} \right)^{\frac{8-3a^2}{4}} \left(\frac{4C^2 b^2 e^{-\frac{a}{m_2} c_2}}{8(8-3a^2)} \right)^{1/2} \right] dt^2 + dx_3^2 \right. \\ &\left. + \frac{(8-3a^2)(2f - rf')^2 f^{\frac{3a^2-6}{2}}}{32 \left[1 - \left(\frac{r^2}{b^2 f} \right)^{\frac{8-3a^2}{4}} \left(\frac{4C^2 b^2 e^{-\frac{a}{m_2} c_2}}{8(8-3a^2)} \right)^{1/2} \right]} \left(\frac{b}{r} \right)^{3a^2} e^{-\frac{a}{m_2} c_2} dr^2 \right) \end{aligned} \tag{57}$$

and solution for dilaton is

$$\begin{aligned} \Phi(r) &= \frac{1}{8-3a^2} \left(- \left(\frac{2\sqrt{6}B}{C} + 3a \right) \ln \left[1 - \left(\frac{r^2}{b^2 f} \right)^{\frac{8-3a^2}{4}} \left(\frac{4C^2 b^2 e^{-\frac{a}{m_2} c_2}}{8(8-3a^2)} \right)^{1/2} \right] + \frac{(8-3a^2)}{m_2} c_2 \right. \\ &\left. - \frac{3a}{2}(8-3a^2) \ln \left(\frac{b^2 f}{r^2} \right) \right) + \phi_0. \end{aligned} \tag{58}$$

We can also see clearly from dilaton profile that dilaton is infinity near $r = 0$ which is not expected for asymptotically AdS solution unless $a = 0$. Recall that the calculation is done for $c_3 = 0$. If $c_3 \neq 0$ the curvature scalar will not be zero and, so there might be AdS black hole solutions with some appropriate fixed remaining constants.

6. Conclusion

We have shown how to construct BPS-like equations for five dimensional gravity-dilaton system in which the dilaton potential is an exponential function parametrized by a constant a. For non-black hole solutions, we obtained the range value of a is $|a| > 4/\sqrt{6}$. It turned out that there are possible black hole solutions for a in the range $|a| < 4/\sqrt{6}$. This in accordance with the black hole solution of the constant dilaton potential, namely AdS-Schwarzschild black hole, in which $a = 0$. We argue there might be some AdS black hole solutions if one of the constants in the solutions is non-zero, or to be precise $c_3 \neq 0$.

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