

# Modeling the dynamics of the chassis of construction machines

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**Abstract.** The article presents the results of a study of the transfer functions of a construction machine as a complex dynamic system. Authors constructed a dynamic model of a construction machine. The paper formulates and solves a system of nonlinear differential equations of motion of the chassis system of a construction machine on the basis of the d'Alembert-Lagrange equation. The numerical values of the transfer function coefficients for the construction machines were determined from the experimentally obtained curves of acceleration, processed by area method. Authors determined the experimental curves of the transition process of chassis system of a construction machine. The results of the study show that the difference of source curves ordinates and calculation of transients is less than 4% on average, which indicates a fairly accurate description of the process. The resulting expressions of transfer system functions of the chassis with sufficient precision can be used for practical purposes in the design and development of new construction machines.

**Keywords:** mathematical modeling of the dynamic model, chassis system, motor construction machine, acceleration curves, Laplace transform.

## 1. Introduction

A construction machine is one of the most massive earth-moving machinery, employed in road construction. Modern market conditions require high-performance, high-speed and powerful heavy transport machine. Constant increase of available power leads to an increase in dynamic loads acting on both the human operator and the machine.

## 2. Construction of dynamic models

For the investigation of dynamics of the construction machine chassis we constructed its dynamic model according to its kinematic scheme [2].

On the basis of the d'Alembert-Lagrange system of equations we made non-linear differential equations of motion of the construction machines chassis system [1]:

- 1) a bent shaft of the engine and a power shaft of a reducer with the general moment of inertia of  $J_1$ ;
- 2) output shaft of a distributing reducer and a gear box with  $J_2$  moment of inertia;
- 3) shaft of running wheels with the given  $J_3$  construction machine moment of inertia.

On the basis of it the system of the differential equations of running system has the following appearance:

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$$\left. \begin{aligned} J_1 \dot{\omega}_1 + M_{c\epsilon 1} + M_{T1} + \frac{M_{1.2}}{i_{1.2} \eta_{1.2}} + \frac{M_{1.5}}{i_{1.5} \eta_{1.5}} &= M_1(\omega_1); \\ J_2 \dot{\omega}_2 + M_{c\epsilon 2} + M_{T2} + \frac{M_{2.3}}{i_{2.3} \eta_{2.3}} &= M_{1.2}(\omega_1 \omega_2); \\ J_3 \dot{\omega}_3 + M_{c\epsilon 3} + M_{T3} + F_x(t) &= M_{2.3}(\omega_2 \omega_3); \end{aligned} \right\} \quad (1)$$

where  $J_1, J_2, J_3$  – respectively the constant given moments of inertia on cranked to an engine shaft with a power shaft of a reducer, clutch with an output in bulk of a gear box, on a shaft of running wheels of the construction machine;  $\omega_1, \omega_2, \omega_3$  – angular speeds of rotation of the corresponding shaft (rad / s);  $M_{c\epsilon 1}, M_{c\epsilon 2}, M_{c\epsilon 3}$  – air drag torques on the corresponding shaft, H·m.

$$M_{c\epsilon 1} = B_1 \omega_1^2; \quad M_{c\epsilon 2} = B_2 \omega_2^2; \quad M_{c\epsilon 3} = B_3 \omega_3^2;$$

$B_1, B_2, B_3$  – the coefficients considering air and other resistance, N·m·s<sup>2</sup>;  $M_{T1}, M_{T2}, M_{T3}$  – the moments of powers of constant friction (N·m) on the corresponding shaft of reduction;  $M_j(\omega_j)$  – the partial or external high-speed characteristic of the engine on a torque on cranked to a shaft which for this case is accepted [2]:

$$M_j(\omega_j) = A - C \omega_j,$$

$M_{1.2}(\omega_1, \omega_2), M_{2.3}(\omega_2, \omega_3)$  – the moments arising in hydro mechanical and friction gears which are accepted in a look [4]:

$$M_{1.2}(\omega_1, \omega_2) = A_1 - C_1 \cdot (\omega_1 / \omega_2) \cdot i_{1.2}; \quad M_{2.3}(\omega_2, \omega_3) = A_2 - C_2 \cdot (\omega_2 / \omega_3) \cdot i_{2.3};$$

$A, A_1, A_2, C, C_1, C_2$  – constant coefficients;  $i_{1.2}, i_{2.3}, i_{1.5}, \eta_{1.2}, \eta_{2.3}, \eta_{1.5}$  – reduction ratios and efficiency of drives;  $F_x(t)$  – external loading (H·m), time-dependent.

We accept the moment  $M_{1.5}$  arising in a drive gear of a distributing reducer to constants.

Let's lead the system of the differential nonlinear equations (1) relatively  $\omega_1, \omega_2, \omega_3$  to a linear look. For this purpose we decompose the components of the equations (1) in ranks and we are limited only to linear elements of increments  $\Delta\omega_1, \Delta\omega_2, \Delta\omega_3$  of rather established values  $\tilde{\omega}_1, \tilde{\omega}_2, \tilde{\omega}_3$  [5]:

$$\Delta M_{c\epsilon 1} = \left( \frac{\partial M_{c\epsilon 1}}{\partial \omega_1} \right) \Delta\omega_1 = I_1 \Delta\omega_1; \quad (2)$$

$$\Delta M_{c\epsilon 2} = \left( \frac{\partial M_{c\epsilon 2}}{\partial \omega_2} \right) \Delta\omega_2 = I_2 \Delta\omega_2; \quad (3)$$

$$\Delta M_{c\epsilon 3} = \left( \frac{\partial M_{c\epsilon 3}}{\partial \omega_3} \right) \Delta\omega_3 = I_3 \Delta\omega_3; \quad (4)$$

$$\Delta M_1 = -C \Delta\omega_1; \quad (5)$$

$$\Delta M_{1.2} = \frac{C_1 i_{1.2}}{\omega_2} \left[ -\Delta\omega_1 + \frac{\omega_1}{\omega_2} \Delta\omega_2 \right]; \quad (6)$$

$$\Delta M_{2.3} = \frac{C_2 i_{2.3}}{\omega_3} \left[ -\Delta\omega_2 + \frac{\omega_2}{\omega_3} \Delta\omega_3 \right], \quad (7)$$

Where  $\Delta M_{c\epsilon 1}, \Delta M_{c\epsilon 2}, \Delta M_{c\epsilon 3}, \Delta M_1, \Delta M_{1.2}, \Delta M_{2.3}$  – increments of the moments;  $I_1 = 2B_1 \tilde{\omega}_1; I_2 = 2B_2 \tilde{\omega}_2; I_3 = 2B_3 \tilde{\omega}_3$ .

Having substituted expressions (2) – (7) in (1), we will receive the following system of the linear equations of the movement of a running gear of the construction machine:

$$\left. \begin{aligned} J_1 \Delta \dot{\omega}_1 + \left[ H_1 - \frac{C_1}{\eta_{1,2} \tilde{\omega}_2} + C \right] \Delta \omega_1 + \frac{C_1 \tilde{\omega}_1}{\eta_{1,2} \tilde{\omega}_2^2} \Delta \omega_2 &= 0; \\ J_2 \Delta \dot{\omega}_2 + \left[ H_2 - \frac{C_2}{\eta_{2,3} \tilde{\omega}_3} - \frac{C_1 \tilde{\omega}_1 i_{1,2}}{\tilde{\omega}_2^2} \right] \Delta \omega_2 + \frac{C_2 \tilde{\omega}_2^2}{\eta_{2,3} \tilde{\omega}_3^2} \Delta \omega_3 + \frac{C_1 i_{1,2}}{\tilde{\omega}_2} \Delta \omega_1 &= 0; \\ J_3 \Delta \dot{\omega}_3 + \left[ H_3 - \frac{C_2 \tilde{\omega}_2 i_{2,3}}{\tilde{\omega}_3^2} \right] \Delta \omega_3 + \frac{C_2 i_{2,3}}{\tilde{\omega}_3} \Delta \omega_2 &= -\Delta F_x(t). \end{aligned} \right\} \quad (8)$$

We will enter designations:

$$G_{11} = H_1 - \frac{C_1}{\eta_{1,2} \tilde{\omega}_2} + C; \quad G_{12} = \frac{C_1 \tilde{\omega}_1}{\eta_{1,2} \tilde{\omega}_2^2}; \quad G_{21} = \frac{C_1 i_{1,2}}{\tilde{\omega}_2}; \quad G_{22} = H_2 - \frac{C_2}{\eta_{2,3} \tilde{\omega}_3} - \frac{C_1 \tilde{\omega}_1 i_{1,2}}{\tilde{\omega}_2^2};$$

$$G_{23} = \frac{C_2 \tilde{\omega}_2^2}{\eta_{2,3} \tilde{\omega}_3^2}; \quad G_{32} = \frac{C_2 i_{2,3}}{\tilde{\omega}_3}; \quad G_{33} = H_3 - \frac{C_2 \tilde{\omega}_2 i_{2,3}}{\tilde{\omega}_3^2}.$$

With the accounting of designations the system of the equations (8) looks like this:

$$\left. \begin{aligned} J_1 \Delta \dot{\omega}_1 + G_{11} \Delta \omega_1 + G_{12} \Delta \omega_2 &= 0; \\ J_2 \Delta \dot{\omega}_2 + G_{22} \Delta \omega_2 + G_{23} \Delta \omega_3 + G_{21} \Delta \omega_1 &= 0; \\ J_3 \Delta \dot{\omega}_3 + G_{33} \Delta \omega_3 + G_{32} \Delta \omega_2 &= -\Delta F_x(t). \end{aligned} \right\} \quad (9)$$

Having  $\Delta \omega_2$  expressed from the third equation of system, having substituted  $\Delta \omega_2$  in the second equation of system (9) we will receive:

$$-\frac{J_2}{G_{32}} \left[ \Delta \dot{F}_x(t) + J_3 \Delta \dot{\omega}_3 + G_{33} \Delta \omega_3 \right] - \frac{G_{22}}{G_{32}} \left[ \Delta F_x(t) + J_3 \Delta \dot{\omega}_3 + G_{33} \Delta \omega_3 \right] + G_{23} \Delta \omega_3 + G_{21} \Delta \omega_1 = 0, \quad (10)$$

From the received ratio we will express  $\Delta \omega_1$  and also we will find a derivative on time from  $\Delta \omega_1$ :

$$\frac{d\Delta \omega_1}{dt} = \frac{J_2}{G_{21} G_{3,2}} \left[ \frac{d^2 \Delta F_x(t)}{dt^2} + J_3 \frac{d^3 \Delta \omega_3}{dt^3} + G_{33} \frac{d^2 \Delta \omega_3}{dt^2} \right] +$$

$$+ \frac{G_{22}}{G_{21} G_{32}} \left[ \frac{d\Delta F_x(t)}{dt} + J_3 \frac{d^2 \Delta \omega_3}{dt^2} + G_{33} \frac{d\Delta \omega_3}{dt} \right] - \frac{G_{23}}{G_{21}} \frac{d\Delta \omega_3}{dt}. \quad (11)$$

Having substituted expressions (10)-(11) in the first equation of system (9) we will receive the differential equation:

$$\frac{J_1 J_2}{G_{21} G_{3,2}} \left[ \frac{d^2 \Delta F_x(t)}{dt^2} + J_3 \frac{d^3 \Delta \omega_3}{dt^3} + G_{33} \frac{d^2 \Delta \omega_3}{dt^2} \right] + \frac{G_{22} J_1}{G_{21} G_{32}} \left[ \frac{d\Delta F_x(t)}{dt} + J_3 \frac{d^2 \Delta \omega_3}{dt^2} + G_{33} \frac{d\Delta \omega_3}{dt} \right] -$$

$$-\frac{G_{23} J_1}{G_{21}} \frac{d\Delta \omega_3}{dt} + \frac{G_{11} J_2}{G_{21} G_{3,2}} \left[ \frac{d\Delta F_x(t)}{dt} + J_3 \frac{d^2 \Delta \omega_3}{dt^2} + G_{33} \frac{d\Delta \omega_3}{dt} \right] + \frac{G_{11} G_{22}}{G_{21} G_{32}} \left[ \Delta F_x(t) + J_3 \frac{d\Delta \omega_3}{dt} + G_{33} \Delta \omega_3 \right] -$$

$$-\frac{G_{11} G_{23}}{G_{21}} \Delta \omega_3 - \frac{G_{12}}{G_{32}} \Delta F_x(t) - \frac{G_{12} J_3}{G_{32}} \frac{d\Delta \omega_3}{dt} - \frac{G_{12} G_{33}}{G_{32}} \Delta \omega_3 = 0.$$

After transformations, we will receive the differential equation of a look:

$$a_3 \frac{d^3 \Delta \omega_3}{dt^3} + a_2 \frac{d^2 \Delta \omega_3}{dt^2} + a_1 \frac{d \Delta \omega_3}{dt} + a_0 \Delta \omega_3 = -b_2 \frac{d^2 \Delta F_x(t)}{dt^2} - b_1 \frac{d \Delta F_x(t)}{dt} - b_0 \Delta F_x(t), \quad (13)$$

where  $a_3; a_2; a_1; a_0; b_2; b_1; b_0$  – movement equation coefficients.

For the purpose of convenience we enter the following dimensionless sizes:

$$X = \frac{\Delta \omega_3}{\tilde{\omega}_3}; t_1 = \tilde{\omega}_3 t; Y = \frac{\Delta F_x(t)}{\tilde{F}_x}; \tilde{a}_3 = \frac{\varepsilon_3 \tilde{\omega}_3^4}{\tilde{F}_x}; \tilde{a}_2 = \frac{\varepsilon_2 \tilde{\omega}_3^3}{\tilde{F}_x}; \tilde{a}_1 = \frac{\varepsilon_1 \tilde{\omega}_3^2}{\tilde{F}_x}; \tilde{a}_0 = \frac{\varepsilon_0 \tilde{\omega}_3}{\tilde{F}_x};$$

$$\tilde{b}_2 = b_2 \tilde{\omega}_3^2; \tilde{b}_1 = b_1 \tilde{\omega}_3; \tilde{b}_0 = b_0,$$

where  $\tilde{F}_x$  и  $\tilde{\omega}_3$  – chosen in advance values of entrance sizes.

We will write down the equation (13) in an operator form, having replaced each member of the equation with the image corresponding to it according to Laplace, under zero entry conditions:

$$[a_3 P^3 + a_2 P^2 + a_1 P + a_0] \cdot Y = -[b_2 P^2 + b_1 P + b_0] \cdot X, \quad (14)$$

where  $X$  и  $Y$  – images according to Laplace entrance  $\alpha$  (rad.) and output  $V$  (m/s) sizes;  $P=d/dt$  – operator of differentiation.

According to (10) transfer function of running system of the construction machine  $W_n(P)$  has an appearance:

$$W_n(P) = -\frac{b_2 P^2 + b_1 P + b_0}{a_3 P^3 + a_2 P^2 + a_1 P + a_0}, \quad (15)$$

where  $a_1, a_2, a_3, a_0, b_2, b_1, b_0$  – the equation coefficients determined by formulas:  $b_2 = \tilde{b}_2 / \tilde{\varepsilon}_0; b_1 = \tilde{b}_1 / \tilde{\varepsilon}_0; b_0 = \tilde{b}_0 / \tilde{\varepsilon}_0; a_3 = \tilde{a}_3 / \tilde{a}_0; a_2 = \tilde{a}_2 / \tilde{a}_0; a_1 = \tilde{a}_1 / \tilde{a}_0; a_0 = \tilde{a}_0 / \tilde{a}_0 = 1$ .

Numerical values of coefficients of transfer function for the construction machine (11) were determined by experimentally received curves of dispersal [4].

When determining curves of dispersal indignation was carried out by sharp change of entrance size – an angle of rotation of the lever steering productivity of the fuel pump of hydromechanical transmission (GMT) on the established corner  $\Delta \alpha$ .

Speed of progress got out in the range of the working speeds of 1,2 - 1,8 m/s [3]. Rotary speed measurement  $\omega_3$  was performed the tacho generator and an induction marker [6,9], and the entrance size  $\alpha$  - the electrometric sensor of the PLP-21 brand.

Experimental curves of transition process of running system of the construction machine are given in figure 1.

With a practical accuracy it is possible to accept that transfer function of simpler look:

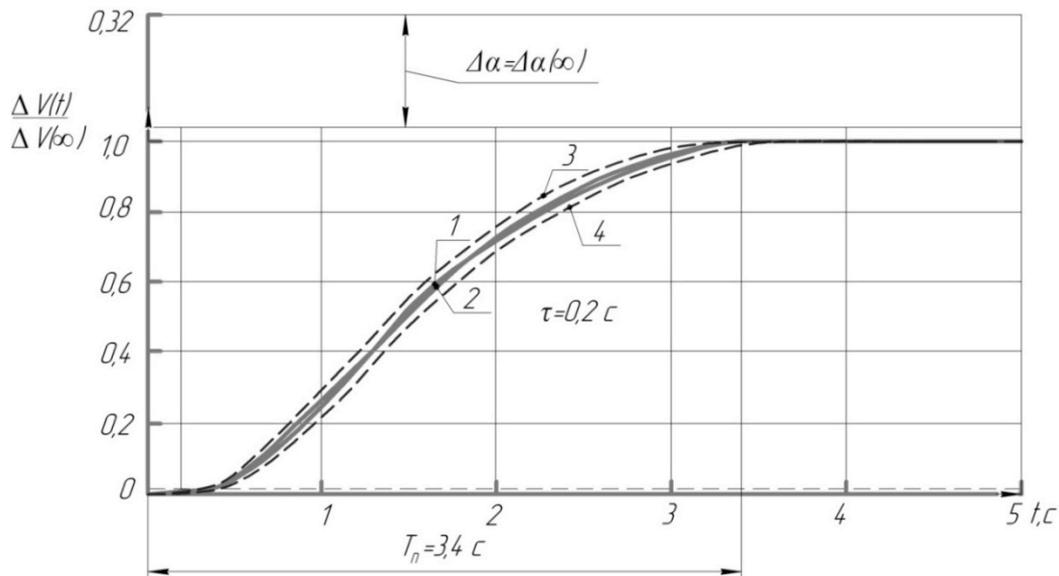
$$W_1(P) = \frac{1}{a_3 P^3 + a_2 P^2 + a_1 P + a_0}, \quad (16)$$

where  $a_3 = F_3 = 0,197 c^3; a_2 = F_2 = 1,209 c^2; a_1 = F_1 = 1,87 c; a_0 = 1$  – for the construction machine.

Then transfer function of running system of the construction machine taking into account time of delay will assume an air:

$$W_1(P) = \frac{K_1 e^{-0,1P}}{0,197 P^3 + 1,209 P^2 + 1,87 P + 1}, \quad (17)$$

where  $K = 3,13$  – coefficient of strengthening of running system;  $\tau = 0,2 c$  – delay time.



**Figure 1.** Transition processes of running system of the construction machine:  
 1 – the experimental; 2 – the settlement; 3, 4 – borders of confidential intervals at confidential probability 0,95

Next we define an error of approximation of transfer functions. For this purpose we will find analytical expressions of transition processes  $\Delta V(t)$  construction machine [8].

We will write down the image of transition process according to Laplace:

$$L[\Delta V_n(t)] = L[\alpha(t)]W_1(P). \quad (18)$$

At single indignation  $\alpha(t) = 1$ ; we have  $L[\alpha(t)] = 1/P$  and

$$L[\Delta V(t)] = \frac{K_1 e^{-tP}}{(a_3 P^3 + a_2 P^2 + a_1 P + a_0)P}, \quad (19)$$

where  $L[\Delta V_n(t)]$  и  $L[\alpha(t)]$  – images according to Laplace of a deviation of speed of progress of the construction machine and the revolting influence.

The roots of denominators of transfer functions of running system (19) found an iterative method [10] the following:  $P_1 = -4,141$ ;  $P_{2,3} = -0,998 \pm 0,479i$ .

We will write down the original of transition process in a general view:

$$\Delta V(t) = \frac{R(0)}{D(0)} + \frac{R(P_1)}{P_1 D'(P_1)} K \cdot e^{-P(t-\tau)} + 2A_k e^{(t-\tau_1)} \cos[\beta_k(t-\tau_1) + \varphi_k], \quad (20)$$

where  $R$  и  $D$  – numerator and denominator of transfer functions;  $P_1$  – valid root of a denominator;  $A_k$ ,  $\varphi_k$  – amplitude and phase of fluctuations which to be determined by formulas:

$$A_k = \sqrt{\delta^2 + \sigma^2}; \quad \varphi_k = \arctg \sigma / \delta,$$

where  $\alpha_k$  and  $\beta_k$  – respectively valid and imaginary parts of complex roots of a denominator;  $\delta$  and  $\sigma$  – respectively valid and imaginary speak rapidly expressions  $R(P_k)/D'(P_k)$  in a case when  $R_k$  is a complex root. After substitution of roots in (20), we will receive expressions of transition processes for the construction machine:

$$\Delta V_n(t) = \{1 - 0,121 e^{-4,141(t-0,1)} - 3,011 e^{-0,998(t-0,1)} \cdot \cos[0,479(t-0,1) - 1,275]\} / \Delta V(\infty), \quad (21)$$

where  $\Delta V_n(t)$  – deviations of speed of the movement of the construction machine from the established value.

We will check correctness of calculations by comparison of the transition processes calculated according to (21) and received experimentally.

### 3. Conclusion

1. Analyzing curves in figure 1 it is possible to say that the toe-out of ordinates of initial curves of 1 and settlement curve 2 transition processes on average doesn't exceed 4% that points to rather exact description of process.

2. Probe of dynamics of chassis of the construction machine showed that its mathematical model is described by the differential equation of the third order. The received expressions of transfer functions of running system are confirmed with pilot studies and with a sufficient accuracy can be used in the form of (21) for practical calculations.

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