Rise-Time Distortion of Signal without Carrying Signal

N S Bukhman
Samara state university of architecture and civil engineering ,
Russian Federation, 443001, Samara, 194 Molodogvardeiskaya str.

E-mail: nik3141rambler@rambler.ru

Abstract. The article deals with one-dimensional problem of rise-time distortion signal without carrying signal, that appears in the starting point intermittently, that is signal distortion at front edge or one of its derivative. The authors show that front edge of signal isn't distorted in case of propagation in unrestricted (including absorbing) area (amplitude of starting signal step or of one of its derivatives doesn't change) and move with the accuracy of vacuum light speed. The paper proves that it is the time interval shortage that causes signal loss with the route extension, but not the reduction of its starting amplitude, during which front edge of signal retains its starting value. The research presents new values for this time interval.

1. Introduction

The matter of rise-time distortion of the signal as it propagates in a material medium has got a long history, but it is still researched today (see [1-7]). The papers quoted above deal with signal with “carrier frequency”; this paper deals with the rising edge of video signal, i.e. signal "without carrier frequency".

Let's consider the signal $E(t)$ as it propagates in a medium with complex dielectric constant $\varepsilon(\omega)$ and refractive index $n(\omega)=\sqrt{\varepsilon(\omega)}$. It's evident that $E(\omega)=\int_{-\infty}^{+\infty} E(t)\exp(-i\omega t)dt$, $E(t)=(2\pi)^{-1}\int_{-\infty}^{+\infty} E(\omega)\exp(i\omega t)d\omega$, where $E(\omega)$ is the frequency spectrum of the signal in the starting point ($z=0$). As the signal propagates, with increase of path length z, both its spectrum $E(\omega,z)$ and its time dependence $E(t,z)$ change.

After the signal goes a path the length of which is z in a homogeneous medium with wave number $k(\omega)=(\omega/c)n(\omega)$, we have the frequency spectrum of the signal: $E(\omega,z)=F(\omega,z)E(\omega)$ where $F(\omega,z)=\exp(-i(\omega/c)z)F_{def}(\omega,z)$ where $\exp(-i(\omega/c)z)$ is the frequency characteristic of the ideal delay line (for the time of “light delay"; further, to reduce writing, we usually imply this delay, but do not explicitly write it), and $F_{def}(\omega,z)=\exp\left(-i(\omega/c)(n(\omega)-1)\right)$ is the additional (with respect to the latter) part of frequency characteristic of the above-mentioned filter, that describes the deformation (distortion) of the signal as it propagates in the medium.

To be specific, let's consider the case of collisional plasma; as it turns out, basically the obtained results are of much more general character. For the plasma, we have the dielectric constant [8.9]

$\varepsilon(\omega) = 1 - \omega_p^2 / (\omega^2 + \nu^2) + i\,\omega_p^2 (\nu/\omega) / (\omega^2 + \nu^2)$, where $\omega_p = (4\pi n e^2/m)^{1/2}$ - is the plasma frequency, ν - is the effective collision frequency.

The first three expansion terms $F_{def}(\omega, z)$ in series in powers of $1/\omega$ are $F_{def}(\omega, z) = 1 + i(k_p z/2)(\omega_p/\omega) - ((\nu/\omega_p)(k_p z/2) + (k_p z)^2/8)(\omega_p/\omega)^2$, where $k_p \equiv (\omega_p/c)$.

To understand further analysis it is necessary to recall that in the theory of complex variable functions there is a theorem on the uniqueness of analytical function (see [10]) which states that two analytic functions coinciding on any finite section of real axis, coincide on all real axis. In particular, the analytical function which is identically zero on some section of the real axis is identically zero on all real axis. In our case, this means that the signal $E(t)$, which is the analytical function of real variable t in point $z = 0$ and is not identically zero, can not become 0 on any finite section of real axis t - in particular, it can not satisfy the condition $E(t) = 0$ if $t \le t_0$. This means that such signal existed, exists and will always exist (if $-\infty < t < +\infty$), and basically it can not be used to transmit information.

Indeed, in any point in space z at any given time t the part of the signal received at all previous times is already available for analysis. By this part of the signal, in accordance with the above-mentioned theorem on the uniqueness of analytical function, its time dependence at any past or future point of time can be basically reconstructed. Therefore, no signal used to transmit information can be the analytical function on all real axis; it can coincide with the analytical function only if $t > t_0$, where t_0 - is the time of the signal's emergence. The point t_0 here is the point of breach of analyticity, which usually manifests itself as a breach of the time dependence of the signal and (or) its derivatives. The main purpose of this paper is to research, how exactly this breach of the time dependence of the signal in the starting point and its near neighborhood look after the signal goes a path the length of which is z in a substance.

2. Results

Let's suppose the signal $E(t)$ with sharp rising edge propagates in the medium, it emerges in the starting point $z = 0$ at the point of time t_0 ($E(t) = 0$ if $t < t_0$, $E(t) \ne 0$ if $t > t_0$). Then for the asymptotics (if $\omega \to \infty$) of the spectrum of such signal in the starting point $z = 0$, with third order accuracy in powers of $1/\omega$, we have (see Appendix) $E(\omega) = \exp(-i\omega t_0)(E(t_0)(i\omega)^{-1} + E'(t_0)(i\omega)^{-2} + E''(t_0)(i\omega)^{-3})$, where $E(t_0), E'(t_0), E''(t_0)$ - is the value of the signal and its first derivatives on the edge, fully describing (See Appendix) the relevant terms of its spectrum's high-frequency asymptotics. The result for the signal's spectrum in point z (with third order accuracy in powers $1/\omega$) is: $E(\omega, z) = \exp(-i\omega t_0)(E(t_0, z)(i\omega)^{-1} + E'(t_0, z)(i\omega)^{-2} + E''(t_0, z)(i\omega)^{-3})$, where $E(t_0, z) = E(t_0)$, $E'(t_0, z) = E'(t_0) - (1/2)E(t_0)(k_p z)\omega_p$,
$E''(t_0, z) = E''(t_0) - (1/2)E'(t_0)(k_p z)\omega_p + E(t_0)\omega_p^2((1/2)(\nu/\omega_p)(k_p z) + (1/8)(k_p z)^2)$ - are the values of the amplitude of the signal and its two first derivatives on the edge (if $t = t_0$) in point *z*. Let us discuss the characteristics of the changes of the rising edge of the signal, that emerges as a jump in point $z = 0$ at the point at time $t = t_0$ ($E(t) = 0$ if $t < t_0$, $E(t_0) \ne 0$), as it propagates in the medium, that is let us make some obvious conclusions from the above formulae.

1. In the case of signal amplitude jumping from 0 to final value $E(t_0) \ne 0$ at the point of time t_0 in starting point $z = 0$, its amplitude also jumps from 0 to the same final value $E(t_0) \ne 0$ in any other

point in space z at the point of time $t_0 + z/c$ corresponding to the light delay time of the signal's edge. In other words, the amplitude of the initial jump of the signal propagating in an arbitrary medium (including an absorbing or an amplifying one) does not change, and the delay time of its rising edge in any medium is exactly equal to the light delay time. We would like to emphasize that the delay time of the initial jump (that is, the rising edge of the signal) can not be less or more than "the light delay", that is, no "early" or "late" appearance of the rising jump is possible.

The fact that this conclusion does not refer to any particular medium (for example, collisional plasma), but it refers to any arbitrary material medium, is connected with the aspect that in the most general case (see [8]), if $\omega \to \infty$, the dielectric constant of an arbitrary medium with an accuracy up to terms of order ω_p^2/ω^2 ($\omega_p = \left(4\pi ne^2/m\right)^{1/2}$ where n - is the total content of free and bound electrons in the medium) is $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$ and, if $\omega \to \infty$, it tends to 1.

Of course, in any dispersive medium signal deformation occurs - it is distorted (usually stretched) and its amplitude decays; in an absorbing medium, in addition to this, the signal is attenuated (i.e., its energy is lost), but all this is not related to the amplitude of the signal's rising edge – the latter never changes. As an example, the figure below shows the numerical results for the time dependence of square wave signal (a) after it goes path $z = 10/k_p$ in collisionless plasma $\nu = 0$ (b) and in collisional plasma $\nu = \omega_p$ (c). "Light delay" is not shown. Line (d) shows the "linear" approximation, used to assess the time interval during which the initial amplitude of the signal remains the same.

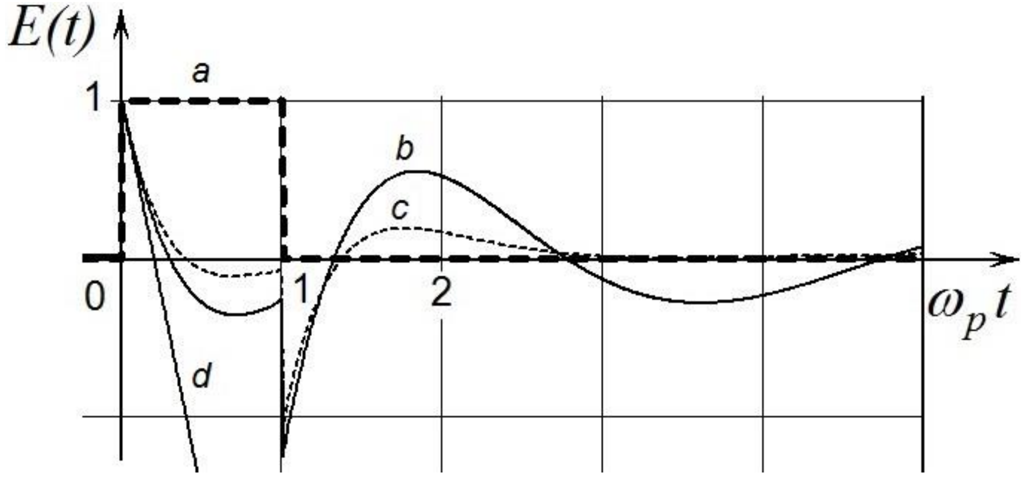

Figure 1. Time dependence of the signal

2. To estimate the duration of the time interval during which the signal which emerges as a jump maintains the amplitude close to that of the initial jump in any arbitrary medium, for the energy of the signal's rising pulse we have the following approximate estimates: $\tau_{fr} \approx 2/\left(\omega_p k_p z\right)$, $W_{fr} \approx E^2(t_0)\tau_{fr}/3 = 2E^2(t_0)/\left(3\omega_p k_p z\right)$. You can get evaluation formula $z_{def} = \left(2mc\varepsilon_0\right)/ne^2\tau_{pulse}$ for the characteristic weak distortion distance of the square-wave pulse.

3. Similarly let us consider the signals, on the rising edge of which there's a breach of time dependence not of the signal as such, but only of its derivatives of order k and higher: $E(t) = 0$ if $t < t_0$, $E(t) \neq 0$ if $t > t_0$, $E''(t_0) = 0, \ldots, E^{(k-1)}(t_0) = 0$, $E^{(k)}(t_0) \neq 0$. It is easy to show that in this case, as the signal propagates in the medium, it is the value of the jump of the derivative $E^{(k)}(t_0)$ that does not change; as a result in *the arbitrary medium* the amplitude of the rising pulse of the signal

slowly decays in accordance with the law $E_{fr} \sim E^{(k)}(t_0)/\left(\omega_p^k (k_p z)^k\right) \sim 1/\left(k_p z\right)^k$, and the duration is still determined by the law $\tau_{fr} \sim 1/\left(\omega_p k_p z\right)$.

Appendix

Let us research the Fourier spectrum $E(\omega)$ that exists for a limited period of time (within the range $[t_1, t_2]$) of smooth signal $E(t)$ with sharp rising edge t_1 and falling edge t_2 ($E(t) = 0$ if $t < t_1$ and $t > t_2$, $E(t) \neq 0$ if $t_1 < t < t_2$). Obviously $E(\omega) = \int_{t_1}^{t_2} E(t)\exp(-i\omega t)dt$,

$E(t) = (2\pi)^{-1} \int_{-\infty}^{+\infty} E(\omega)\exp(i\omega t)d\omega$. After n-fold integration by parts ($n \to \infty$), the first of these formulae can be written as an asymptotic formula $E(\omega) = -\sum_{k=0}^{\infty} \left(\exp(-i\omega t)(i\omega)^{-k-1} E^{(k)}(t)\right)\Big|_{t_1}^{t_2}$. The latter formula demonstrates a direct link between the breaches of the signal on the rising and falling edges and the asymptotic behaviour (if $\omega \to \infty$) of its spectrum. Of course, the noted connection between the asymptotics of the signal spectrum, if $\omega \to \infty$, and the character of its breaches can be used "reverse" too - the presence of relevant terms in the asymptotics of the signal's spectrum indicates that the signal itself (or its derivatives of the corresponding order) are experiencing breaches of the corresponding rank and amplitude at the relevant points of time. This fact allows, while researching the rising edge of the signal, limiting yourself to the minimum amount of information about the properties of the medium, i.e. the first few expansion terms of its refractive index in series in powers of $1/\omega$.

References

[1] Sommerfeld A 1914 *Ann. Phys.* **44** 177
[2] Brillouin L 1914 *Ann. Phys.* **44** 203
[3] Maske B and Segard B 2012 *Phys. Rev.* A **86** 013837
[4] Maske B and Segard B 2013 *Phys. Rev.* A **87** 043830
[5] Akulshin A M, McLean R J 2010 *Journal of Optics* **12** 104001
[6] Malykin G B, Romanets E A 2012 *Optics and Spectroscopy* **112** 920
[7] Bukhman N 2007 *J. Commun. Technol. Electron.* **52** 555
[8] Landau L D and Lifshits E M 1960 *Electrodynamics of Continuous Media* (Oxford: Pergamon Press).
[9] Vinogradova M B, Rudenko O V and Sukhorukov A P 1979 *The Wave Theory* (Moscow: Nauka)
[10] Smirnov V I *The higher mathematics course* 1974 vol 3 part 2 (Moscow: Nauka) p 72