Mathematical modeling of a non-Newtonian fluid flow in the main fracture inside permeable porous media

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Abstract. This paper describes a mathematical model of the main fracture isolation in porous media by water-based mature gels. While modeling injection, water infiltration from the gel pack through fracture walls is taking into account, due to which the polymer concentration changes and the residual water resistance factor changes as a consequence. The salutation predicts velocity and pressure fields of the non-Newtonian incompressible fluid filtration for conditions of a non-deformable formation as well as a gel front trajectory in the fracture.
The mathematical model of agent injection into the main fracture is based on the fundamental laws of continuum mechanics conservation describing the flow of non-Newtonian and Newtonian fluids separated by an interface plane in a flat channel with permeable walls. The mathematical model is based on a one-dimensional isothermal approximation, with dynamic parameters pressure and velocity, averaged over the fracture section.

Keywords: mathematical modeling, non-Newtonian fluid, the main fracture, porous media, the SIMPLE algorithm

1. Introduction

In current situation, one of the major oilfield problems is the water inflow restriction into the wells draining naturally fractured reservoirs and the isolation of a high-conductivity single fracture connecting production and injection wells or aquifer. The instant water-cut increase in the produced fluid is caused by formation and injected water breakthrough via high-permeability layers and fractures. Under such conditions, the objective to shut-off water inflow to production wells becomes extremely important [1-9]. The problem is particularly acute in the case of complex fractured oil reservoirs. For isolation of fractures acting as the main channels of water flow and breakthrough to production wells, a crosslinked (mature) gel is applied. Numerous experimental studies are carried out to investigate the mechanism of gel propagation through the fracture and determine the rheological and filtration properties of crosslinked polymer compositions.

The studies [3-6] shows that in the process of gel propagation through the fracture the gel dehydration, i.e., water removal, takes place. Water, separated from gel, infiltrates through the fracture walls into a porous matrix. Under the severe flow exposure conditions, gel concentration as well as gel stability increases. The gels, used for fracture water-shutoff, are crosslinked aqueous polymer solutions of low concentrations. According to rheological studies [7], a crosslinked gel is a non-Newtonian fluid which apparent viscosity decreases with a shear rate increase. The rheological characteristics of its composition are close to the pseudoplastic fluid model. Study [8] experimentally proves that the gel injection into a fracture forms two flows - gel flow and formation fluid flow-separated by a moving interface. The gel spreads in the fracture as a piston and the gravity force does not affect the shape of its front. Placement of immature and mature gels and their ability to block fractures during subsequent waterfloods were investigated in paper [9]. The immature gel and fully formed (mature) polymer gel show different behavior during placement in a fractured system, and the gels deposit differently in the fracture volume. Injection of different maturity gels at into a fracture may therefore influence the ability of gel treatment to block fractures, and hence its performance during conformance-control operations. At the same time, the problem of numerical modeling of non-Newtonian agent injection into the main fracture is very important for predicting the distribution of final injected volumes and creating stable gel barriers. This paper describes a mathematical model of the main fracture water-shutoff in the porous reservoir by the crosslinked gel. The mathematical model is based on the fundamental laws of continuum mechanics conservation describing the flow of non-Newtonian and Newtonian fluids separated by an interface in a flat channel with permeable walls. This study was targeted to optimize the design of gel placement in the fracture and to determine subsequently optimal technological parameters of the process and the size of a gel screens ensuring their stability under the excessive exposure to flow from Water-Shut-Off operations.

2 Mathematical model

Let us consider a rectangular zone in the oil reservoir of permeability k and porosity m, where a vertical main fracture of length L, width w_f and height h_f, connecting the injection and production wells, is symmetrically parallel to the reservoir boundaries. The relation between the width, height and length of the fracture can be described as $w_f << h_f << L$. The coordinate system is selected in such way that its origin coincides with the outer radius of the injection well and axis x goes along the fracture. In the main fracture, initially filled with formation water, gel injection forms two flow regions separated by moving interface $x_f(t)$. In the first region $0 \le x \le x_f(t)$, gel density is ρ_g, gel effective viscosity is μ_g. In the second region $x_f(t) \le x \le L$, formation water of constant density ρ_w and constant viscosity μ_w is displaced by gel in a piston-like manner. In the first region water infiltration from gel into the reservoir takes place, and in the second region - formation water losses or inflow, depending on the sign of pressure gradient in the direction perpendicular to the fracture walls.

The mathematical model of gel injection into a vertical fracture with permeable walls was built on the following assumptions: no mixing occurs on the formation water and gel interface; water stream lines from fracture to reservoir are straight lines in the reservoir region under review; as per rheology, the crosslinked gel is a non-Newtonian pseudoplastic liquid. The fluid flow inside the reservoir is isothermal and follows the Darcy's law; fluids in the fracture and reservoir are incompressible; and the reservoir rock matrix is rigid. The mathematical model is calculated under the hydraulic approximation with hydrodynamic parameters - pressure and velocity - averaged over the fracture cross section. It is assumed, that the main fracture is flushed out by previous flows, so its boundaries are affected only by fluid pressure and friction (the fracture walls are free of geomechanical stress).

2.1. Mass conservation equation.

The numerical modeling of gel injection into the main fracture with permeable walls requires to write down the basic laws of continuum mechanics conservation (CMC) in the hydraulic approximation describing a viscous incompressible fluid flow in the vertical fracture with permeable walls for the averaged fields:

$$\langle v \rangle = \frac{1}{S}\int_S v ds, \qquad \langle v^2 \rangle = \frac{1}{S}\int_S v^2 ds = (1+\beta)\langle v \rangle^2, \qquad \langle p \rangle = \frac{1}{S}\int_S p ds, \tag{1}$$

where β is the Coriolis correction [10].

Let us review a specific fluid volume V in the fracture, which is bounded by a piecewise-smooth surface $\Sigma = S_0 + S_1 + \Sigma_l + \Sigma_r + \Sigma_{hd} + \Sigma_{hup}$, where: S_0, S_1 are inflow and outflow boundaries of the specific volume, respectively; Σ_l, Σ_r are left and right (flow-wise) porous boundaries of the specific volume; $\Sigma_{hd}, \Sigma_{hup}$ are lower and upper (reservoir base and top) impermeable boundaries of the specific volume. All boundaries in the reviewed volume are assumed as flat.

To derive the integral mass conservation law for the selected specific volume V of a rectangular parallelepiped shape we applies the known continuum mechanics theorem about the time-derivative of the tensor quantities integral over a mobile volume [11]:

$$\frac{d}{dt}\int\limits_{V(t)}\rho\, dV = \int\limits_{V(t)}\frac{\partial\rho}{\partial t}dV + \int\limits_{S_1}\rho v_n\, d\Sigma + \int\limits_{S_0}\rho v_n\, ds = J_m = -\int\limits_{\Sigma_l}\rho u_l\, ds - \int\limits_{\Sigma_r}\rho u_r\, ds, \tag{2}$$

where ρ is fluid density; v_n is the fluid particle velocity component, normal to the cross section; J_m is fluid mass flow into the reservoir through fracture sidewalls; u_r, u_l are the rate components of the outflow (inflow) from (into) the reservoir, normal to the left and right fracture walls. Equation (2) assumes flow absence at the reservoir top and base. For the one-dimensional flow of gel in the region $0 \le x \le x_f(t)$ and the symmetric infiltration $u_l = u_r = u_g$ equation (2) can be written as

$$\frac{\partial \rho_g}{\partial t} + \frac{\partial(\rho_g v)}{\partial x} = -\frac{2\rho_g u_g}{w_f}. \tag{3}$$

For water infiltration from gel, the rate is determined from the experimental data [3], approximation of which resulted in the power law of infiltration rate vs. time with a negative exponent. The infiltration rate in the International System of Units is defined as:

$$u_g = At^{-\alpha}, \quad A = 1{,}764 \cdot 10^{-7}\ m \cdot c^{\alpha-1}, \quad \alpha = 0{,}55. \tag{4}$$

The next assumption is that the crosslinked gel is a binary water solution of polymer. Then, by definition, the "reduced" density of polymer ρ_p and water ρ_w in aqueous polymer solution is:

$$\rho_p = \frac{M_p(t)}{V(t)}, \qquad \rho_w = \frac{M_w(t)}{V(t)}, \quad \rho_g = \rho_p + \rho_w, \tag{5}$$

where $M_p(t)$ and $M_w(t)$ are polymer and water masses in the gel material volume, respectively, and ρ_g is gel density. The experimental data [3] show that the crosslinked gel being injected behaves like a "sponge", from which into the reservoir only water infiltrates while polymer retains its mass in the specific volume of crosslinked gel: $M_p(t)$ = const.

Using the mass concentrations for the binary mixture components:

$$c_p = \frac{M_p}{M(t)} = \frac{\rho_p V(t)}{\rho V(t)} = \frac{\rho_p}{\rho_g}, \quad c_w = \frac{M_w}{M(t)} = \frac{\rho_w V(t)}{\rho V(t)} = \frac{\rho_w}{\rho_g}, \quad c_p + c_w = 1. \tag{6}$$

The reduced water and gel density can be described as:

$$\rho_w = (1-c)\rho_g(c), \quad \rho_p = c\rho_g(c), \quad c = c_p. \tag{7}$$

Using the reduced water and gel densities (7), the equation of continuity (3) for gel only in the flow region $0 \le x \le x_f(t)$ is reduced to an equivalent system for a binary mixture:

$$\frac{\partial \rho_g(c)}{\partial t} + \frac{\partial \rho_g(c) v}{\partial x} = -\frac{2(1-c)\rho_g(c) u_g}{w_f}, \tag{8}$$

$$\frac{\partial c \rho_g(c)}{\partial t} + \frac{\partial c \rho_g(c) v}{\partial x} = 0, \tag{9}$$

where equation (8) is the continuity equation for a binary mixture (gel), and equation (9) is the continuity equation for the polymer component of gel.

The application of continuity equation (2) to the specific volume, occupied by formation water in the flow region $x_f(t) \le x \le L$, leads to the equation:

$$\frac{\partial v}{\partial x} = -\frac{2u_w}{w_f}, \tag{10}$$

where water incompressibility in the fracture is $\rho_w =$ const, u_w is the rate of formation water infiltration through the flat channel walls.

The formation water infiltration rate through the fractures face is determined from the Darcy's law:

$$u_w(x,t) = \frac{k}{\mu_w L_k}(p(x,t) - p_k), \tag{11}$$

where k is reservoir permeability; μ_w is formation water viscosity; p_k is constant pressure at the external reservoir boundary at distance L_k from the fracture walls.

2.2. Equation of a fluid momentum conservation in a flat channel

Given that the momentum change in the specific liquid volume equal to the sum of all external surface forces and mass forces [10], we have

$$\int_{V(t)} \frac{\partial(\rho\vec{v})}{\partial t} dV + (1+\beta)\left[\int_{S_0} \rho\vec{v}(\vec{v}\cdot\vec{n})d\Sigma + \int_{S_1} \rho\vec{v}(\vec{v}\cdot\vec{n})d\Sigma\right] = \int_{S_0} \vec{p}_n d\Sigma + \int_{S_1} \vec{p}_n d\Sigma + \int_{\Sigma_l} \vec{p}_n d\Sigma + \int_{\Sigma_r} \vec{p}_n d\Sigma, \tag{12}$$

where $\vec{p}_n -$ tension vector on the surface Σ at the point with normal $\vec{n}$; $\vec{g}$ $-$ gravitational acceleration.

The momentum equation (12) for the unsteady gel one-dimensional flow in a flat channel $0 \le x \le x_f(t)$ with permeable walls is reduced to a form:

$$\frac{\partial(\rho_g v)}{\partial t} + (1+\beta_g)\frac{\partial(\rho_g v^2)}{\partial x} = -\frac{\partial p}{\partial x} - \frac{(1-m)C_g \rho_g v^2}{w_f}. \tag{13}$$

Similarly, given the formation water incompressibility in the formation water flow region $x_f(t) \le x \le L$, the equation of motion is:

$$\frac{\partial v}{\partial t}+(1+\beta_w)\frac{\partial v^2}{\partial x}=-\frac{1}{\rho_w}\frac{\partial p}{\partial x}-\frac{(1-m)C_w v^2}{w_f}\ , \tag{14}$$

In equations (13) and (14), β_g, β_w are the Coriolis corrections for gel and water, C_g, C_w are hydraulic friction coefficients of gel and water, respectively. The viscous stress on the walls of the flat channel is determined by hydraulic friction coefficients.

2.3. Key model equations

The experimental data show [3] that if the initial polymer concentrations are $c = 0.001 \div 0.005$ then resulting from water infiltration from crosslinked polymer the gel concentration in aqueous solution can increase by an order of magnitude and achieve $c = 0.01 \div 0.05$. The aqueous polymer solution density varies in the range of $1-2\,\%$. If the initial solution concentration is $\rho_g = 1000$ kg/m^3, the solution density upon water infiltration changes by $\Delta\rho = 10 - 20$ kg/m^3. Therefore, it can be estimated that

$$\Delta\rho \big/ \rho_g << 1, \tag{15}$$

allowing to apply an incompressible fluid approximation for describing gel injection. Thus, according to equation (15) for the crosslinked gel flow region $0 \le x \le x_f(t)$ in the fracture, equations (8), (9), (13) are reduced to the following form:

$$\frac{\partial v}{\partial x}=-\frac{2(1-c)At^{-\alpha}}{w_f}, \tag{16}$$

$$\frac{\partial c}{\partial t}+\frac{\partial (cv)}{\partial x}=0, \tag{17}$$

$$\frac{\partial v}{\partial t}+(1+\beta_g)\frac{\partial v^2}{\partial x}=-\frac{1}{\rho_g}\frac{\partial p}{\partial x}-\frac{(1-m)C_g v^2}{w_f} \tag{18}$$

For the formation water flow region $x_f \le x \le L$ in the fracture according to equation (15), equations (10) and (14) are reduced to the following form:

$$\frac{\partial v}{\partial x}=-\frac{2k(p(x,t)-p_k)}{w_f \mu_w L}\,. \tag{19}$$

$$\frac{\partial v}{\partial t}+(1+\beta_w)\frac{\partial v^2}{\partial x}=-\frac{1}{\rho_w}\frac{\partial p}{\partial x}-\frac{(1-m)C_w v^2}{w_f}\,. \tag{20}$$

When the crosslinked gel injecting into the fracture the regions, occupied by gel and formation water, change over time, which can be fully described by the law of gel - formation water interface movement. Therefore, the differential equation for the interface paths is defined to close the model using the following initial assumption at fracture inlet (walls of the injection well):

$$\frac{dx_f(t)}{dt}=v(x_f(t))\,. \tag{21}$$

2.4. Closing relations, initial and boundary conditions

The experimental studies on the crosslinked gel injection into the fracture [3-8] show that under the applicable gel injection rates the gel flow in fractures is generally characterized by the laminar flow regime.

To solve the gel flow equation (18), the hydraulic friction coefficient was calculated using equation [12]:

$$C_g(\mathrm{Re}) = \frac{16}{\mathrm{Re}_g} 2^{n-3} \left(\frac{3n+1}{n} \right)^n, \qquad \mathrm{Re}_g = \rho D_H^n v^{2-n} / K \tag{22}$$

where K is a consistency index for power fluid, $n \in (0,1)$ is the non-Newtonian degree of pseudoplastic fluid.

Equation (20) is closed by defining the friction coefficient C_w for the laminar flow regime ($R_e \le 2000$) [13]:

$$C_w = \frac{24}{\mathrm{Re}_w}, \quad \mathrm{Re}_w = \frac{\rho_w v D_H}{\mu_w}, \quad D_H = \frac{4 w_f h_f}{2(w_f + h_f)} \approx 2 w_f \tag{23}$$

The approximate equality in equation (23) follows from the assumption that $w_f << h_f$.

In our study we assume that the production well is shut down and therefore the liquid in the main fracture initially is at rest, and the fracture pressure is equal to pore pressure. The initial conditions for the equations (16) - (21) are:

$$v(x,0) = 0, \qquad p(x,0) = p_k, \qquad c(x,0) = 0, \qquad x_f(0) = 0. \tag{24}$$

Gel of initial polymer concentration c_0 is injected into the fracture at a constant rate Q. The boundary conditions at the main fracture entry are:

$$v(0,t) = \frac{Q}{w_f h_f}, \qquad c(0,t) = c_0, \quad p(L,t) = p_k, \qquad \frac{\partial c(L,t)}{\partial x} = 0. \tag{25}$$

3. Results of numerical calculations

For the numerical solution of equations (16) - (25) by the finite volume approach the SIMPLE algorithm was updated [14] to be applied to the problem of non-stationary Newtonian fluid displacement by non-Newtonian power fluid in a flat channel with permeable walls. The numerical calculations used the following Coriolis parameters $\beta_g = \beta_w = 0$. The results of calculations for the reservoir of permeability $k = 32.7$ mD and porosity $m = 0.2$ are shown on Figures 1-4.

For the numerical calculations the accepted length of main fracture is L = 300 m, and its height is $h_f = 27$ m. The fracture is symmetrical with respect to the boundaries of the considered rectangular reservoir region and the distance from the fracture walls to the reservoir boundaries is $L_k = 50$ m. Reservoir pressures at the boundaries of the reviewed region, left and right of the fracture, are constant and equal $p_k = 7$ MPa. Formation water density is $\rho_w = 1010$ kg/m^3, its viscosity is taken as μ_w = 0.001 Pa·s. Gel consistency index is $K = 0.178$, non-Newtonian exponent is $n = 0.72$. The average fracture width is $w_f = 0.005$ m. The gel injection rate is $Q = 88.6$ m^3/day. The initial mass concentration of crosslinked polymer is $c_0 = 0.003$.

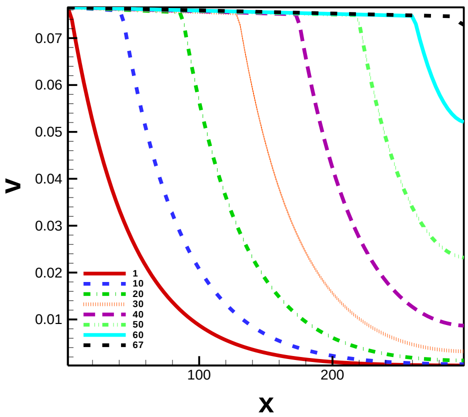

Fig. 1. Velocity (m/s) vs. fracture length (m) curves at time intervals $t = 10$ min.

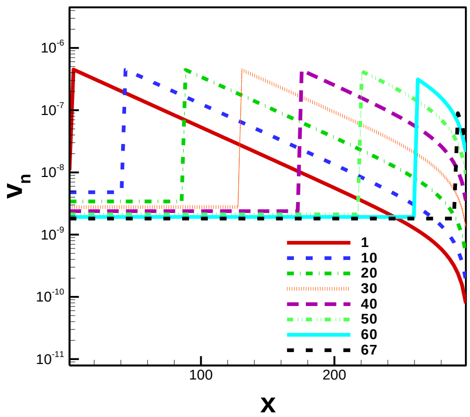

Fig. 2. Water infiltration velocity (m/s) vs. fracture length (m) at time intervals $t = 10$ min.

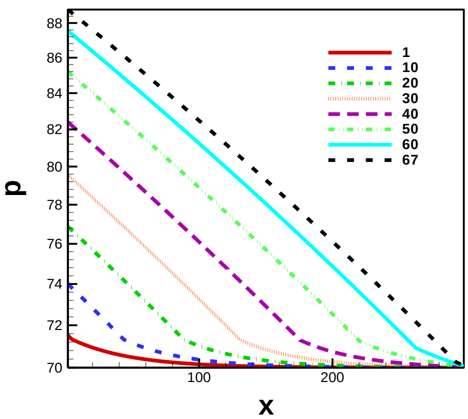

Fig. 3. Pressure (atm) vs. fracture length (m) at time intervals $t = 10$ min.

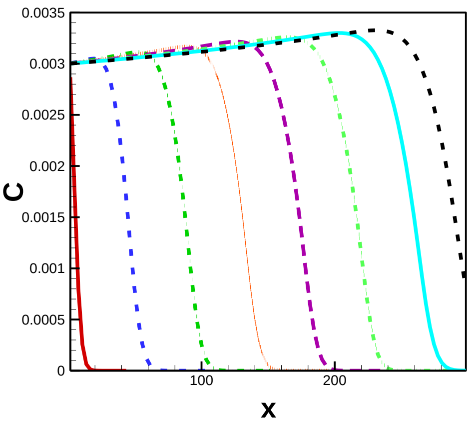

Fig. 4. Polymer concentration vs. fracture length (m) at time intervals $t = 10$ min.

Figure 1 shows the flow velocity behavior along the fracture length. In the gel flow region the rate is almost constant, but in the formation water region the rate drops due to its infiltration into the formation. The sharp bend of the velocity profile corresponds to the gel - formation water contact in the fracture. Figure 2 shows the rates of infiltration into formation both for water from gel and for formation water. Here, the shift of velocity gaps is also consistent with movement of the gel-water interface. It can be seen from the Figure 2 that in the process of fluid flow though the fracture the fluid loss velocity decreases. Figure 3 shows the profile of flow velocity vs. fracture length. At the first time points of the flow the pressure profile has a curved shape, but then, as gel fills the fracture, pressure profile is straightened and, when gel reaches the production well, it becomes almost linear. Figure 4 shows the crosslinked polymer concentration variation in gels. This calculation gave about 10% difference in initial and final concentrations of the crosslinked polymer. Even so, changes in crosslinked polymer concentrations depends on many factors including the injection rate, initial polymer concentration in the aqueous solution, crosslinker concentration, and rock permeability. Affecting by all these factors, the change in crosslinked polymer concentration with the flow time in the fracture can be significant, and in this case we must take into account the effect of gel rheological properties on the on-going crosslinked polymer concentration. The numerical simulation for the above-described mathematical model allows providing optimal technological parameters for successful gel placement in the main fractures.

4. Conclusion

We presented a mathematical model of pseudoplastic gel injection into the main fracture for the purpose of its isolation. The mathematical model can be easily generalized for considering the formation fluid and rock matrix compressibility, as well as the dependence of effective gel viscosity on the ongoing crosslinked polymer concentration and destruction of gel. The gel placement in the fracture is an important problem, based on it solutions the stability of gel barriers in the fracture can be solved if the limiting shear gradients are known for the applied gels.

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