The relationship between trading volumes, number of transactions, and stock volatility in GARCH models

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Abstract. We examine the relationship between trading volumes, number of transactions, and volatility using daily stock data of the Tokyo Stock Exchange. Following the mixture of distributions hypothesis, we use trading volumes and the number of transactions as proxy for the rate of information arrivals affecting stock volatility. The impact of trading volumes or number of transactions on volatility is measured using the generalized autoregressive conditional heteroscedasticity (GARCH) model. We find that the GARCH effects, that is, persistence of volatility, is not always removed by adding trading volumes or number of transactions, indicating that trading volumes and number of transactions do not adequately represent the rate of information arrivals.

1. Introduction

The empirical properties of asset returns have been intensively studied, and some universal properties are classified as "stylized facts" [1]. The notable stylized facts include (1) no significant autocorrelation in returns, (2) long autocorrelation in absolute returns, (3) fat-tailed return distributions, and (4) volatility clustering. The return dynamics explaining these stylized facts have been the subject of numerous studies. Assuming that the return dynamics can be described by a Gaussian random walk with time-varying volatility, one possible explanation is that $r_t = \sigma_t \epsilon_t$, where r_t is a return, σ_t^2 represents volatility, and ϵ_t is a random variable from $N(0,1)$ at time t. Several studies have verified this assumption [2]–[8] by examining whether r_t/σ_t is consistent with the random variable $\sim N(0,1)$.

Yet another unresolved issue relates to volatility dynamics. Under the mixture of distributions hypothesis (MDH) proposed by Clark [9], volatility dynamics is related to the rate of information arrivals to the market. Since the rate of information arrivals is latent and unobservable, Clark used trading volume as a proxy for the rate of information arrivals. Empirical evidence indicates the existence of a contemporaneous correlation between volatility and trading volume; see, for example, [10].

On the other hand, the dynamic behavior of volatility is well captured by the autoregressive conditional heteroscedasticity (ARCH) model [11] and its extension, the generalized ARCH (GARCH) model [12]. In particular, the GARCH model successfully captures the persistence of volatility variation, referred to as GARCH effects. In the GARCH model, the volatility process is described by a function of past volatilities and returns. The MDH also implies that the volatility process is described by a function of trading volume. Lamourex and Lastraps [13] inserted trading volume into the GARCH process by using individual stocks in the US market and found that the GARCH effects disappear, supporting the MDH. Some subsequent studies,

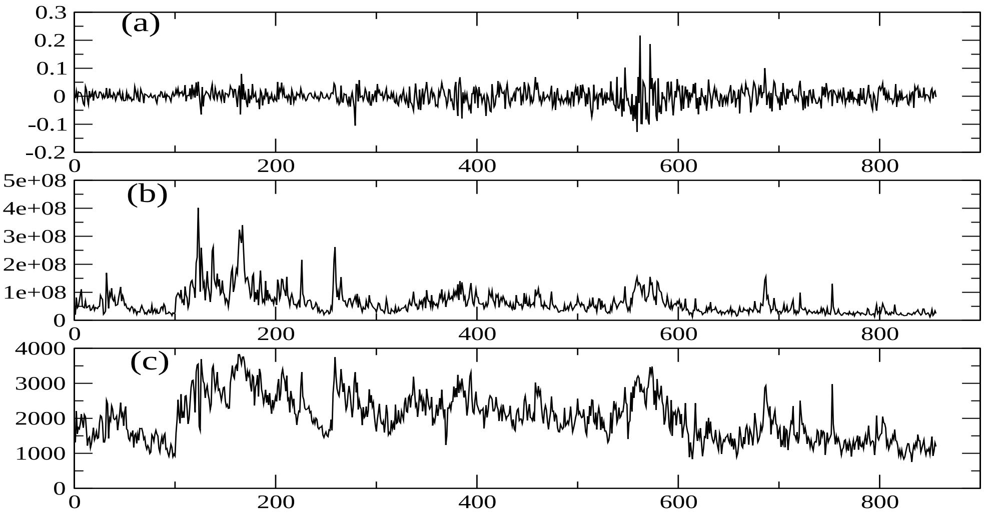

Figure 1. Time series of (a) stock returns, (b) trading volume, and (c) number of transactions for Nippon Steel Co.

for example, [14, 15, 16, 17], also support their finding, that is, that the inclusion of trading volume in the GARCH model reduces the GARCH effects. On the other hand, some other studies, such as [18, 19, 20, 21, 22], report that the inclusion of trading volume in the GARCH model does not completely remove the GARCH effects; thus, the MDH is not supported.

In order to elaborate the volatility dynamics, we examine the relationship between trading volume and stock volatility by using the daily stock data of the Tokyo Stock Exchange from June 3, 2006, to December 30, 2009. Specifically, by including trading volumes into the GARCH process, we can infer the GARCH parameters and examine whether the GARCH effects can be explained by trading volume. We also use the number of transactions as a proxy for the rate of information arrivals and examine its effect on GARCH volatility.

2. GARCH Test

We focus on the GARCH(1,1) model [12] described by $r_t = \sigma_t \epsilon_t$, and

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{1}$$

where α, β and ω are the GARCH parameters to be determined. The magnitude of persistence of volatility, that is, the GARCH effects, is measured by $\alpha + \beta$, and for high persistence of volatility, we observe that $\alpha + \beta$ is close to 1. The effect of trading volume or the number of transactions is examined by adding a term to the GARCH process, as

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma N_t, \tag{2}$$

where N_t stands for either the trading volume or number of transactions at time t. We infer the GARCH parameters by the Bayesian inference conducted using the Markov Chain Monte Carlo (MCMC) method [23]–[28].

3. Empirical Study

Our analysis is based on four individual stock data, (1) Astellas Pharma Inc., (2) JFE Steel Co., (3) Nippon Steel Co., and (4) Seven & i Holdings Co., on the Tokyo Stock Exchange. The sample period of our data is from June 3, 2006, to December 30, 2009. The stock return is defined by the log-price difference: $r_t = 100 \times (\ln P_t - \ln P_{t-1})$, where P_t is the closing stock price at day t. Figure 1 shows the time series of (a) returns, (b) trading volume, and (c) number

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Table 1. GARCH parameter results without trading volume and number of transactions. SD stands for standard deviation.

	α	β	ω	$\alpha+\beta$
Astellas Pharma Inc.	0.095	0.857	0.188	0.952
SD	0.018	0.026	0.066	
JFE Steel Co.	0.096	0.895	0.126	0.991
SD	0.019	0.020	0.065	
Nippon Steel Co.	0.159	0.825	0.202	0.984
SD	0.031	0.032	0.086	
Seven & i Holdings Co.	0.117	0.873	0.0761	0.990
SD	0.027	0.028	0.0324	

Table 2. GARCH parameter results with trading volume.

	α	β	ω	γ	$\alpha+\beta$
Astellas Pharma Inc.	0.232	0.251	0.043	1.97	0.483
SD	0.045	0.093	0.043	0.38	
JFE Steel Co.	0.108	0.851	0.046	0.373	0.959
SD	0.025	0.046	0.045	0.232	
Nippon Steel Co.	0.177	0.790	0.102	0.241	0.967
SD	0.037	0.043	0.081	0.131	
Seven & i Holdings Co.	0.166	0.786	0.0233	0.230	0.952
SD	0.045	0.064	0.0257	0.136	

of transactions for Nippon Steel Co. as a representative case. We find no strong correlation between the returns and trading volume or number of transactions. The correlation coefficient ρ between the returns and trading volume (number of transactions) is estimated to be 0.14 (0.02). On the other hand, we find a strong correlation between trading volume and number of transactions, $\rho \sim 0.84$.

For parameter estimations, we use the trading volume normalized by its average. Similarly, we use the number of transactions normalized by its average. We perform our GARCH parameter estimation in this study using the MCMC method based on the Bayesian inference. The MCMC method we use is the Metropolis–Hastings algorithm with a multivariate Student's t-proposal density, which has been shown to be particularly efficient for GARCH parameter estimations [23]-[28]. After the first 5000 Monte Carlo samples are discarded as "burn-in" or "thermalization" process, we collect 50000 samples for analysis.

Table 1 shows the GARCH parameter results. For all stocks, we find that $\alpha+\beta$ is close to 1, implying that a strong persistence of volatility, in other words, the GARCH effect, exists.

Table 2(3) shows the GARCH parameter results with trading volume (number of transactions). We find that even after including the trading volume or number of transactions in the GARCH process, the value of $\alpha+\beta$ does not change much, except for Astellas Pharma Inc. Moreover, we find that γ is always positive, indicating positive correlations between volatility and the trading volume or number of transactions.

4. Conclusions

We examined the relationship between stock volatility and trading volumes or number of transactions by using four individual stock data of the Tokyo Stock Exchange from June 3, 2006, to December 30, 2009. We find that including the trading volume or number of transactions in the GARCH process does not always reduce the value of $\alpha+\beta$, that is, the magnitude of the GARCH effects. Thus, the mixture of the distributions hypothesis using trading volumes or

Table 3. GARCH parameter results with number of transactions.

	α	β	ω	γ	$\alpha+\beta$
Astellas Pharma Inc.	0.194	0.588	0.052	0.827	0.782
SD	0.049	0.127	0.054	0.370	
JFE Steel Co.	0.100	0.872	0.050	0.233	0.972
SD	0.021	0.028	0.047	0.132	
Nippon Steel Co.	0.168	0.797	0.088	0.249	0.965
SD	0.034	0.041	0.079	0.150	
Seven & i Holdings Co.	0.132	0.846	0.0313	0.099	0.978
SD	0.033	0.037	0.0317	0.061	

number of transactions as a proxy for the rate of information arrivals is not completely verified. Since our findings are based on only four individual stock data, it might be interesting to further investigate the robustness of the volatility dynamics using other stock data.

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