

Comparative Study of Shrinkage and Non-Shrinkage Model of Food Drying

N. Shahari¹, N. Jamil¹, KA. Rasmani¹

¹Faculty of Computer and Mathematical Sciences,
Universiti Teknologi MARA Negeri Sembilan, Malaysia

Email:norazni@ns.uitm.edu.my

Abstract. A single phase heat and mass model has always been used to represent the moisture and temperature distribution during the drying of food. Several effects of the drying process, such as physical and structural changes, have been considered in order to increase understanding of the movement of water and temperature. However, the comparison between the heat and mass equation with and without structural change (in terms of shrinkage), which can affect the accuracy of the prediction model, has been little investigated. In this paper, two mathematical models to describe the heat and mass transfer in food, with and without the assumption of structural change, were analysed. The equations were solved using the finite difference method. The converted coordinate system was introduced within the numerical computations for the shrinkage model. The result shows that the temperature with shrinkage predicts a higher temperature at a specific time compared to that of the non-shrinkage model. Furthermore, the predicted moisture content decreased faster at a specific time when the shrinkage effect was included in the model.

1. Introduction

The single phase heat and mass model, which is modelled as a homogeneous medium, has been widely used to describe the movement of water and heat during the drying of food [1-6]. In this case, moisture transport involves two dependent processes: the evaporation of moisture (in terms of liquid water) at the solid surface that needs heat from the drying air and the internal diffusion of liquid from the centre of the fruit to the surface. Overall, there are many advantages to using a single phase heat and mass model to capture the behaviour of heat and mass during drying for use in the development of enhanced food processing. Firstly, these models are generally fairly simple and contain few parameters. It is also relatively straightforward to include changes in shape and size as a result of water removal and internal collapse during the drying process. This may arise from a moisture gradient within the particle that induces microstructure stresses, leading to shrinkage [7].

Appearance analyses made by consumers on dried fruits relate to visual and physical appearance [8]. One of the physical characteristics is shrinkage: it is rarely possible to ignore shrinkage in the food material and this physical change must be taken into account in evaluating the profiles of moisture and temperature. Since these structural changes and the transport process are involved simultaneously and continuously during the drying process, several researchers have developed a shrinkage equation by transforming the space coordinate in the equation of heat and mass transport (for example see [6, 9, 10]). Other researchers have developed the shrinkage equation



experimentally and then conducted the calculation at each time step in the equation of the heat and mass transport model [2, 11, 12]. In recent years there have been many studies of the heat and mass transfer model with the inclusion of shrinkage, such as the reviews by Katekawa and Silva [13] and Mayor and Sereno [7]. However, a comparison of shrinkage and non-shrinkage that could affect the accuracy of the distribution of the moisture and temperature profile has been little investigated.

The purpose of this present study is to make a comparative study by neglecting or considering the shrinkage effect subject to the convective drying process, based on a single phase continuum model of heat and mass transport [1, 14]. With respect to the importance of obtaining a product with the desired quality, the effect of the drying air temperature to evaluate the moisture and temperature profile is also investigated.

2. Mathematical Model Formulation

A one-dimensional model is used: in this case, an infinite slab of finite thickness L , with moisture content $M(x,t)$ across the slab and unsteady temperature $T(x,t)$ is expressed by the well-known system of partial differential equations (PDEs) for moisture and energy transport [2]. The fruit sample is hygroscopic porous media, yet is assumed to be on a fictitious continuum. Weak internal evaporation and the transport of vapour within the dehydrated fruits towards the external food surface have not been considered. Non-dimension scaled variables are used in the formulation: a representative diffusion timescales, scaled moisture content, and scaled temperature are defined by

$$\tau = \frac{D_0 t}{L_0^2}, \quad \bar{M} = \frac{M}{M_0}, \quad \bar{T} = \frac{T - T_0}{T_{air} - T_0}, \quad \xi = \frac{x}{L_0} \text{ and } s(\tau) = \frac{L(t)}{L_0} \quad (1)$$

2.1. Continuum model (Fixed Volume Model)

Shahari and Hibberd [1] consider models with no shrinkage ($x = L_0$) associated with isothermal and non-isothermal diffusion and constant moisture diffusivity ($D = D_0$), with the governing of heat and mass transfer model as,

$$\frac{\partial \bar{M}(\xi, \tau)}{\partial \tau} = \frac{\partial^2 \bar{M}(\xi, \tau)}{\partial \xi^2}, \quad \frac{\partial \bar{T}(\xi, \tau)}{\partial \tau} = Le \frac{\partial^2 \bar{T}(\xi, \tau)}{\partial \xi^2} \quad (2)$$

$$\text{at } \tau = 0: \quad \bar{M} = 1 \quad \text{and} \quad \bar{T} = 0 \quad (3)$$

$$\text{at } \xi = 0: \quad \frac{\partial \bar{M}}{\partial \xi} = 0, \quad \text{and} \quad \frac{\partial \bar{T}}{\partial \xi} = 0, \quad (4)$$

$$\text{at } \xi = 1: \quad \frac{\partial \bar{M}}{\partial \xi} = -Bi_m (\bar{C}_{sur} - 1) \quad \text{and} \quad \frac{\partial \bar{T}}{\partial \xi} = -Bi(T_{sur} - 1) + \bar{\lambda} \frac{\partial \bar{M}}{\partial \xi} \quad (5)$$

2.2. Continuum model with shrinkage effect

During drying, the shape and size of the food particles are constantly changing as a result of water removal and internal collapse. Shahari and Hibberd [6] consider a model with interface $L(t)$ decreasing with drying and, using the transformation $\xi = \frac{x}{L(t)}$, the surface interface corresponds to a fixed value $\xi = 1$. Using the same non-dimension scale variable as above, the resulting system for moisture, temperature and shrinkage becomes

$$\frac{\partial \bar{M}}{\partial \tau} = \frac{1}{s^2(\tau)} \frac{\partial^2 \bar{M}}{\partial \xi^2} + \xi \frac{1}{s(\tau)} \frac{ds}{d\tau} \frac{\partial \bar{M}}{\partial \xi}, \quad (6)$$

$$\frac{\partial \bar{T}}{\partial \tau} = \frac{1}{s^2(\tau)} Le \frac{\partial^2 \bar{T}}{\partial \xi^2} + \xi \frac{1}{s(\tau)} \frac{ds}{d\tau} \frac{\partial \bar{T}}{\partial \xi}, \quad (7)$$

$$\frac{ds}{d\tau} = \frac{\rho_s}{\rho_w} \frac{M_0}{s(\tau)} \frac{\partial M}{\partial \xi}. \quad (8)$$

Initial conditions associated with constant conditions

$$\tau = 0: \bar{T}(\xi, 0) = 0, \quad \bar{M}(\xi, 0) = 1, \quad s(0) = 1. \quad (9)$$

Taking symmetry boundary conditions in the mid-plane of the drying slice gives

$$\xi = 0: \quad \frac{\partial \bar{M}}{\partial \xi} = 0, \quad \text{and} \quad \frac{\partial \bar{T}}{\partial \xi} = 0. \quad (10)$$

At the surface, moisture and temperature boundary conditions of the drying body in contact with drying air become

$$\xi = 1: \frac{\partial \bar{M}}{\partial \xi} = -s(\tau) Bi_m C_{air} \left(\frac{C_{sur}}{C_{air}} - 1 \right), \quad (11)$$

$$\frac{\partial \bar{T}}{\partial \xi} = s(\tau) Bi(1 - \bar{T}_{sur}) + \lambda \frac{\partial \bar{M}}{\partial \xi}. \quad (12)$$

In the above, several dimensionless controlling parameters are defined by

$$Bi_m = \frac{h_m L_0}{DM_0}, \quad Bi = \frac{hL_0}{k}, \quad Le = \frac{\alpha}{D_0}, \quad \lambda = \frac{H_v M_0 D_0 \rho_s}{k(T_{sur} - T_0)}.$$

Where M is the local moisture content (dry base), T is the local temperature, D_0 represents the effective diffusion coefficient, ρ_s is the density of dry solid, C_p heat capacity, k thermal conductivity, x is thickness and t the time. H_v is the heat of vaporization, T_{air} is air temperature and T_{sur} is the surface temperature of the fruit, h is the heat transfer coefficient, h_m is the mass transfer coefficient, T_0 is the initial temperature, M_0 is the initial moisture. C_{air} is the concentration of moisture in the air and C_{sur} is the concentration in the form of liquid water film at the food surface (example: see[15]) and

$\alpha = \frac{k}{\rho_s C_p}$ is thermal diffusivity.

3. Numerical procedures

A numerical solution for equation (2) was obtained using the method of lines [16]. Discretizing the fixed integration region $0 < \xi < L$ into N subinterval gives $N+1$ ordinary differential equation. This system was then solved numerically using MATLAB ODE45 solver, which is based on an order-5 Runge Kutta-Fehlberg method (RK45).

Equations (6) and (7) were solved together with (8), which involves a shrinkage equation obtained using the method of lines [16]. This is the case of the Stefan problem in which the interface condition is associated with melting at a moving interface. Discretising equations (6) and (7) with a central difference approximation gives,

$$\frac{d\bar{M}_j}{d\tau} = \frac{1}{s(\tau)^2} \left(\frac{\bar{M}_{j-1} - 2\bar{M}_j + \bar{M}_{j+1}}{(\Delta\xi)^2} \right) + \xi_j \frac{1}{s(\tau)} \frac{ds}{d\tau} \left(\frac{\bar{M}_{j+1} - \bar{M}_{j-1}}{2\Delta\xi} \right) \quad j = 1, 2, 3, \dots, N+1 \quad (13)$$

$$\frac{d\bar{T}_j}{d\tau} = \frac{1}{s(\tau)^2} \left(\frac{\bar{T}_{j-1} - 2\bar{T}_j + \bar{T}_{j+1}}{(\Delta\xi)^2} \right) + \xi_j \frac{1}{s(\tau)} \frac{ds}{d\tau} \left(\frac{\bar{T}_{j+1} - \bar{T}_{j-1}}{2\Delta\xi} \right) \quad j = 1, 2, 3, \dots, N+1 \quad (14)$$

A higher order discretization of the gradient term at the boundary is taken in line with the recommendation for the analogous Stefan problem by Furzeland (in Crank [17]). Discretization of the shrinkage condition in equation (8) is given by

$$\frac{ds}{d\tau} = \frac{\rho_s}{\rho_w} \frac{M_0}{s(\tau)} \left(\frac{3\bar{M}_N - 4\bar{M}_{N-1} + \bar{M}_{N-2}}{2\Delta\xi} \right) \quad (15)$$

For drying simulation, the input parameter is given in table 1 for generic drying conditions for fruit. Based on this input parameter, the following reference non-dimension parameter was obtained.

Table 1. Input parameter used in the simulation

$Bi=0.3$	$Bi_m=20$	$M_0=0.8$ kg water/kg moist sample
$\lambda=0.3$	$Le=5$	
$T_{air}=60^\circ\text{C}$	$H_v=2345\text{KJ/kg}$	$D_0=8 \times 10^{-10} \text{m}^2/\text{s}$
$RH=20\%$	$\rho_s=1080 \text{kg/m}^3$	$h=20\text{-}250 \text{W/m}^2\text{K}$
$L_0=5 \text{mm}$	$\rho_w=1000 \text{kg/m}^3$	$h_m=2 \times 10^{-6} \text{m/s}$
$C_p=2.344\text{KJ/kg K}$	$\alpha= k/\rho_s C_p= 4 \times 10^{-9} \text{m}^2/\text{s}$	$k=0.475\text{W/mK}$

4. Result and Discussion

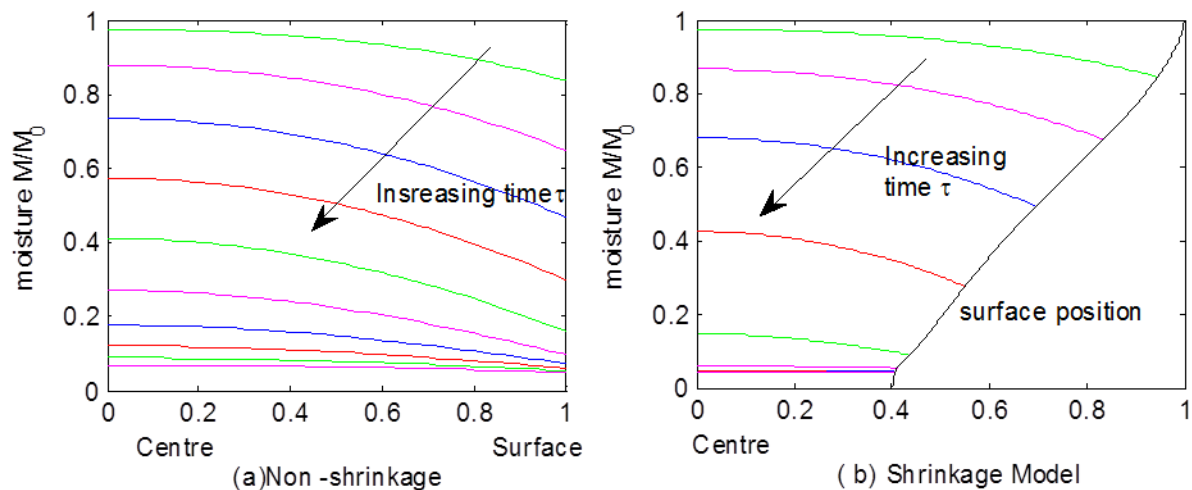


Figure 1: Profile of moisture through the sample with elapsed time $\tau=0\text{-}3$ in step 0.25 for (a) Non-shrinkage model (b) shrinkage model

Figure 1 shows the moisture profile through the sample of the fruit with increasing time for the shrinkage and non-shrinkage models. Moisture decreased as time increased, but this process was a little slower for the non-shrinkage model compared to the shrinkage model. Figure 1(b) also includes the location of the free surface during shrinkage; the shrinkage of the product is shown by the change in the thickness of the fruit. The final value of shrinkage in this model can be determined readily as only solid mass is left at the end of drying, which in this case is 0.4. A similar pattern profile of shrinkage has been shown by other authors (for example see Chemkhi et al. [9], Crapiste et al. [18]).

However, Chemkhi et al [9] consider elastic deformation and Crapiste et al. [18] employ density values to account for shrinkage velocity.

Figure 2 shows a temperature profile in the food as a function of both position and time during drying for the non-shrinkage and the shrinkage model. The temperature at each location increases with the drying time, which is due to the higher drying temperature. It was found that the temperature profile rises rapidly in the early period of heating due to the difference between the air temperature and the food temperature. As the heating period progresses, the rise in temperature attains an almost uniform profile. At the end of drying, the temperature rises rapidly throughout the food as only small quantities of heat are needed for evaporation. The overall dimension varies with time depending on the amount of moisture loss for each section, which can be seen in figure 2(b). The temperature distribution increased much faster for the shrinkage model compared with the non-shrinkage model.

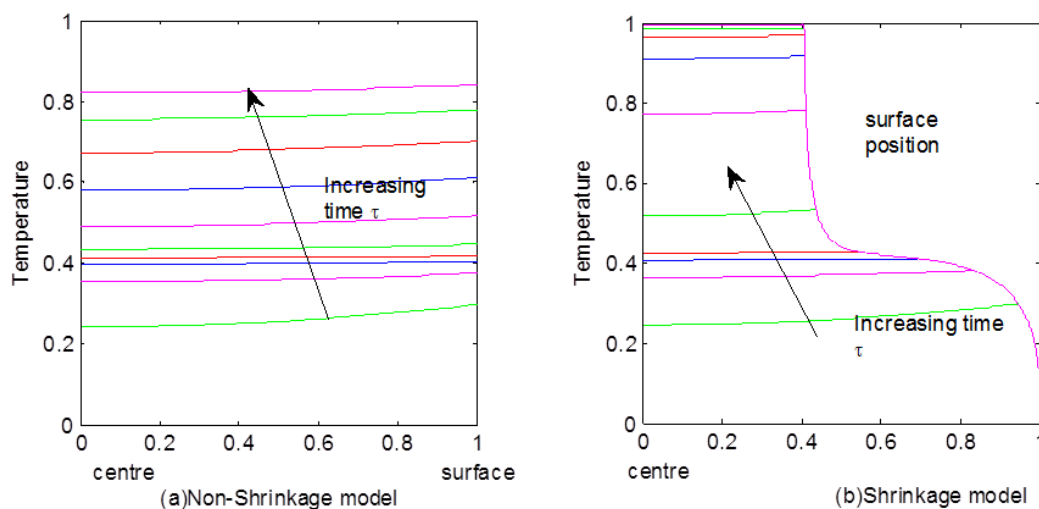


Figure 2: Temperature profile inside the fruit with increasing time $\tau=0-3$ instep 0.25 for (a)Non-shrinkage model (b)shrinkage model

Figure 3 represents the comparison of moisture and temperature gradients at the centre of the fruits, with and without the shrinkage effect. It can be seen that the temperature with shrinkage predicts a higher temperature at a specific time, compared to that with non-shrinkage. However, the predicted moisture content decreased faster when the shrinkage effect was included in the model. As a result of the thickness of the sample and decreases due to shrinkage, the moisture has less distance to cover and hence it reaches the surface faster, with the result that the time taken to dry a food becomes shorter. Therefore, the model with shrinkage needs a shorter time for drying. When shrinkage occurs, there are volumetric and dimensional changes to the product. This occurs because the tissues in the food are unable to hold their structure when the space taken by the water is continuously removed. This suggests that there are significant differences in predicted moisture and temperature gradients.

Figure 4 shows the profile of moisture and temperature at the centre of the fruit when different air temperatures are used during drying. The distribution of moisture and temperature at the centre is different at a specific time during drying and the detection of this region is a very important aspect in terms of food safety. Locating the area of potential improvement will improve product quality while reducing product development and prototype cost.

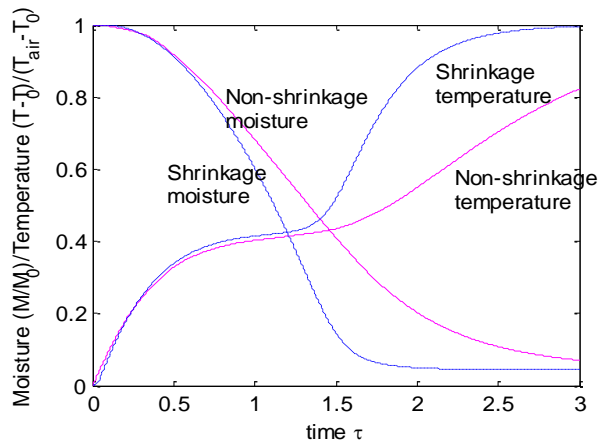


Figure 3: Comparison of moisture and temperature at the centre for shrinkage and non-shrinkage model, drying at 60°C and 20%RH

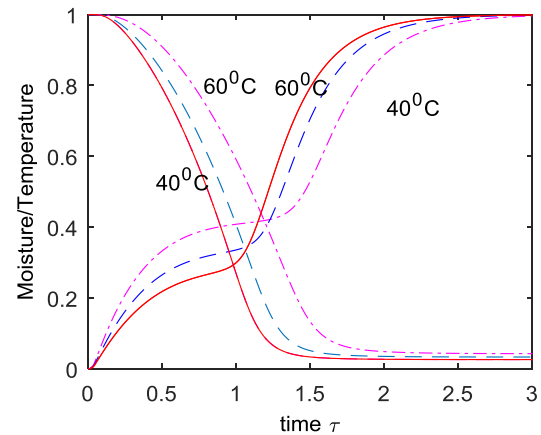


Figure 4: Comparison of moisture and temperature at the centre of the food with different air temperatures 40°C, 50°C and 60°C (for shrinkage model)

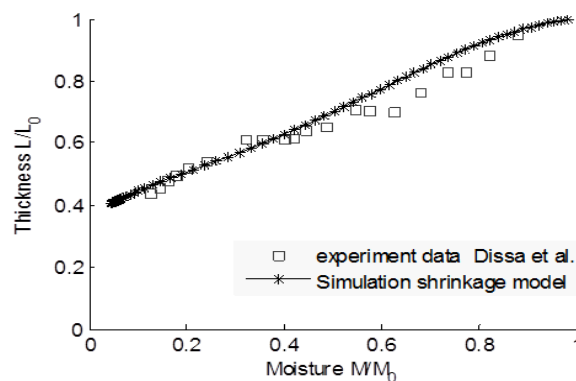


Figure 5: Comparison between experiment data [19] with simulation study of variation of shrinkage (V/V_0) with respect to the moisture content

Figure 5 shows the experiment measurement of shrinkage (V/V_0) taken from experiment drying of mango by Dissa et al. [19] at 60°C, which was compared with the simulation model of shrinkage. The thickness is assumed to decrease linearly with a reduction in moisture content. The trend deviates with higher moisture content but there is good agreement at lower moisture content: when moisture content is very low, the decrease in moisture is influenced by drying air and a very small decrease in thickness can be observed

5. Conclusion

This paper adopts a simple one dimensional model of heat and mass transfer that regards fruit as a homogenous structure, as discussed in [1, 6]. Using a numerical solution, the behavior of moisture and temperature subject to shrinkage was predicted for constant diffusivity. Shrinkage has been included in this study by including a velocity equation in the differential equation without adopting the separated procedure by means of regression analysis of the shrinkage data. Moisture content decreases more quickly in the shrinkage model, which concludes that the model with shrinkage effect needs a shorter time for drying, consistent with the experiment finding [20]. The result of the simulation study shows that the water in the food is removed, amounting to about 60% of the initial volume. This is consistent

with the experiment finding by [21] which shows mango shrank by 83% of the initial volume, 77% of the initial volume of banana drying [22] and 62% of dried onion at 60°C [23].

Acknowledgments

This work was supported by Ministry of Higher Education, Malaysia under Grant No 600-RMI/RAGS 5/3 (147/2014) and Universiti Teknologi MARA, Malaysia.

References

- [1] Shahari N and Hibberd S 2013 Analysis of Single Phase Moisture and Heat Model of Food Drying, *4th International Conference of Mathematical Models in Engineering and Computer Science*,(Brosov, Romania) 137
- [2] Wang N and Brennan J 1995 *Journal of Food Engineering* **24** 47
- [3] Villa-Corrales L, Flores-Prieto J, Xaman-Villasenor J and Garcia- Hernandez E 2010 *Journal of Food Engineering* **98** 198
- [4] Janjai S, Lamlert N, Intawee P, Mahayothee B, Haewsungcharern M, Bala B and Muller J 2008 *Biosystem Engineering* **99** 523
- [5] Barati E and Esfahani J A 2012 *International Journal of Thermal Sciences* **56** 86
- [6] Shahari N and Hibberd S 2012 Mathematical Modelling of Shrinkage Effect during Drying of Food, *2012 IEEE Colloquium on Humanities, Sciences & Engineering Research (CHUSER 2012)*,(Kota Kinabalu, Sabah Malaysia) 242
- [7] Mayor L and Sereno A 2004 *Journal of Food Engineering* **61** 373
- [8] Shahari N, Nursabrina M and Suhairah A Z 2015 The effects of drying on physical properties of bilimbi slices (*Averrhoa bilimbi* L.), *AIP Conference Proceedings*,(Penang, Malaysia, 6-8 Nov 2013) 070
- [9] Chemkhi S, Zagrouba F and Bellagi F 2005 *Brazilian Journal of Chemical Engineering* **22** 153
- [10] Hussain M M and Dincer I 2003 *Journal of Heat and Mass Transfer* **46** 4033
- [11] Aprajeeta J, Gopirajah R and Anandharamakrishnan C 2015 *Journal of Food Engineering* **144** 119
- [12] Ramallo L A and Mascheroni R H 2013 *Journal of Food Process Engineering* **36** 66
- [13] Katekawa M E and Silva M A 2006 *Drying Technology* **24** 5
- [14] Pavon-Melendez G, Hernandez J, Salgado M and Garcia M 2002 *Journal of Food Engineering* **51** 347
- [15] Dunn G J, Wilson S K, Duffy B R, David S and Sefiane K 2008 *Colloids and Surfaces A : Physicochemical and Engineering Aspects* **323** 50
- [16] Schiesser W E 1991 *The Numerical Method of Lines*. (San Diego: Academic Press)
- [17] Crank J 1984 *Free and moving boundary problem*. (New York: Clarendon-Oxford Science Publication)
- [18] Crapiste G H, Whitaker S and Rotstein E 1988 *Chemical Engineering Science* **43** 2919
- [19] Dissa O A, Desimorieux H, Bathiebo J and Koulidiati J 2008 *Journal of Food Engineering* **88** 429
- [20] Simal A, Femenia A, Garau M C and Rossello C 2005 *Journal of Food Engineering* **66** 323
- [21] Yan Z, Sousa-Gallagher M J and Oliveira F A R 2008 *Journal of Food Engineering* **84** 340
- [22] Hassini L, Azzouz S, Peczalski R and Belghith A 2007 *Journal of Food Engineering* **79** 47
- [23] Abasi S, Mousavi S M, Mohebi M and Kiani S 2009 *Iranian Journal Of Chemical Engineering* **6** 57