

Analysis of the Solar magnetograms evolution using a spectral gap of critical nets

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Abstract. Analyzing the dynamics of the photospheric magnetic field of the Sun is one of the most important problems in Solar Physics. Different estimates of the complexity of magnetograms of the Sun Active Regions (AR) are used to predict the time and the strength of the solar flares, but the quality of the forecasts are still insufficient. A magnetogram is a highly variable discrete image with a very large number of local extrema. We use an idea of extraction of stable critical points within a framework of the scale-space theory. Two sequential convolutions of the image with the same Gaussian kernel and calculating the difference between the produced images allow to get a stable estimation of the Laplacian of the image. A critical graph is constructed using maxima and minima of the Laplacian. Dynamics of critical graphs can be used for diagnostics of dynamical regimes of ARs. The so-called spectral gap is proposed to be used as a numerical descriptor. This is the difference between the two largest eigenvalues of the discrete Laplacian of the graph constructed on critical networks. We investigated several ARs and found that there was a sudden increase in the spectral gap values one or two days before the flares.

1. Introduction

An early forecast of solar eruptive events remains an important objective of extreme space weather studies. Most of the failures of spacecrafts are caused by impact of the solar wind particles in the near-Earth space. Possible physical mechanisms in the magnetic field of active regions are known, each of which, or a combination of which, in certain circumstances, can induce a large solar flare [1]. However, these conditions are still uncertain. There are a large number of the so-called solar flare precursors, which preceded or accompanied the solar flares in the past. But it is admitted that a specific precursor does not necessary precede a solar flare and the appearance of the precursor does not necessarily cause the solar flare. There are different machine learning systems, based on different precursors, for example [2, 3].

Traditional precursors are based on the estimation of complexity of the observed patterns in the magnetic field of active regions. As a measure of complexity some morphological characteristics are commonly used, namely: the area, the number of spots in the active region, the maximum gradient, the length and complexity of a neutral line, the number of singular points of the field, the current helicity and others (see references in [4]). As a rule, these



characteristics have been measured on the line-of-sight magnetograms of solar active regions, which includes only one component of the magnetic field. It is important to mention that vector magnetograms with three components of the magnetic fields are now becoming available [5].

Existing models of solar flares do not contradict the theoretical conception of the physical mechanisms leading to a flare, but they are not a direct consequence. Statistical properties of the observed sequence of events make the situation even more complicated. Reappearance of a flare at the same AR is described by the laws of the heavy-tailed Pareto type [6]. This means that the probability of occurrence of the large events is exponentially small. Therefore, searches of new approaches to describe different dynamic regimes in the dynamics of fields before the emergent events is an actual and still unsolved problem. The main idea of this project is to find a simple and stable descriptor of the magnetogram, which could be easily tracked during the time. In our previous works [4, 7] we computed different topological characteristics of the active regions, and we now continue to explore some of them. To get a descriptor let's use some interesting points, such as local extrema of solar magnetic fields, that do not change under transformations. We picked up the idea of constructing such points from [8]. The authors considered only such critical points that are stable under the consecutive smoothing.

In the process of the AR evolution a number and values of local extrema are changing. Therefore, the structure of a critical network is also changing. So, the dynamics of critical graphs can be used to diagnose dynamical regimes of ARs. As a numerical descriptor we propose to use the so-called spectral gap. It is the difference between the two largest eigenvalues of the discrete Laplacian of the graph constructed on critical networks. We tracked of changes in the spectral gap for three different active regions and found that there is a sharply increase of its values from 20 to 40 hours before the flare.

2. Method description

In our study, we have used a sequence of HMI magnetograms made with the Helioseismic and Magnetic Imager (HMI) on-board Solar Dynamics Observatory (SDO) (hmi.stanford.edu). A magnetogram of Solar magnetic fields is a matrix of data varying between -5000 and 5000 , but most of the values are in the range of -500 to 500 . Changes between the pixels not smooth, which makes the extraction of stable patterns from magnetograms very difficult.

Due to high variability of a magnetogram it is characterized by a very large number of local extrema. We use the idea of extraction of stable critical points within a framework of the scale-space theory, proposed in [8]. The authors use not all extrema of the image, because there are too many such extrema in the case of high variability of contrasts, but only the so-called beta-resistant features. The idea of the beta-stability is based on the properties of the Laplacian of the image calculated by the design, which is called scale-space. Scale-space was introduced in a number of papers [9, 10] and subsequently became widespread in image analysis. In the framework of this approach an image is represented as a set of one-parameter smooth images, depending on the size of the smoothing window used for the suppression of small-scale features. Smoothing is performed using a Gaussian kernel

$$g_t(x) = \frac{1}{\sqrt{2\pi t}} \exp(-x^2/2t), \quad I_t(x) \equiv f * g_t = \int I(y)g_t(y-x)dy, \quad (1)$$

where t is a scale parameter.

On the other hand, the scale-space description can be presented as a solution of the heat equation with the Neumann boundary conditions, where temperature plays the role of the width of the smoothing kernel. In addition, it can be shown that the Laplacian of the image is quite accurately approximated by the difference between the image smoothed t times with a Gaussian

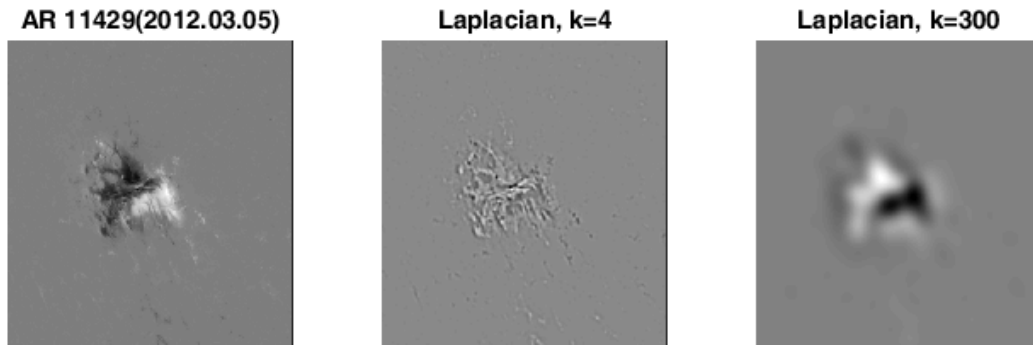


Figure 1. AR 11429 and two Laplacians with a different level of smoothing

kernel and the image smoothed $t + 1$ times with the same Gaussian kernel:

$$\delta_t L = \frac{1}{2} \nabla^2 L, \quad L(x, y, 0) = I(x, y) \quad \Rightarrow \quad L_t = \nabla^2 I_t(x) \approx I_{t+1} - I_t. \quad (2)$$

2.1. Beta-stable Laplacian

We can convolve the input image I with the Gaussian kernel G_σ a large number of times to give a scale space representation $\{I_k\}$:

$$I_k = \underbrace{G_\sigma * G_\sigma * \dots * G_\sigma}_k * I,$$

where σ is the width of the smoothing kernel and k is the index of the scale.

For each fixed scale the Laplacian of the image divides the whole image into convex areas of brightness (with the positive Laplacian) and concave areas of brightness (with the negative Laplacian). It turns out that when we zoom in, the number of such regions is stabilized. When the number of areas with positive and negative Laplacians does not vary, we say about its stability at this scale. A scale at which the number of regions, mentioned above, does not change any more is called a sustainable scale. Thus, we have to determine the stable singular points of the image. The maxima and minima of a sustainable Laplacian are the stable singular points of the image.

2.2. Nets construction

After the set of critical points is defined, we construct a graph in accordance with the following rules: the nodes of the graph are the critical points; a local minimum is connected to a local maximum, if there is an upward path between them, that is the pixels values must increase along this path. Example of two possible ascending path computed by simple matrix is presented at Fig. 2 This graph is called a critical network. The authors [8] showed that the critical net is invariant under affine transformations and various distortions of images, and the graph has a minimum number of self-intersections, that is, almost-planar.

2.3. The laplacian spectrum of graph

Full information about spectral graph theory could be found in the books [11, 12]. Recall, that $G(V, E)$ be a graph with a vertex set V of cardinality n and an edge set E of cardinality m . We will assume that it is undirected and finite. Let d_j be the degree of a vertex j . Let A be

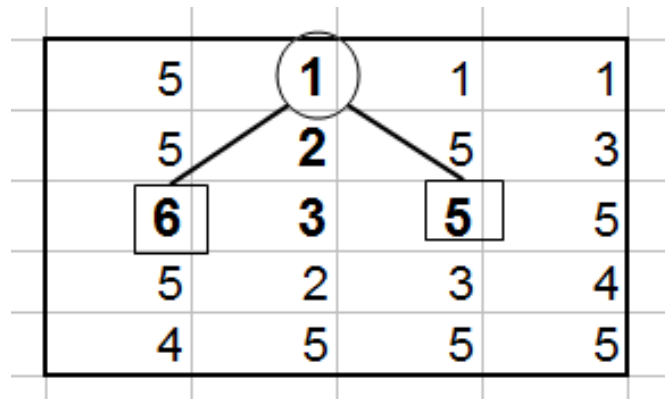


Figure 2. AR 11429 and two Laplacians with a different level of smoothing

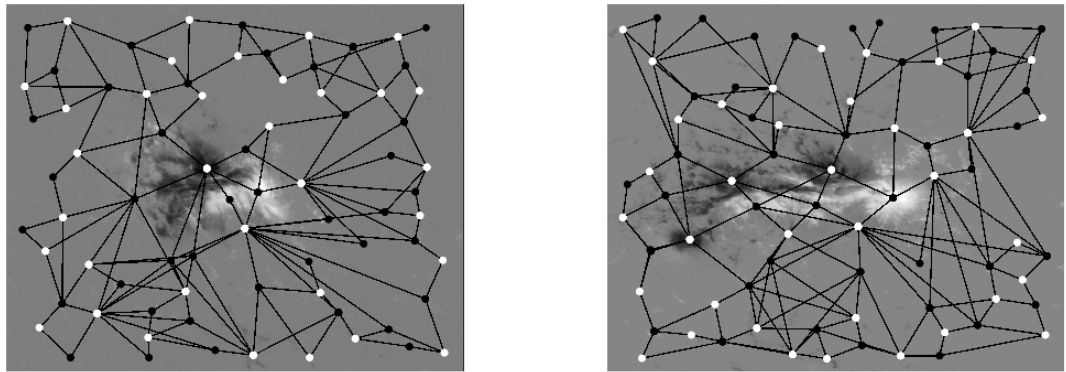


Figure 3. Critical nets for AR 11429 in two different time moments

the $n \times n$ $\{0, 1\}$ adjacency matrix, such that $A_{ij} = 1$ if and only if $ij \in E(G)$. Let D be the $n \times n$ diagonal matrix with $D_{jj} = d_j$. Then $L = D - A$ will be a combinatorial Laplacian matrix associated with the graph.

For this Laplacian matrix L of the graph (not to be confused with the Laplacian of the image!), the eigenvalues λ of the graph can be found by solving the equation

$$Lx = \lambda x.$$

As this matrix is symmetric along its diagonal, then, by definition, the eigenvalues are real. The set of eigenvalues of the Laplacian matrix is known as the spectrum of the graph, $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$. Any given Laplacian matrix has a unique set of eigenvalues. The spectrum of a graph is also known to be invariant, as the eigenvalues do not depend on the labeling of vertices. The first largest eigenvalue λ_1 of the graph is a kind of "average degree" of graph, the second λ_2 eigenvalue is a characteristic of algebraic connectivity of the graph, which depends from the number of vertices and the way of connection. The spectral gap Υ is defined as the difference between two largest eigenvalues of the spectrum,

$$\Upsilon = \lambda_1 - \lambda_2.$$

If the number of edges of the graph is close to the maximum, when every vertex is connected to every other vertex, and the graph is said to be dense. A graph in which the number of edges is close to the minimal number of edges is called a sparse graph. If we talk about evolution of complex network, increase in spectral gap means decreasing of algebraic connectivity, so network become more sparse, or growth of the critical points.

3. Results

Three Active Regions of the Sun have been examined, namely, AR 11158, AR 11429, and AR 11520. This choice was motivated by the criteria of complexity of magnetic fields of these active regions and the solar flare activity. The activity of the regions is measured as the amount of flares produced. The Solar flares are classified as *A*, *B*, *C*, *M* and *X* according to peak flux of *X*-rays near the earth measured on the GOES spacecraft. For prediction only the strongest *M* and *X* flares are usually considered, because they are the most dangerous for near-space systems. The location on the solar disk, where a flare released, is also important, as the most dangerous flares happen at the center of the solar disk. All three observed regions, listed above, produced *X* and *M*-Class flares at the center of the disk. We have already used these regions in our previous works, where dynamics of topological features have been investigated [4].

Each active region is represented by a set of fragments of the line-of-sight magnetogram having the size of 600×600 pixels. We took a time step between the fragments as 1 hour. An active region passes through the solar disk within 5–6 days. So, the whole set of images analyzed for each region consists about 130 fragments. For each fragment we computed the beta-stable Laplacian of the image, then constructed the critical nets using the beta-stable Laplacian, and, finally, estimated the spectral gap of the Laplacian matrix associated with the critical net.

As a result for each active region we received a time series describing the spectral gap evolution. Fig. 3-5 shows the evolution of the spectral gap for the active regions AR 11158, AR 11429, and AR 11520.

By the abscissa axis, time t in dates is shown, by the ordinate axis the dimensionless quantity of the spectral gap is shown. The flare strength is indicated by the stems with the point in all figures. We translated the class of a flare into the numerical quantity using the standard procedure: the *M*-flares and *X*-flares were multiplied by 10 and 100, correspondingly.

As we see, in all three cases there is the sharp rise in the values of the spectral gap withing 24 – 48 hours before the flare. From the physical point of view this means that either the degree of vertexes decreases or the number of vertexes increases, or both. In term of critical nets this means that the number of beta-stable extrema raises significantly, what corresponds to the increase in topological complexity. This result agrees with the hypothesis that before the solar flares a new emerging flux arises in the magnetic field and forms some new stable structures.

Let's consider the dynamics of each ARs.

AR 11429. The group appeared at the eastern limb on 3 March 2012 as a simple compact group, but next day it evolved to the large complex group (see Fig. 2). *X*5.4 flare released on 7 March at 00:24 UT, which was the second largest flare in magnitude since 2010. From March 9 the activity in the region began to decay slowly. This behaviour is accurately traced on a graph of the spectral gap in Fig. 3. We see the significant rise in the gap during the first three day, with sharp increase by the 25 hours before the *X*5.4 flare occurred, and another increase after the flare, which was followed by a slow decay.

AR 11520. The group 11520 appeared on the solar disk on 8 July 2012. Just after the appearance of the group, the magnetic class FHC has been assigned to it, which corresponds to a large, complex group with a high probability of flares. Strong activity and powerful flares were expected from this group, but only one *X*1.4 flare was on 12 July just at the center of disk. After that the activity started to decrease. Two *M*-Class flares released near the West limb of the Sun. There is no increase at the beginning of the graph in the dynamics of the spectral

gap (see Fig. 4). This corresponds to the situation that the group appeared was already very complex and developed, although there is a jump within two days before the flare, followed by a reduction in the spectral gap. It could be considered as a predictor of the $X1.2$ flare which released on 12 July. The similar increase was on 14 July, before the flare $M1.7$ on 17 July.

AR 11158. AR 11158 appeared on 12 February 2011 not from the eastern limb, but just near the center of the disk in the form of a compact group. After that it developed rapidly and a day later it reached the next class of complexity δ and the $M6.6$ flare was produced on 13 February. Two days later the $X2.2$ flare released and further activity of this region started to decrease. Again, on the plot of the spectral gap (Fig. 5), we see a rather big rise started two days before the X flare, and then the decrease in the spectral gap just before the flare, as well as several peaks at the second part of evolution graph coinciding with the weaker flares.

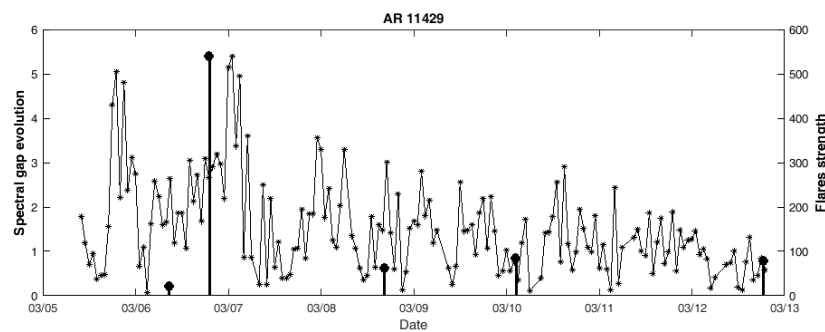


Figure 4. Evolution of the spectral gap for AR 11429.

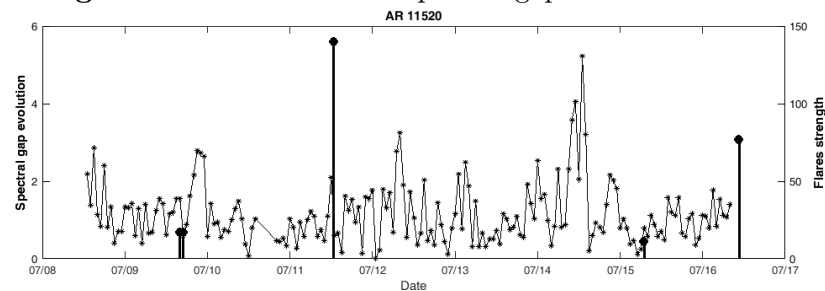


Figure 5. Evolution of the spectral gap for AR 11520.

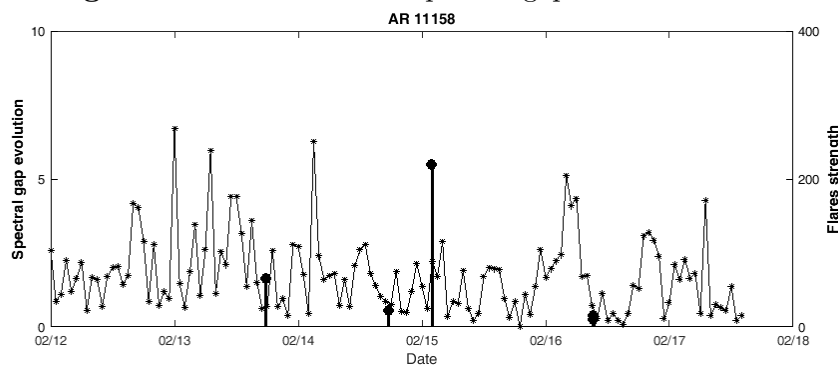


Figure 6. Evolution of the spectral gap for AR 11158.

4. Discussion

The aim of our research is developing effective descriptors of magnetic fields in the Solar active regions. The spectral gap can be considered as one of them. Combination of different descriptors might be used for predictive system construction. There are different machine learning systems by the present date, which have already been used for the solar magnetograms processing and the solar flare predictions. But there are an unlimited scope to make the predictions more accurate. The area of mathematics, connected with computational topology and complex network analysis, has developed significantly recently due to a huge number of practical problems and development of scientific computing. The powerful mathematical approaches for analyzing the distinguishing features of magnetograms can significantly improve the accuracy of the predictions of the solar flares. Our next step will be testing of some topological features. We developed this technique in our previous work for the features of critical nets in machine learning systems of solar flare prediction.

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References

- [1] Longcope D W 2005 *Living Rev. Solar Phys* **2**
- [2] Ahmed O W, Qahwaji R, Colak T, Higgins P A, Gallagher P T and Bloomfield D S 2013 *Solar Physics* **283** 157–175
- [3] Bobra M G and Couvidat S 2015 *The Astrophysical Journal* **798** 135
- [4] Knyazeva I and Makarenko N 2014 *Astronomy Reports* **58** 145–155
- [5] Scherrer P H, Schou J, Bush R, Kosovichev A, Bogart R, Hoeksema J, Liu Y, Duvall Jr T, Zhao J, Schrijver C *et al.* 2012 *Solar Physics* **275** 207–227
- [6] Boffetta G, Carbone V, Giuliani P, Veltri P and Vulpiani A 1999 *Physical review letters* **83** 4662
- [7] Knyazeva I, Makarenko N and Livshits M 2011 *Astronomy reports* **55** 463–471
- [8] Gu S, Zheng Y and Tomasi C 2010 *Computer Vision–ECCV 2010* (Springer) pp 663–676
- [9] Witkin A P 1984 *Acoustics, Speech, and Signal Processing, IEEE International Conference on ICASSP'84*, vol 9 (IEEE) pp 150–153
- [10] Lindeberg T 2013 *Scale-space theory in computer vision* vol 256 (Springer Science & Business Media)
- [11] Chung F R 1997 *Spectral graph theory* vol 92 (American Mathematical Soc.)
- [12] Newman M 2010 *Networks: an introduction* (OUP Oxford)