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Formalization of an environmental model using formal concept analysis - FCA

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Abstract. Nowadays, there is a huge necessity to generate novel strategies for social-ecological systems analyses for resolving global sustainability problems. This paper has as main purpose the application of the formal concept analysis to formalize the theory of Augusto Ángel Maya, who without a doubt, was one of the most important environmental philosophers in South America; Ángel Maya proposed and established that Ecosystem-Culture relations, instead Human-Nature ones, are determinants in our understanding and management of natural resources. Based on this, a concept lattice, formal concepts, subconcept-superconcept relations, partially ordered sets, supremum and infimum of the lattice and implications between attributes (Duquenne-Guigues base), were determined for the ecosystem-culture relations.

1. Introduction

This paper has as its foundation the model of environmental interpretation of Augusto Ángel Maya, a very important philosophical conception for the understanding of the current environmental problems and possible solutions. Augusto Ángel Maya's model presents two dimensions related with environmental problems, the cultural system and the ecosystem, as follows:

First of all, the two orders are recognized as independent. The environmental problem consists of both, the ecosystem and the cultural dimensions as they have their own laws of operation. If mankind had to adapt to play a role in the ecosystem, there would not be any environmental problems... The environmental problem arises since the human species does not occupy any niche within the ecosystem. The fact that the human species has achieved a certain independence ... is the result of the evolutionary process... The relationship between ecosystem and culture does not happen only through technology, but it also involves the way men relate with one another [1].

The ecosystem [1] is one of the main dimensions of Ángel's model and it consists of the functional components such as energy flow, trophic levels, biogeochemical cycles, ecological niche, ecological balance and resilience and areas of life on earth (biomes) that explain multiple forms of the existing ecosystem and the succession in it. In addition, besides the ecosystem, there is also a cultural dimension or a cultural system [1], where its components are, the cosmovision of nature or symbolic world, population, technological paradigm and ultimately, both economic


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and political social relations, as productive and reproductive, established between different populations and within themselves.

Augusto Ángel identifies at least three phases of relations between the ecosystem and the cultural system by appealing to their theoretical differences, which in practice are closely linked. A first **A** relation is recognized which goes from the ecosystem to cultural systems. Culture has to adjust to the external environment and it is known that a culture of arid land is different from one built in the middle of the rainforest. The **B** relation goes from the cultural system to ecosystems. It is called the ratio of impact. Every culture, in its formation process, transforms the ecosystem. The third relation, or ratio **C**, is given the name nemesis. A culture beyond the limits of the external environment must change or disappear due to the pressure of its impacts. Therefore, relation **C** is the nature's answer against non-adaptive cultures. Ángel aims to synthesize the patterns of Ecosystem-Culture relations more than to exhaustively describe the environmental components of the two dimensions. Our goals were to update and to formalize the Augusto Ángel Maya's environmental model. In figure 1 there is a representation of Augusto Ángel Maya's Model (AAM's Model).

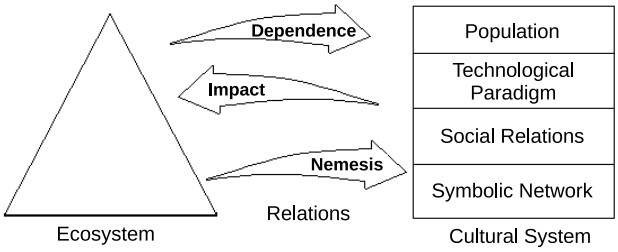


Figure 1. Augusto Ángel Maya's Model of Ecosystem-Culture relations. Based on the image included in [1]. It doesn't any have restriction of use.

2. Theory and Calculations

2.1. Augusto Ángel Maya's updated model

The ecosystem, culture and its components were updated by us, while retaining part of the definitions and connotations already submitted by Ángel. From the general systems theory and some principles of complexity, we classified the ecosystem and cultural system as evolutionary complex systems. The AAM's model was updated, refined and expanded in its environmental dimensions and timescale, to cover the entire biosphere as much as possible (life systems of planet earth), so that it could be formalized as a model of operating networks. In the Ecosystem, the biogeochemical cycles were separated into two parts: circulation through the ecosystem or biochemical circulation (nutrient cycling) and geochemical circulation (environment's fluids, natural resources and mineral deposits); this is due to the fact that biochemical and geochemical circulations have different circulation times through a biogeochemical cycle or full replacement (e.g. nitrogen has a replacement time in the atmosphere and sediments of a scale of 10^7 years, which is very different from the replacement time on the soil which is 2000 years old [2]) and its main reservoir in the world (for example, Nitrogen has 4×10^{21} g in its main reserve in the atmosphere, Sulfur has 2×10^{22} g in its main reserve in the lithosphere and Carbon has $7,7 \times 10^{19}$ g in sediments and rocks [2]) is found outside the ecosystem.

The components of the updated ecosystem with [2, 3, 4, 5, 6, 7, 8] are: Energy flow, food webs, nutrient cycles (biochemical circulation), fundamental and realized niche, morphostasis and morphogenesis; the environment's fluids constituted by the atmosphere and water bodies (geochemical circulation); natural resources and mineral deposits, both organic and inorganic (geochemical circulation); and the living areas or biomes and aquatic ecosystems. On the other hand, cultural system updated components are: The community formed by human populations and domesticated species, as well as animals and plants, their needs and their satisfaction [9]; the

symbolic network or human symbols represented on the art, the letters and oral tradition; the technological paradigm composed of science, technology and technical, traditional and ancestral knowledge; social networks are produced by both, transverse and vertical relations [10]; and the sociocultural spatiality or culture's development space [11, 12].

2.2. Formal concept analysis and representation

The study of relations, between environmental dimensions (ecosystem and culture) as a network, is addressed from the formal concept analysis (FCA), since its versatility doesn't need a minimal number of concepts (nodes), nor does it lose information. It is even possible to recover additional information [13]. The FCA theory was developed by Wille in 1982 and the central notion is the *formal context* [14], which is defined as a structure set $\mathbb{K} := (A, B, I)$ where A is a set of *objects*, B is a set of *atttibutes* and I is a binary relation between A y B, i.e. $I \subseteq A \times B$; aIb or $(a, b) \in I$ indicates that object a has attribute b. The *formal context* can be represented by a cross table where rows are headed by the *objects*' names and its columns headed by the *attibutes*' names. A cross (X) in row a and column b means that, in the formal context, it holds aIb.

The following derivation operators are defined for arbitrary X $\subseteq$ A y Y $\subseteq$ B:

$$X \to X^I := \{b \in B \mid aIb \text{ for all } a \in X\}$$
$$Y \to Y^I := \{a \in A \mid aIb \text{ for all } b \in Y\}$$

A *formal concept* of the *formal context* (A, B, I) is defined as a pair (A_1, B_1) with $A_1 \subseteq A$, $B_1 \subseteq B$, $A = B^I$ y $B = A^I$, respectively. Set A_1 is called the *extent* and set B_1 is called the *intent* of the formal concept (A_1, B_1).

Once the components are determined to formalize the updated model, set A of objects and set B of attributes are defined starting from the components of the ecosystem and the cultural system presented above. For instance, the biogeochemical cycles were divided into nutrient cycles, in the ecosystem, and out of the ecosystem by the environment's fluids, natural resources and mineral deposits. Each one is presented in [15]. By using the properties of the substances' cycles in the ecosystem (nutrient cycles), the concepts and attributes of table 1 were defined.

Table 1. Nutrient cycles object and its attributes.

OBJECT	**ABBREVIATION** (OBJECT, attribute)	**MEANING**
Nutrient cycles	CIRCBIO	
	recicsust	Recycling of the elements through the food chain under different escales
	medivida	Processing cycles chemicals mediated a vital process
	sustavida	Combination of elements that make up all living organisms
	procevida	Chemical processes that form all living organisms
	pericorto	With short periods and means of circulation and recirculation of substances (hours, days, months, years, decades or hundreds of years)

For example, attribute *recicsust* belongs to CIRCBIO which belongs to the ECOSISTEM. Therefore, attribute *recicsust* belongs to these objects for inclusion and it was indicated with a cross (X) on the formal context [15] for the intersection between attribute *recicsust* and

CIRCBIO and ECOSISTEM objects. For others attributes, the same procedure was done. By using all A objects (36) and B attributes (176), the formal context of the environment (AAM's updated model) was obtained [15].

3. Results and Discussion

Starting from the formal context, the conceptual lattice was built and for this, the ConExp software [16] was used. An excerpt of the conceptual lattice (PART II) is presented in figure 2 where nodes represent the environment's formal concepts (A_i, B_i).

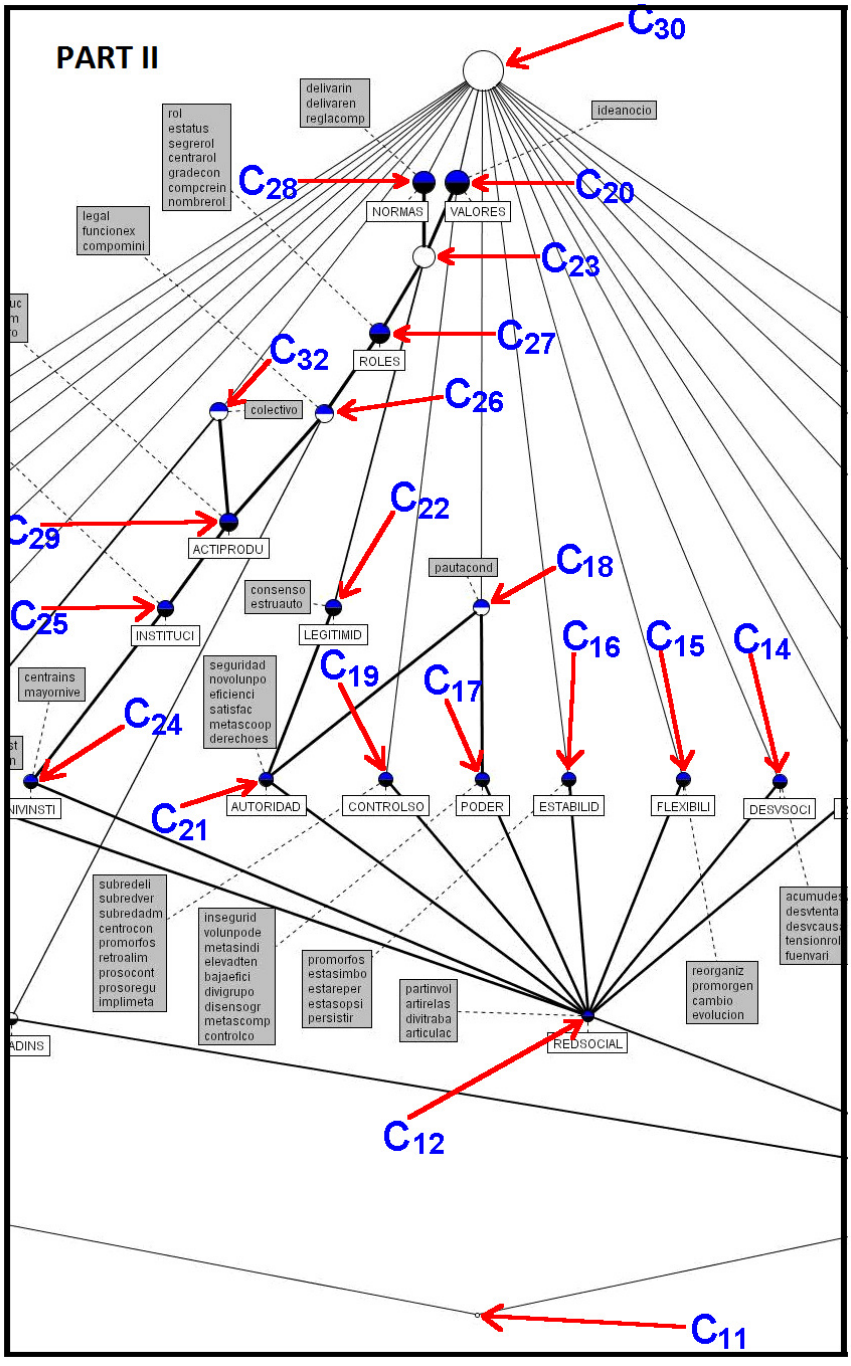


Figure 2. Sociocultural Compartment (PART II) of the Conceptual Lattice of the Environment.

For the lattice, the objects can be identified due to the white boxes below each node and the attributes due to the gray boxes associated with each other. In the whole conceptual lattice of environment [15], one has separated its components (Ecosystem and Cultural System), concepts, objects, attributes and relations, but both components connected by common concepts or attributes. However, the Formal Concept Analysis allows to separate the environmental components and to review each one independently.

Now to get a *formal concept*, from the lattice, *...each concept [formal] is represented by a little circle [in the lattice] so that its extension (intension) consists of all the objects (attributes) whose names can be reached by a descending (ascending) path from the circle* [14].

The total number of concepts found was 44 (C_0 - C_{43}) and for relations, it was 70, but the formal context has over 200 binary relations [15], in fact, the lattice representation condenses the relations because it avoids redundancy on the binary relations. The height of the lattice is 10 because there are 10 different hierarchical levels of concepts or a maximum number of subconcept-superconcept relations in a partially ordered set of the lattice. The hierarchy of concepts tells us about the connections between concepts and its levels or importance. 30 partially ordered sets of the environment were obtained, the one with the lowest number of concepts was $C_{11} < C_{10} < C_8 < C_{30}$ (or $C_{11} < C_{10} < C_9 < C_{30}$) and the one with the largest number of concepts was $C_{11} < C_{10} < C_{12} < C_{24} < C_{25} < C_{29} < C_{26} < C_{27} < C_{23} < C_{20} < C_{30}$ (or $C_{11} < C_{10} < C_{12} < C_{24} < C_{25} < C_{29} < C_{26} < C_{27} < C_{23} < C_{28} < C_{30}$).

In the conceptual lattice of the environment, the infimum of the lattice (Inf**MA**) is concept C_{11} and it contains the empty set in its extension and all attributes of the context in its intension, it is, of course, the smallest element of the lattice and it corresponds to the formal concept of the environment, because it is the only one that contains all the attributes of context. In addition, the lattice has supreme (Sup**MA**), which is the C_{30} concept and contains the set of all objects in the environment in its extension and the empty set in its intension. It is, of course, the largest element of the lattice as there is no common attribute in the context. The only element that shares intention in the lattice is the one with the empty set.

Finally, the implications between attributes or Duquenne-Guigues base were determined. An implication between attributes $P \Rightarrow C$ for the (A, B, I) context, consists of two P and C subsets of B attributes set of (A, B, I) context, where the P set is called the premise and C is called the conclusion of that involvement. The $P \Rightarrow C$ implication for the (A, B, I) context is valid (kept in) as long as what's next is true:

For every **a** by A: If all attributes of the P premise apply to the object, then all attributes of the C conclusion also apply to **a**.

In total, 297 implications between attributes for the context of the environment were found. The implications have the following format:

< Number of objects > Premise ⇒ Conclusion ; # it simply refers to the number of involvement in the list; the Number of objects means, how many objects of the context, the implication is valid for; Premise and Conclusion are usually lists of names of attributes (or intentions) that take place in the premise and in the conclusion as well.

It was necessary to reorganize the implications that apply to the same attributes and to determine the general condition that may be necessary, necessary and sufficient, etc. The reorganization of the implications between attributes is exemplified by implications 34 - 38 from the Duquenne-Guigues base of the environmental formal context [15].

34 < 2 > lenguaes ⇒ tradioral tradiarti prerrohum cosmovisi; 35 < 2 > tradioral ⇒ lenguaes tradiarti prerrohum cosmovisi; 36 < 2 > tradiarti ⇒ lenguaes tradioral prerrohum cosmovisi; 37 < 2 > prerrohum ⇒ lenguaes tradioral tradiarti cosmovisi; 38 < 2 > cosmovisi ⇒ lenguaes tradioral tradiarti prerrohum. 34- 38 implications are accomplished for the following objects: CULTURA (cultural system) and SIMBOLOS (symbolic network). All these attributes are located in the premise and conclusion. This means that every attribute is a necessary and

sufficient condition for the rest of the attributes and this is a necessary and sufficient condition for all attributes. The two attributes can be interpreted as follows:

lenguaes $\Rightarrow$ tradioral requires, tradioral $\Rightarrow$ lenguaes; then, lenguaes $\Leftrightarrow$ tradioral

This necessary and sufficient condition is met for all attributes of symbols or symbolic network and can be interpreted as follows: The complex structures of written language (lenguaes), the oral (tradioral) and artistic traditions (tradiarti) must be submitted simultaneously, since they are a prerogative of human species (prerrohum) and they reflect their cosmovision of nature (cosmovisi).

Next, the most important implications between attributes for the conceptual lattice of the environment are discussed. In [15] there are all implications between attributes.

63 - 68 implications are valid for the following objects: CULTURA, REDSOCIAL (social networks), NIVINSTI (institutional network) and INSTITUCI (institutions). They are interpreted as follows: on the institutional network, the existence of productive activities determines the existence of organized groups of people in institutional units with specific functions, with purposes, goals and specific mechanisms for developing social function and also that they present explicit rules. *This means that without productive activities, civilizations can not exist.* Before we can even consider colonizing another planet (like Mars), we need to establish productive activities.

106 - 111 implications are valid for the following objects: CULTURA, REDSOCIAL and AUTORIDAD (authority). These conditions are interpreted as follows: if the character pattern of social behavior and the legitimacy of authority take place, the imposed duty ocurrs either within a social dimension, cooperative and collective goals, high satisfaction in people and social security or high group efficiency and it is possible for power not to be manifested in people.

120 - 122 implications are valid for the following objects: CULTURA, REDSOCIAL, NIVINSTI (institutional level), ACTIPRODU (productive activities), INSTITUCI (institutional networking) and UNIDADINS (institutional unit). The interpretation is that if there are social roles, there will be a minimum institutional component with explicit and legalized functions. It can be synthesized that in the sociocultural system, performances of roles are critical to shaping the legal minimum institutional units.

206 - 209 implications apply to the following objects: CULTURA and REDSOCIAL which are interpreted as follows: If there are components of social networks, vertical and transverse articulation of human beings through social relations, there is social division of labor and participation of individuals too, starting from the formal context of the environment used.

4. Conclusions

To make Ángel's environmental interpretation model quantitatively operative, it was necessary to include more realism, that is, to secure the greatest possible degree of correspondence between the environmental concepts of the model and mathematical statements. This was the primary task undertaken in this paper by means of formal concept analysis and network approach that culminated in the design of a formal network model, which enabled clarification in the relations or implications between components of environment.

It was not intended to exhaustively cover all aspects of the environment in terms of interactions, but to present a path of work for the elucidation of relations and interactions. The current environmental researchers (and their trends), despite the criticism, do not present proposals to approach interactions, but sometimes admit that the focus of interactions is a better proposal with real processes.

With the use of FCA, it was possible to get novel hypotheses and relations about the environment from its formal context, in order to be verified with the real world.

The FCA allowed clear out the connections or relations between the environmental components in a graphic sense with the conceptual lattice and it condensed the binary relations of the formal context. Therefore, the FCA is recommended to do the first formal representation of any phenomenon because it allows to clear out the phenomenon in objects, attributes and concepts related.

The constraints and relations between attributes, objects and concepts were determined for 191 implications. These 191 implications or association rules were interpreted into text, so that they could become accurate. Full results can be found at Bourdon's dissertation [15].

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